Fractional Gaussian Noise and Network Traffic Modeling

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Abstract: - Fractional Gaussian noise (fGn) is a commonly used model of network traffic with long-range dependence (LRD). This paper revisits the basic results of fGn towards noticing its limitation in traffic modeling.

Key-Words: - Fractional Gaussian noise; Long-range dependence; Network traffic.

1 Introduction

Fractal time series model of traffic belongs to a class of statistical models of traffic. As early as 1920th, A. K. Erlang presented his statistical works based on his experimental research regarding the statistics of traffic on telephony networks (Erlang [1], Brockmeyer et al. [2]). Briefly, the probability distribution functions (PDFs) he investigated are those that fast decay, such as the Poisson distribution and the binomial distribution.

The Erlang's results were so successful for characterizing the old telephony traffic to be used in queuing theory that the following PDF of the Erlang's distribution was taken as a law in the field of traffic engineering (see e.g., Akimaru and Kawashima [3], Gibson [4], Cooper [5]),

$$p_{\text{Erlang}}(x;k,\lambda)=\frac{\lambda^k x^{k-1}e^{-\lambda x}}{(k-1)!}\ \text{for } x>0, \qquad (1)$$

where e is the base of the natural logarithm and ! is the factorial function. The parameter k is called the shape parameter and λ is called the rate parameter. This situation lasts to the early age of the Internet, see e.g., Pitts and Schormans [6].

The Internet is obvisouly the most remarkable modern communication network. Before the day of the Internet becoming popular, actually, in the seventies of the last century, Tobagi et al. [7] reported a noticeable behavior of traffic, which is called "burstiness" defined by peak to average transmission rate [8, p. 45]. It simply implies that there would be no packets transmitted for a while, then flurry of transmission, no transmission for another long time, and so forth if one observes traffic over a long period of time. This also means that traffic has intermittency. In 1986, Jain and Routhier [9] further described the intermittency or burstiness of traffic using the term "packet trains". One of the significant results concluded in [9] is that traffic is neither a Poisson process nor a compound Poisson one. The results in [7] and [9] are quite qualitative but they may be taken as pioneering work in the field.

The early literature that quantitatively describes the statistical properties of traffic from a view of fractals refers to Leland et al. [10], Beran et al. [11], Csabai [12], Paxson and Floyd [13]. Those scientists convincingly revealed some of the main fractal properties of traffic, such as self-similarity (SS), long-range dependence (LRD), power-law type autocorrelation function (ACF), power-law type power spectrum density (PSD) function, i.e., $1/f$ noise, and heavy-tailed distribution. The mathematical model of traffic described in [10,11,13] is the fractional Gaussian noise (fGn) that was introduced by Mandelbrot and van Ness [14].

Though fGn has been regarded as a common model of traffic, see e.g., Tsybakov and Georganas [15], Adas [16], Michiel and Laevens [17], Li et al. [18], Li [19], Lee and Fapojuwo [20], Karagiannis et al. [21], Gong et al. [22], computer scientists feel unsatisfactory in a way with it, see e.g., [13,15] due to highly local irregularity of traffic (Feldmann et al. [23], Willinger et al. [24]). This paper discusses the basic results of fGn and points out its limitation in traffic modeling.

The rest of paper is organized as follows. In Section 2, the preliminaries of conventional 2-order stationary random processes are briefed. FGn is explained in Section 3. Its limitation in traffic modeling is discussed in Section 4, which is followed by conclusions.

2 Conventional 2-order Stationary

Random Processes

Let $\{x_l(t)\}$ ($l = 1, 2, ...$) be a 2-order stationary random process, where $x_l(t)$ is the lth sample function of the process. Usually, one simply uses $x_l(t)$ to represent the process without confusion causing. Its mean is given by

$$\mu_x^s(t) = \lim_{N\to\infty} \frac{1}{N}\sum_{l=1}^{N} x_l(t) = \text{const.} \quad (2)$$

Its ACF is given by

$$R_x^s(t, t+\tau) = \lim_{N\to\infty} \frac{1}{N}\sum_{l=1}^{N} x_l(t)x_l(t+\tau) = R_x^s(\tau). \quad (3)$$

In (2) and (3), the superscript s implies that the mean and the ACF are computed by using spatial average. The mean and ACF of a process expressed by time average are expressed by

$$\mu_x^t(t) = \lim_{T\to\infty} \frac{1}{T}\int_0^T x_l(t)dt = \text{const}, \quad (4)$$

$$R_x^t(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T x_l(t)x_l(t+\tau)d\tau, \quad (5)$$

where the superscript t indicates that the mean and the ACF are computed by time average.

The process $\{x_l(t)\}$ is said to be ergodic if (6) and (7) hold,

$$\mu_x^s(t) = \mu_x^t(t) \triangleq \mu_x = \text{const.}, \quad (6)$$

$$R_x^s(\tau) = R_x^t(\tau) \triangleq R(\tau). \quad (7)$$

Note that a real-traffic trace is a series of single history. In what follows, we just use $x(t)$ to represent a traffic process.

The probability of $x(t)$ is given by

$$P(x_2) - P(x_1) = \text{Prob}[x_1 < \xi < x_2] = \int_{x_1}^{x_2} p(\xi)d\xi, \quad (8)$$

where $p(\xi)$ is PDF.

The mean and the ACF of $x(t)$ based on PDF are written by (9) and (10), respectively,

$$\mu_x = \int_{-\infty}^{\infty} xp(x)dx, \quad (9)$$

$$R_x(\tau) = \int_{-\infty}^{\infty} x(t)x(t+\tau)p(x)dx. \quad (10)$$

Let σ_x^2 be the variance of x. Then, x is said to follow the Gaussian distribution if

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}}. \quad (11)$$

The Poisson distribution is a discrete probability distribution that expresses the probability of a number of events occurring in a fixed period of time if these events occur with a known average rate and independently of the time since the last event. In communication networks, one is interested in the work focused on certain random variables N that count, among other things, a number of discrete occurrences (sometimes called "arrivals") that take place during a time-interval of given length. Denote the expected number of occurrences in this interval by a positive real number λ. Then, the probability that there are exactly k occurrences (k being a non-negative integer, $k = 0, 1, 2, ...$) is given by the Poisson distribution below

$$p(x;\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}. \quad (12)$$

One thing worth noting in the conventional statistics is that either (11) or (12) fast decays, more precisely, exponentially decays. Therefore, according to (9) and (10), μ_x and R_x are convergent, which is actually a defaulted assumption in the traditional theory of communication networks. This assumption is natural from a view of 2-order statistics. However, actual traffic data challenges such an assumption.

Computer scientists based on processing real-traffic data measured in the Internet claim that a traffic series is heavy-tailed, see e.g., Paxson and Floyd [13], Resnick [26], Willinger et al. [27], Abry et al. [28], Cappe et al. [29]. The tail of the PDF of traffic may be so heavy that its ACF decays slowly or hyperbolically. On the one hand, because of slowly decaying of the ACF, a random variable that represents a traffic series can be no longer considered to be independent. Hence, LRD. On the other hand, the Fourier transform

$$S_x(\omega) = \int_{-\infty}^{\infty} R_x(\tau)e^{-j\omega\tau}d\tau, \quad (13)$$

of a slowly decayed ACF implies that the PSD of traffic with LRD obeys a power law. Hence, $1/f$ noise. These contents are actually in the domain of fractal time series.

3 FBM and FGN

3.1 ACF of fGn

Let $B(t)$ be a random process. Then, $B(t_{n+1}) - B(t_n)$ ($n = 0, 1, 2, ...$) is called increment process. If $B(t)$ has the following characteristics:

1) The increments $B(t + t_0) - B(t_0)$ are Gaussian,
2) $\text{E}[B(t + t_0) - B(t_0)] = 0$
   and $\text{Var}[B(t + t_0) - B(t_0)] = \sigma^2 t$,
3) in non-overlapping intervals $[t_1, t_2]$ and $[t_3, t_4]$, the increments $B(t_4) - B(t_3)$ and $B(t_2) - B(t_1)$ are independent,
4) $B(0) = 0$ and $B(t)$ is continuous at $t = 0$,

where $\sigma^2 = E\{[B(t+1) - B(t)]^2\} = E\{[B(1) - B(0)]^2\} = E\{[B(1)]^2\}$. Then, $B(t)$ is called Brownian motion (Papoulis and Pillai [30], Hida [31]).

Let $B_H(t)$ be the fractional Brownian motion (fBm) with the Hurst parameter $H \in (0, 1)$. Let $\Gamma(\cdot)$ be Gamma function. Then,

$$B_H(t) - B_H(0) = \frac{1}{\Gamma(H+1/2)}\left\{\begin{array}{l}\int_{-\infty}^{0}[(t-u)^{H-0.5} - (-u)^{H-0.5}]dB(u) + \\ \int_{0}^{t}(t-u)^{H-0.5}dB(u)\end{array}\right\}. \quad (14)$$

The function $B_H(t)$ has the following properties.
1). $B_H(0) = 0$,
2). the increments $B_H(t + t_0) - B_H(t_0)$ are Gaussian,
3). $\mathrm{Var}[B_H(t + t_0) - B_H(t_0)] = \sigma^2 t^{2H}$,
where

$$\sigma^2 = E\{[B_H(t+1) - B_H(t)]^2\} = E\{[B_H(1) - B_H(0)]^2\} = E\{[B_H(1)]^2\}.$$

According to the properties of fBm,

$$E\{[B_H(t_2) - B_H(t_1)]^2\} = E\{[B_H(t_2 - t_1) - B_H(0)]^2\} = E\{[B_H(t_2 - t_1)]^2\} = \sigma^2(t_2 - t_1)^{2H}. \quad (15)$$

In addition,

$$\begin{aligned} &E\{[B_H(t_2) - B_H(t_1)]^2\} = \\ &E\{[B_H(t_2)]^2\} + E\{[B_H(t_1)]^2\} - \\ &2E[B_H(t_2)B_H(t_1)] \\ &= \sigma^2(t_2)^{2H} + \sigma^2(t_1)^{2H} - 2r[B_H(t_2), B_H(t_1)] \\ &= \sigma^2(t_2 - t_1)^{2H}. \end{aligned} \quad (16)$$

Thus, the ACF of $B_H(t)$ is given by

$$r[B_H(t_2), B_H(t_1)] = \frac{\sigma^2}{2}[(t_2)^{2H} + (t_1)^{2H} - (t_2 - t_1)^{2H}]. \quad (17)$$

Eq. (17) implies that fBm is non-stationary.

The increment series, $B_H(t + s) - B_H(t)$, is fGn. Let us consider the ACF of fGn. Note that

$$\begin{aligned} &E\{[B_H(t_4) - B_H(t_3)][B_H(t_2) - B_H(t_1)]\} \\ &= r\{[B_H(t_4) - B_H(t_3)], [B_H(t_2) - B_H(t_1)]\} \\ &= E\{[B_H(t_4)B_H(t_2) - B_H(t_4)B_H(t_1) \\ &\quad - B_H(t_3)B_H(t_2)] + B_H(t_3)B_H(t_1)\} \\ &= E[B_H(t_4)B_H(t_2)] - E[B_H(t_4)B_H(t_1)] \\ &\quad - E[B_H(t_3)B_H(t_2)] + E[B_H(t_3)B_H(t_1)] \\ &= r[B_H(t_4), B_H(t_2)] - r[B_H(t_4), B_H(t_1)] \\ &\quad - r[B_H(t_3), B_H(t_2)] + r[B_H(t_3), B_H(t_1)]. \end{aligned} \quad (18)$$

According to (17), one has

$$r[B_H(t_4), B_H(t_2)] = \frac{\sigma^2}{2}[(t_4)^{2H} + (t_2)^{2H} - (t_4 - t_2)^{2H}], \quad (19)$$

$$r[B_H(t_4), B_H(t_1)] = \frac{\sigma^2}{2}[(t_4)^{2H} + (t_1)^{2H} - (t_4 - t_1)^{2H}], \quad (20)$$

$$r[B_H(t_3), B_H(t_2)] = \frac{\sigma^2}{2}[(t_3)^{2H} + (t_2)^{2H} - (t_3 - t_2)^{2H}], \quad (21)$$

$$r[B_H(t_3), B_H(t_1)] = \frac{\sigma^2}{2}[(t_3)^{2H} + (t_1)^{2H} - (t_3 - t_1)^{2H}]. \quad (22)$$

Replacing the right hand of (18) by (19) ~ (22) yields

$$\begin{aligned} &E\{[B_H(t_4) - B_H(t_3)][B_H(t_2) - B_H(t_1)]\} \\ &= r\{[B_H(t_4) - B_H(t_3)], [B_H(t_2) - B_H(t_1)]\} \\ &= \frac{\sigma^2}{2}[(t_4 - t_2)^{2H} + (t_3 - t_2)^{2H} - (t_4 - t_2)^{2H} - (t_3 - t_1)^{2H}]. \end{aligned} \quad (23)$$

In the discrete case, we let

$$t_1 = n,\ t_2 = n + 1,\ t_3 = n + k,\ t_4 = n + k + 1.$$

Then,

$$r\{[B_H(t_4) - B_H(t_3)], [B_H(t_2) - B_H(t_1)]\} = \frac{\sigma^2}{2}[(k+1)^{2H} - 2k^{2H} + (k-1)^{2H}]. \quad (24)$$

Therefore, the ACF of the discrete fGn (dfGn) is given by

$$r(k) = \frac{\sigma^2}{2}[(k+1)^{2H} - 2k^{2H} + (k-1)^{2H}], \quad (25)$$

where

$$\sigma^2 = (H\pi)^{-1}\Gamma(1-2H)\cos(H\pi), \quad (26)$$

is the intensity of fGn. Since the ACF is an even function, (25) can be rewritten as

$$r(k) = \frac{\sigma^2}{2}\left[\left(|k|+1\right)^{2H} + \left||k|-1\right|^{2H} - 2|k|^{2H}\right], \quad (27)$$

where $k \in \mathbb{Z}$. In the continuous case, the ACF of fGn is given by

$$r(\tau) = \frac{\sigma^2}{2}\left[\left(\frac{|\tau|}{\varepsilon}+1\right)^{2H} + \left|\frac{|\tau|}{\varepsilon}-1\right|^{2H} - 2\left|\frac{\tau}{\varepsilon}\right|^{2H}\right], \quad (28)$$

where $\tau \in \mathbb{R}$ and $\varepsilon > 0$ is a constant, which smoothes fBm so that the smoothed fBm is differentiable, see [14, p. 427-428] for details.

3.2 PSD of fGn and 1/*f* Noise

The PSD of dfGn was derived out quite early by Sinai [32]. It is given by

$$S(\omega) = 2C_f[1 - \cos(\omega)]\sum_{n=-\infty}^{\infty}\left|2\pi n + \omega\right|^{-2H-1}, \quad \omega \in [-\pi, \pi], \quad (29)$$

where $C_f = \sigma^2(2\pi)^{-1}\sin(\pi H)\Gamma(2H+1)$ and ω is the angular frequency. It has the applications to practice, e.g., analysis of fGn, simulation of fBm as well as fGn, see e.g., Beran [33, Eq. (2)], Beran [34, Proposition 2.1 on p. 53], Pipiras [35, p. 59], Purczyński and Włodarski [36, Eq. (10)], Ledesma and Liu [12]. Nevertheless, the PSD of fGn seems more difficult to obtain than that of dfGn.

Kou and Xie [38] presented the PSD of fGn. Late, Li and Lim [39] introduced a rigorous derivation of the PSD of fGn, which is given by

$$S(\omega) = \sigma^2 \sin(H\pi)\Gamma(2H+1)|\omega|^{1-2H}, \tag{30}$$

which exhibits that fGn is a type of $1/f$ noises.

3.3 LRD and SS

We say that $f(t)$ is asymptotically equivalent to $g(t)$ under the limit $x \to c$ if $f(t)$ and $g(t)$ are such that $\lim_{x\to c} \frac{f(t)}{g(t)} = 1$ (Murray [40]). We write

$$f(t) \sim g(t)\ (t \to c) \text{ if } \lim_{x\to c} \frac{f(t)}{g(t)} = 1, \tag{31}$$

where c can be infinity. It is proved that if $f(t) \sim g(t)$ $(t \to c)$ and $g(t) \sim h(t)$ $(t \to c)$ then $f(t) \sim h(t)$ $(t \to c)$ [40]. That is,

$$f(t) \sim g(t) \sim h(t)\ (t \to c). \tag{32}$$

In this sense, $f(t)$ is called slowly varying function if $\lim_{u\to\infty} \frac{f(ut)}{f(u)} = 1$ for all t.

Following the work of Beran [34], a random series $x(i)$ is said to be of LRD if

$$r(k) \sim ck^{-\beta}\ (k \to \infty) \text{ for } c > 0,\ \beta \in (0, 1), \tag{33}$$

where c can also be a slowly varying function.

Eq. (33) implies that the ACF of a series with LRD is non-summable. That is,

$$\sum_k r(k) = \infty. \tag{34}$$

Replacing β by the Hurst parameter H with

$$\beta = 2 - 2H, \tag{35}$$

yields another expression of (33), which is written by

$$r(k) \sim ck^{2H-2}\ (k \to \infty) \text{ for } c > 0,\ H \in (0.5, 1). \tag{36}$$

On the other side, if $\beta > 1$ or $H \in (0, 0.5)$, $r(k)$ is summable, corresponding to the case of short-range dependence (SRD) [33,34].

Without loss of generality, we consider traffic series y in the discrete case. By dividing y into non-overlapping blocks of size L and averaging over each block, we obtain another series given by

$$y(i)^{(L)} = \frac{1}{L}\sum_{j=iL}^{(i+1)L} y(j). \tag{36}$$

According to the analysis in Beran [34], Willinger and Paxson [43], Mandelbrot [44], [6,10,11], in the sense of fGn, one has

$$\mathrm{Var}(y^{(L)}) = L^{2H-2}\mathrm{Var}(y), \tag{37}$$

where Var implies the variance operator. Thus, fGn has the self-similarity measured by H.

3.4 Asymptotic Expressions

Note that $0.5[(\tau+1)^{2H} - 2\tau^{2H} + (\tau-1)^{2H}]$ can be approximated by $H(2H-1)(\tau)^{2H-2}$. As a matter of fact, $0.5[(\tau+1)^{2H} - 2\tau^{2H} + (\tau-1)^{2H}]$ is the finite 2-order difference of $0.5(\tau)^{2H}$ (Mandelbrot [41]). Approximating it with 2-order differential of $0.5(\tau)^{2H}$ yields

$$0.5[(\tau+1)^{2H} - 2\tau^{2H} + (\tau-1)^{2H}] \approx H(2H-1)(\tau)^{2H-2}. \tag{38}$$

From (37), one immediately sees that fGn contains three subclasses of time series. In the case of $H \in (0.5, 1)$, the ACF is non-summable and the corresponding series is of LRD. For $H \in (0, 0.5)$, the ACF is summable and fGn in this case is of SRD. FGn reduces to white noise when $H = 0.5$.

Applying the techniques of the Fourier transform of generalized functions as discussed in Lighthill [42] to the right side of (38), one has

$$S(\omega) \sim |\omega|^{1-2H} \text{ for } \omega \to 0, \tag{39}$$

implying $1/f$ noise.

4 Discussions

Among LRD processes, fGn has its advantage in the theory of fractal time series. For example, it can be used to easily represent two types of fractal time series, namely, SS processes and processes with LRD or SRD. However, Tsybakov and Georganas [15, Paragraph 1, Section II] noticed that “the class of exactly self-similar processes (i.e., fGn) is too narrow for modeling actual network traffic”. Li [45] demonstrates the error order of magnitude for modeling ACF of interarrival times of network traffic using fGn. In this section, a possible reason why fGn is limited in traffic modeling is explained.

From the discussions of H in Section 3.3, we see that LRD is a global property of traffic. However, in principle, SS is a local property of traffic. It is measured by fractal dimension, see e.g., Mandelbrot [44], Hall and Roy [46], Chan et al. [47], Adler [48], Kent and Wood [49]. Following [46-49], if the ACF $R(\tau)$ of $X(t)$ is sufficiently smooth on $(0, \infty)$ and if

$$R(0) - R(\tau) \sim c|\tau|^{\alpha} \text{ for } |\tau| \to 0, \tag{40}$$

where c is a constant, then one has the fractal dimension of $X(t)$ as

$$D = 2 - \frac{\alpha}{2}. \tag{41}$$

Denote D_{fGn} the fractal dimension of fGn. Then, according to the asymptotic expression (38), one has

$$r_{\text{fGn}}(0) - r_{\text{fGn}}(\tau) \sim c|\tau|^{2H} \text{ for } |\tau| \to 0. \tag{42}$$

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According to (41) and (42), therefore, one immediately gets

$$D_{\text{fGn}} = 2 - H. \quad (43)$$

Hence, for fGn, the local properties happen to be reflected in the global ones as noticed by Mandelbrot [50, p. 27].

The above discussions exhibit that fGn has its limitation in traffic modeling because it uses a single parameter H to characterize two different phenomena, saying, local property and global one. Recently, Li and Lim [25] discusses a traffic model called the generalized Cauchy (GC) process, which may be taken as an advance in traffic modeling to release the limitation of fGn.

5 Conclusion

The results regarding fGn are revisited in its ACF, PSD and their asymptotic expressions. Its limitation in traffic modeling has been discussed.

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