New Chromo theory of Canonically Conjugate fuzzy Subset

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Abstract: - The new chromo theory of canonically conjugate fuzzy subsets is presented. The Heisenberg’s principle’s analog principle is established. In Hilbert space the informational functions and joint membership functions are defined. In the work Zadeh operators, color operators and corresponding commutatively relations are presented.

Key-Words: - Fuzzy sets, Membership Function, Color Operator, Zadeh operators.

1 Introduction
In the work a new chromo theory of canonically conjugate fuzzy subsets is presented [1]. In many cases there exists unlimited number of ways of inter-action of a subject with the object. As a result of this, the controlled inter-action is almost always incomplete. It is based on limited (generally small) number of attributes (color) of the object which corresponds to the interests of the subjects and which can be recognized. When the set of colors are defined as a result of our interaction with the object we say, that there is defined the system on the object with the given structure of uncertainty.

Data in informatics are the set of so-called informational units [2]. Each of informational units is the four: (object, sign, value, plausibility). It’s important to make difference the notion of inaccuracy and uncertainty. Inaccuracy belongs to information content (corresponding to the component “value”), and uncertainty – to its confidence, understandable in terms of compatibility with reality (component “plausibility”). For the given data there exist the contradiction between inaccuracy of expression content and its uncertainty, expressed in that with the increase of expressions accuracy, its uncertainty rise as well and vice versa, uncertain character of information leads to some inaccuracy of the final conclusions, received from the data. We see that from one side these notions are in certain contradiction, and from another side – complete each other.
We offer to model such situation by means of new concept of optimal pare of canonically conjugate fuzzy subsets. We offer the method of construction of the informational unit membership function taking into account the both canonically conjugated components simultaneously and hence describing this unit in the most complete and optimal way.

2 Problem Formulation

Let \( \varphi \) denotes some color, \( \xi_\varphi \) - its numerical value is a random variable. In the referential system \( \Omega \) (universal set) is the latent parameter.

Let probability distribution of \( \xi_\varphi \) is characterized by density \( \rho_\varphi (x_\varphi) \):

\[
\int_\Omega \rho_\varphi (x_\varphi) dx_\varphi = 1
\]

(1)

Quantity

\[
x_\varphi = M\xi_\varphi = \int_\Omega x_\varphi \rho_\varphi (x_\varphi) dx_\varphi
\]

(2)

will be called calculated value of color \( \varphi \) in point \( \omega \) of universal set \( \Omega \). Note that formula (2) establishes one to one correspondence of \( \Omega \) and \( R \) (set of real numbers).

Except the quantity \( M\xi_\varphi \), the color of \( \omega \) is characterized by dispersion also:

\[
\sigma^2_\varphi (\omega) = D(\xi_\varphi) = \int_\Omega (x_\varphi - x^*_\varphi)^2 \rho_\varphi (x_\varphi) dx_\varphi
\]

(3)

In our model exactly \( \sigma^2_\varphi (\omega) \) is connected with definition of presence of \( \varphi \) color for \( \omega \). If \( \sigma^2_\varphi (\omega) \rightarrow 0 , \) we will say, that \( \varphi \) has certain value in \( \omega \). The more is \( \sigma^2_\varphi (\omega) \) the more is the uncertain \( \varphi \) in \( \omega \). If \( \sigma^2_\varphi (\omega) \rightarrow \infty , \) we say \( \omega \) has no color \( \varphi \). Thus, if \( \mu(\varphi) (\omega) = 1 , \) we say that \( \omega \) possesses color \( \varphi \), if \( \mu(\varphi) (\omega) = 0 , \) than \( \omega \) does not possess color \( \varphi \). leave two blank lines between successive sections as here.

2 Problem Solution

For \( \forall \omega \in \Omega \) introduce some interval of \( \varphi \) values \( I_\varphi (\omega) \subseteq R \) by relation:

\[
\mu_\varphi (\omega) = \int_{I_\varphi (\omega)} \rho_\varphi (x_\varphi) dx_\varphi = \int_\Omega I_\varphi (\omega) \rho_\varphi (x_\varphi) dx_\varphi
\]

(4)

Where \( I_\varphi (\omega) \) is defined in such a way, that (4) will be true when \( \mu_\varphi (\omega) \) is a membership function established by expert. In the theory of information representation main role plays the notion of informational function which is defined by

\[
x_\varphi (\omega) = \sqrt{\rho_\varphi (\omega)} e^{i\alpha} = \sqrt{\rho_\varphi (\omega)} e^{i\alpha}
\]

(5)

where \( \sqrt{\rho_\varphi (\omega)} \) is the arithmetic root and \( \alpha \) is random phase (real quantity). It is evident that:

\[
\rho_\varphi (\omega) = |x_\varphi (\omega)\|^2 = |x_\varphi (\omega)|^2
\]

Here \( |x_\varphi (\omega)\|^2 \equiv \{ x_\varphi, \varphi \} \) and \( |x_\varphi (\omega)\) are bra- and ket-vectors [3] correspondingly.

Let consider the Fourier transformation of the ket-vector:

\[
| x_{\varphi c} \rangle = \hat{F} | x_\varphi, \varphi \rangle = \frac{1}{\sqrt{2\pi}} \int \{ x_\varphi, \varphi \} e^{-i x_{\varphi c} x_{\varphi c}} dx_{\varphi c}
\]

(6)

where \( c \) is a constant. Integral (6) converges in mean-quadratic sense to the function from \( L^2 (R) \) (Hilbert space). Correspondence under \( | x_\varphi (\omega) \rangle \) and \( \hat{F} | x_\varphi (\omega) \rangle \) is reciprocal:

\[
\exists \hat{F} : F^{-1} \{ x_{\varphi c}, \varphi c \} = | x_\varphi (\omega) \rangle . \]

If in \( L^2 (R) \) the scalar product is defined, than

\[
\langle \{ x_{\varphi c}, \varphi c \}, \{ x_{\varphi c}, \varphi c \} \rangle = \langle \{ x_{\varphi c}, \varphi c \} \rangle \langle \{ x_{\varphi c}, \varphi c \} \rangle
\]

(7)

From formula (7) we can conclude that in parallel with color \( \varphi \) exists other color \( \varphi_c \) with informational function \( | x_{\varphi c} ; \varphi_c \rangle \) and density \( \rho_{\varphi c} (\omega) \) which determines corresponding membership function:

\[
\chi_{\varphi c} (\omega) = \int_{I_{\varphi c} (\omega)} | x_{\varphi c} ; \varphi_c \rangle^2 d\omega_{\varphi c} =
\]

(8)

where \( I_{\varphi c} (\omega) \) is the interval corresponding to membership function of canonically conjugate fuzzy subset. Usually fuzzy subset is constructed on the basis of expert estimation of one of the non-commuting component. Fuzzy subset, constructed in this way, characterize the information unit incompletely. We offer the method of construction of the information unit membership function taking into account the both canonically conjugated components simultaneously and hence describing this unit in the most complete and optimal way.

The joint distribution of two canonically conjugate colors is given by the following formula:

\[
\rho_{\varphi c \times \varphi c} (x_\varphi, x_{\varphi c}) =
\]
From formulas (9) and (10) follows very important relation between dispersion of the canonically conjugate colors:

\[
\left(\sigma_\varphi^2(\omega) + \sigma_{\varphi^*}^2(\omega)\right)\left(\sigma_\varphi^2(\omega) + \sigma_{\varphi^*}^2(\omega)\right) \geq \frac{c^2}{2}
\]

(13)

where \(\varphi^D\) and \(\varphi_e^D\) denotes dual colors.

This relation is analogue of Heisenberg’s uncertainty principle [5]. Generalized information theory leaves to interpret the constant \(c\) in terms of the Shannon measure of uncertainty.

When in formula (13) one has equality this means that corresponding membership function is optimal.

In presented work the form of optimal informational and membership functions are established.

Formula, which corresponds to (9), with \(\hat{A} = \hat{M}(\alpha_1, \alpha_2)\) have following form:

\[
W_{\varphi_1\varphi_1^*}(x_\omega, x_{\omega^*}) = \frac{1}{2\pi} \int |x_\omega - \pi v| \exp(-ivx_{\omega^*}) |x_\omega + \pi v| dv
\]

(14)

Or

\[
W_{\varphi_1\varphi_1^*}(x_\omega, x_{\omega^*}) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{c^2}{2} \frac{\partial^2}{\partial x_\omega^2} \frac{\partial^2}{\partial x_{\omega^*}^2} \right)|x_\omega - \pi v||x_\omega + \pi v| \exp\left(-\frac{1}{2\pi} x_\omega^2 x_{\omega^*}^2\right)
\]

(15)

Corresponding optimal compatibility function has the form:

\[
W_{\varphi_1\varphi_1^*}^{opt}(x_\omega, x_{\omega^*}) = (\frac{\pi}{c})^{-1} \exp\left(-\frac{(x_\omega - x_{\omega^*})^2}{2\sigma_{\varphi^2}} - \frac{2(x_{\omega^*} - x_\omega)^2}{c^2}\right)
\]

(16)

And

\[
\rho_{\varphi_1\varphi_1^*}^{opt}(x_\omega) = \frac{1}{\sqrt{2\pi} \sigma_{\varphi_1}} \exp\left(-\frac{(x_\omega - x_{\omega^*})^2}{2\sigma_{\varphi_1}^2}\right)
\]

where

\[
\sigma_{\varphi}^2 = \sigma_{\varphi_1}^2 + \sigma_{\varphi_1^*}^2 \quad \text{and} \quad \sigma_{\varphi_1}^2 = \sigma_{\varphi_1^*}^2 + \sigma_{\varphi_2}^2 = \frac{c^2}{4\sigma_{\varphi_1}}
\]

The characteristic intervals

\[
I_{\varphi_1} = \left[x_{\omega^*} - \alpha_1(\omega)\sigma_{\varphi_1}, x_{\omega^*} + \alpha_1(\omega)\sigma_{\varphi_1}\right]
\]

and

\[
I_{\varphi_2} = \left[x_{\omega^*} - \alpha_2(\omega)\sigma_{\varphi_2}, x_{\omega^*} + \alpha_2(\omega)\sigma_{\varphi_2}\right]
\]

In these formulas \(\alpha_1(\omega), \alpha_2(\omega), \alpha_1(\omega), \alpha_2(\omega) \in R\) and are determined by the structure of supports of canonically conjugate fuzzy subsets.

As illustration what we said previously, consider arithmetic operations on canonically conjugate optimal fuzzy real numbers. In the basis of optimal arithmetic laid in correspondence under mean error and standard deviation:

\[
\Delta x_\omega = |x_\omega - x_{\omega^*}|
\]

(17)

Math. Expectation of this quantity calls “error” of counted value:

\[
\Delta x_\omega = \frac{1}{\sqrt{2\pi} \sigma_{\varphi_1}(\omega)} \int |x_\omega - x_{\omega^*}| \exp\left(-\frac{(x_\omega - x_{\omega^*})^2}{2\sigma_{\varphi_1}(\omega)^2}\right) dx_{\omega^*} = \frac{2}{\sqrt{\pi} \sigma_{\varphi_1}(\omega)}
\]

(18)
Errors of arithmetic operations as it is known are defined by formulas:

\[
\Delta(x_1^* \pm x_2^*) = \Delta(x_1^*) \pm \Delta(x_2^*)
\]  
(19)

\[
\Delta(x_1^* \cdot x_2^*) = |x_1^*\Delta x_2^* + x_2^*\Delta x_1^*|
\]  
(20)

\[
\Delta\left(\frac{x_1^*}{x_2^*}\right) = \frac{x_1^*\Delta x_2^* - x_2^*\Delta x_1^*}{x_2^*}
\]  
(21)

Because of (18) for arithmetic operations on optimal F numbers we have following regulations:

\[
\sigma_{\varphi}(\tilde{x}_1\ast \tilde{x}_2^*) = \sigma_{\varphi}(\tilde{x}_1^*) + \sigma_{\varphi}(\tilde{x}_2^*)
\]  
(22)

\[
\sigma_{\varphi}(\tilde{x}_1\ast \tilde{x}_2^*) = |\tilde{x}_1^*\sigma_{\varphi}(\tilde{x}_2^*) + \tilde{x}_2^*\sigma_{\varphi}(\tilde{x}_1^*)|
\]  
(23)

\[
\sigma_{\varphi}(\tilde{x}_1^* \otimes \tilde{x}_2^*) = \frac{\tilde{x}_1^*\sigma_{\varphi}(\tilde{x}_2^*) - \tilde{x}_2^*\sigma_{\varphi}(\tilde{x}_1^*)}{\tilde{x}_2^*}
\]  
(24)

With help of these formulas are demonstrated the usual rules of arithmetic: commutativity of sum and product, associativity of sum and product, distributive rule.

On the basis of these formulas \(\tilde{x}_c^*\) we can derive corresponding laws for canonically conjugate variables:

\[
\sigma_{\varphi}(\tilde{x}_c^* \oplus \tilde{x}_c^*) = \sigma_{\varphi}(\tilde{x}_c^*) + \sigma_{\varphi}(\tilde{x}_c^*)
\]  
(25)

\[
\sigma_{\varphi}(\tilde{x}_c^* \otimes \tilde{x}_c^*) = \frac{\tilde{x}_c^*\sigma_{\varphi}(\tilde{x}_c^*) - \tilde{x}_c^*\sigma_{\varphi}(\tilde{x}_c^*)}{\tilde{x}_c^*}
\]  
(26)

\[
\sigma_{\varphi}(\tilde{x}_c^* + \tilde{x}_c^*) = \frac{\tilde{x}_c^*\sigma_{\varphi}(\tilde{x}_c^*) - \tilde{x}_c^*\sigma_{\varphi}(\tilde{x}_c^*)}{\tilde{x}_c^*}
\]  
(27)

Remark: Formulas (22) - (27) corresponds to real number in \(\varphi\) calibration. Calculation can be made in \(\varphi_{\varphi}\) calibration by changing \(x^* \rightarrow x_c^*\) and \(\sigma_{\varphi} \rightarrow \sigma_{\varphi_{\varphi}}\).

Constructed rules permits calculation of opposite and inverse fuzzy numbers:

\[
\tilde{a}^* \ast \tilde{x}^* = \tilde{0}^*, \quad (\tilde{0}^* = 0)
\]

Condition of existing opposite fuzzy number has the form: \(\sigma_{\varphi}(\tilde{0}^*) > \sigma_{\varphi}(\tilde{a}^*)\). Selection can be presented as follows:

\[
x^* = -\tilde{a}^*, \quad \sigma_{\varphi}(\tilde{x}^*) = \sigma_{\varphi}(\tilde{0}^*) - \sigma_{\varphi}(\tilde{a}^*)
\]

For the inverse fuzzy number we can write:

\[
\tilde{a}^* \ast \tilde{y}^* = \tilde{1}^*, \quad (\tilde{1}^* = 1)
\]

\[
y^* = \frac{1}{\tilde{a}^*}
\]

and \(\sigma_{\varphi}(\tilde{y}^*) = \frac{\sigma_{\varphi}(\tilde{1}^*) - \sigma_{\varphi}(\tilde{a}^*)}{\tilde{a}^*}\)

Established rules permit to receive solutions for algebraic equations and the system of equations, and also for all problems, which are connected with these equations.

References:


[6]. W.Haisenberg, Physical Principle of Quantum Mechanics, Clarandon Press , 1930