Quarterly Accounting Earnings Forecasting: A Grey Group Model Approach

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Abstract: - Forecasting quarterly accounting earnings is an important task in prospective analysis and seasonality is an important phenomenon in those data’s behavior. In this paper, we investigate the efficiency of applying grey group model to forecast the earning per share. Unlike traditional statistical model such as Foster Model, grey group model inheres not only easy to calculate but also few observations needed than statistics methods when model building. Furthermore, the model liberates the users from the sample data assumptions from statistics analysis such as identically independent distribution. A data sample with fifty firms trading on Taiwan Stock Exchange is employed here and the forecasting performances are compared with those obtained by Foster Model. The results demonstrate that the grey group model is a competitive and competent one in prospective analysis.

Key-Words: - Quarterly Accounting Earnings, Forecasting, Grey Group Model.

1 Introduction
Forecasting sales, earnings, and earnings per share are important tasks in prospective analysis. Quarterly accounting earning forecasts are of significant interest to investors, managers, financial analysts, and capital markets researchers [3]. However, for those who are unfamiliar with the behavior of earning data to understand how it tends to evolve through time, some statistical methods, for example, Box and Jenkins ARIMA models [2], Foster models [8] and Griffin models [9], are usually employed for the empirical study of forecasting quarterly earning per share. However, the classical technique for predicting a continuous target variable is the statistical ordinary least squares (OLS) regression model. The statistical regression model is assumption-full such as the target variable is normally distributed. When the latter is not met, the predictions are questionable.

With rapid development in the study of forecasting, several comparative methodologies have been evolved in the past decades. One special technique, grey model (denoted as GM) [5, 6], has been applied successfully in forecasting the behavior of a system or a series [4, 10, 11, 12, 13, 15]. The series is considered as “partly known”, “partly unknown”, “partly certain” and “partly uncertain in grey model. There are several advantages employing GM model to time series, such as easy to calculate and few observations needed than statistics methods when model building [5].

Notice that seasonal is an important phenomenon in those quarterly earning data’s behavior. Although [7] and proposed a hybrid method that combines the GM(1,1) model and the ratio-average deseasonalization method to forecast seasonal time series, no application of the grey forecasting model was reported on forecasting quarterly earning data. Thus, the purpose of this research is to investigate the efficiency and ability of applying GM(1,1) group model to forecast the earning per share. Furthermore, the empirical evidence to date indicates that the Foster or Griffin models dominate the forecasting abilities of other general Box-Jenkins models [1]. Therefore, the study compares the relative forecasting ability of the parsimonious Foster model with the forecasting ability of hybrid grey model in quarterly accounting earnings for a sample of firms trading on Taiwan Stock Exchange (TWSE).

The organization of this paper is as follow. In section 2, we make a brief describe of data, the linear models and grey group model. Section 3 compares the forecast accuracy of grey model to that of the Foster model. In section 4, some concluding remarks are made.
2 Quarterly earnings forecast models

The model used in this study consists of three stages: data preparation, Grey Group model and Foster model established, and validation. Data preparation includes task such as data cleaning, completeness and rearranging. Then the second stage of models construction can be conducted by utilized those observations. The validation step confirms with validations data set.

2.1 Data

Quarterly earnings per share data were collected from the value line data base for firms actively trading on the Taiwan Stock Exchange (TWSE). There are two criteria to choose these firms. First, the firms must be listed in the TWSE Taiwan Index Series Compiled by FTSE group, such as TSEC Taiwan 50 Index, TSEC Taiwan Mid-Cap 100 Index, TSEC Taiwan Technology Index and TSEC Taiwan Dividend Index. FTSE is a world-leader in the creation and management of over 120,000 equity, bond and alternative asset class indices, thus the firms listed in these compiled indexes are of clear rules that are publicly available and which govern index management in all foreseeable eventualities. The second criterion dictated that each firms have a sufficiently lengthy time series of quarterly earnings per share data for estimation and valuation purposes. Furthermore, the data had to include the entire period from the first quarter of 1996 until the fourth quarter of 2007. According to the criteria described above, 50 firms listed in TWSE were selected with a total of 48 observations for each firm. In this research, all data are collected from the database of CMoney Investment Decision Support System.

2.2 The Foster model

In this subsection, we first describe the Foster model, which is usually employed for the empirical study of forecasting quarterly earning per share.

Let \( Q_t \) to denote earnings per share (or EPS) for quarter \( t \), then the Foster models for EPS forecasting take the following form:

\[
(\dot{Q}_t - \dot{Q}_{t-4}) = \alpha + \beta Q_{t-4} + \varepsilon_t, \quad (1)
\]

where \( \alpha \) and \( \beta \) are parameters to be estimated and \( \varepsilon_t \) is random noise on the quarter \( t \).

Note that equation (1) is a simple regression model with the dependent variable \((\dot{Q}_t - \dot{Q}_{t-4})\) and the independent variable \((Q_{t-1} - Q_{t-5})\). Thus, to estimate those parameters, prior earnings data must first be expressed in term of year-to-year changes. Then using the processed data, the coefficients can be estimated by a least square approach or standard maximum likelihood approach.

Research has produced models that forecast earning per share based only on prior quarters’ observations. Although such linear regression models are not used by many analysts, who have access to much more information than such simple models contain. However, the models are useful for helping those unfamiliar with the behavior of earnings data to understand how it tends to evolve through time [14].

In this study, the sample data described in subsection 2.1 will be used to compare the forecasts obtained from grey group model.

2.3 Grey group model

Given the series \( y^{(0)} = \{x^{(0)}(k), k = 1,2,\ldots,N\} \).

A GM(1,1) is the solution of the pseudo difference equation formed as

\[
\frac{dy^{(1)}}{dt} + az^{(1)} = b \quad (2)
\]

where

\[
z^{(1)}(i) = \frac{1}{2}(y^{(1)}(i) + y^{(1)}(i-1)) \quad \text{for} \quad 2 \leq i \leq n,
\]

and \( y^{(1)}(i) \) is accumulated generating sequence by using the accumulated generating operation (AGO). The parameters \( a \) and \( b \) in (2.1) can be obtained by least square method as

\[
\begin{bmatrix}
    a \\
    b \\
\end{bmatrix} = (B^T B)^{-1} B^T y
\]

(3)

and

\[
\begin{bmatrix}
    y^{(0)}(2) \\
    y^{(0)}(3) \\
    \vdots \\
    y^{(0)}(N) \\
\end{bmatrix}
\]

Then \( \hat{x}^{(0)}(k) \) is obtained by

\[
\hat{x}^{(0)}(k) = (x^{(0)}(1) - \frac{b}{a})e^{-(k-1)} + \frac{b}{a}, \quad k = 2,3,\ldots,N
\]

(4)

and the estimate of \( x^{(0)}(k) \), denoted by \( \hat{x}^{(0)}(k) \), is

\[
\hat{x}^{(0)}(k) = \hat{x}^{(0)}(k) - \hat{x}^{(0)}(k-1), \quad k = 2,3,\ldots,N
\]

(5)

where
\[ x^{(0)}(1) = x^{(1)}(1) = x^{(0)}(1) \]

Assume \( y^{(0)} \) is seasonal with period \( p \). To analyze the series, it is helpful to arrange the series in a two-dimensional table according to the period and season. If \( N = n \times p \), then a GM(1,1) group model for modeling \( y^{(0)} \) includes the following steps:

1. Arrange \( y^{(0)} \) into a two-dimensional table according to its seasonal period \( p \) as shown in Table 1.

2. Define the sub-sequence \( s_{i,ss} \) as follow:
   \[
   s_{i,ss} = \{ x^{(0)}(k), k = (m-1) \cdot p + i, \quad m=1,2,\ldots,n-1 \} , \quad 1 \leq i \leq p .
   \]  

3. Re-index subsequence \( s_{i,ss} \) as
   \[
   s_{i,ss} = \{ x^{(0)}(k), x^{(0)}(p+k), x^{(0)}(2p+k), \ldots, x^{(0)}((n-1)p+k) \}
   \]
   \[
   = \{ x^{(0)}_{s_{i,ss}}(1), x^{(0)}_{s_{i,ss}}(2), x^{(0)}_{s_{i,ss}}(3), \ldots, x^{(0)}_{s_{i,ss}}(n) \}
   \]
   , where \( 1 \leq i \leq p . \)

4. Utilizing \( n \) observations in each subsequence \( s_{i,ss} \) to construct GM(1,1). Then \( p \) GM(1,1)s are constructed and named as GM(1,1) group model. The forecasts of \( x^{(0)}(t) \), \( N < t \leq N + p \) thus can be obtained by the following formula:
   \[
   \hat{x}^{(i)}_{s_{i,ss}}(t) = \left\{ \begin{array}{ll}
   \hat{x}_{s_{i,ss}}^{(i)}(t) - \hat{x}_{s_{i,ss}}^{(i)}(t-1), & \text{if } \text{REM}(\frac{t}{p}) = 1 \\
   \hat{x}_{s_{i,ss}}^{(i)}(t) - \hat{x}_{s_{i,ss}}^{(i)}(t-1), & \text{if } \text{REM}(\frac{t}{p}) = 2 \\
   \vdots & \\
   \hat{x}_{s_{i,ss}}^{(i)}(t) - \hat{x}_{s_{i,ss}}^{(i)}(t-1), & \text{if } \text{REM}(\frac{t}{p}) = 0
   \end{array} \right.
   \]
   , where
   \[
   \hat{x}_{s_{i,ss}}^{(i)}(t) = (x^{(0)}_{s_{i,ss}}(1) - \frac{b_{s_{i,ss}}}{a_{s_{i,ss}}})e^{-a_{s_{i,ss}}(t-1)} + \frac{b_{s_{i,ss}}}{a_{s_{i,ss}}}.
   \]  

3 Comparing quarterly earnings forecasts

In order to compare the forecasting abilities and forecasting performance, each of the linear time series Foster model and the grey group model are established by using the in-sample data from the first quarter of 1996 until the fourth quarter of 2006. The in-sample data are used to estimate the parameters of Foster model and grey group model. After the parameters are obtained, the series from the first quarter of 2007 until the fourth quarter of 2007 are reserved as a post-sample, which is used to evaluate and compare the one step ahead (one year) forecasting abilities of both models at each company.

And the following statistics are used to compare the forecasting performance:

1. Mean square error
   \[
   MSE = \frac{1}{n} \sum_{i=1}^{n} (x^{(0)}(i) - \hat{x}^{(0)}(i))^2
   \]  

2. Mean absolute error
   \[
   MAE = \frac{1}{n} \sum_{i=1}^{n} \left| x^{(0)}(i) - \hat{x}^{(0)}(i) \right|
   \]  

To show the availability of grey group model, the in-sample series from the first quarter of 1997 to the fourth quarter of 2006 and those predicted with grey group model and traditional Foster model are compared in the left hand side of Table 2. Note that the in-sample series are used to parameters estimated.
and the in-sample MSE and MAE indicate the performance of model fitting. The MSE and MAE of one-year-ahead forecasts are also showed on the right hand side of Table 2.

Table 2 shows the average in-sample square error and absolute error of Foster model are 0.3455 and 0.3313 respectively. On the other hand, the results predicted by grey group model have the in-sample mean square error 0.1629 and mean absolute error 0.2108, which are significantly smaller than those obtained by Foster model. The results clearly show that the linear Foster model does not fit the series well.

For the post-sample forecasts, the Mean square error and the Mean absolute error of Foster model are smaller than those of grey group model. However, the post-sample errors differences between grey group model and Foster model are not significant and could be roughly accepted by the users who are unfamiliar in quarterly earnings forecasts. Note that the observations we used to establish grey group model and Foster model are 44 (11 years) respectively for each firms.

As we know, there are several advantages employing grey model to time series forecasting, such as easy to calculate and few observations needed than statistics methods when model building. Four observations are the smallest requirement for GM(1,1) model construction\[5\]. Too many observations way increase the model building cost. Thus the chosen of a proper amount of observations while constructing the grey model is an important issue.

To investigate the proper amount of observations and the forecasts performance while grey model building, we discard the longest four observations (one year observations) step by step to establish the grey group models. The mean square error and mean absolute error of the forecasting performances are calculated according to the different data length. Figure 2 and Figure 3 show the in-sample and post-sample mean square error and mean absolute error with different data lengths respectively. In Figure 1, one may find that the mean square error and mean absolute error look like reversed V. We may conjecture that the improper amount of data lengths in grey group quarterly earnings forecasting models construction lie in between 6 to 8 years. On the other hand, Figure 2 indicates that the post-sample mean square error and mean absolute error are lowest with 9 years data lengths. In this research, we may conclude that the proper amount of data lengths in grey group quarterly earnings forecasting model are nine years. Table 2, Figure 1 and Figure 2 demonstrate that the grey group model is a compatible one in prospective analysis.

| Table2 Forecasting Comparisons of Grey Group Model and Foster Model |
|-----------------|----------------------|-----------------|-----------------|
|                 | In-sample            | Post-sample      |
| Grey group Model| Foster Model         | Grey group Model| Foster Model |
| MSE             | 0.1629               | 0.3455          | 0.2296          |
| MAE             | 0.2108               | 0.3313          | 0.3324          |

Fig. 1 Grey group model in-sample MSE (solid line), MAE (dashed line) vs. Different data length
4 Conclusion
Forecasting represents the first step of prospective analysis, and serves to summarize the forward-looking view that emanates from business strategy analysis, accounting analysis, and financial analysis. In this study, we investigate the efficiency of GM(1,1) group model on forecasting the earnings-per-share. Empirical study indicates improper amount of observations is not helpful to reduce the forecasting errors but increasing the cost of model construction. Furthermore, the in-sample and post-sample forecast performances also show the grey group model with the proper amount of observations is a competitive and competent method for the non-stationary seasonal time series analysis, forecast and control. Not only do the advantages of the grey group model inhere the ability of forecasting but also the characteristics of easy calculating and few observations necessary. The grey group model has also liberated the users from the procedures of the model-based selection and the sample data assumptions in statistics analysis. Analyzing the seasonal time series, we can conclude that the grey group model suggests a competitive method for the system analysis, forecast and control. Despite of the efficiency of the grey group model, there remain some problems to be solved. For example: (i) What are the appropriate mean factors in grey group model? (ii) How many observations are appropriate for model building in different application cases?

However, in order to get an appropriate seasonality for quarterly accounting earnings time series, we hope the grey group model will be a worthwhile approach and will stimulate more future empirical work in time series analysis.

References:


