The Combined Decision Making Method based on the statistical and fuzzy Analysis

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Abstract: - The following paper offers a combined Decision Making method, which is based on four fuzzy-statistical methods. This combined novel method proposes two stages of decision-making. At the first stage three fuzzy-statistical methods independently make a decision for one object. These methods are statistical method of fuzzy grades’ analysis, fuzzy discrimination analysis and case based reasoning. Each of the methods makes a decision with its own approach to a problem and works with historical and expert data. These initial data are known cases of correctly made decisions with exhibited activities. The fourth method applied at the second stage makes the final decision. It carries out an expertons method that works with expert data only. The expertons method chooses the most believable decision from those offered by the above-named methods. The Decision Support system is constructed completely based on proposed combined method. The paper provides an example clearly illustrating the work of the constructed system.

Key-Words: - Decision Support System, Fuzzy Relative Frequencies, Membership Functions, most believable Decision, the similarity Measure between two Cases, Experton, Possibility Distribution.

1 Introduction

As is well known, in tasks of decision-making the deterministic or probabilistic approaches are traditional. However, for complex enough object, its description in traditional mathematical terms, likewise, construction of its exact mathematical model becomes impossible. The description of such objects is impossible without introduction of fuzzy representations. Many authors clearly support use of the fuzzy sets theory and soft computing methods to expand human ability in making optimal decisions involving uncertainty ([1,2,4-7,10,12-20] and so on). In particular, the use of fuzzy sets theory is considered to be effective enough for construction of decision-making support systems, because often the construction of such systems is based on expert knowledge and representations.

Thus, for complex by nature object it is expedient to construct a fuzzy model of the object. Moreover, construction of several fuzzy models makes it possible to reflect expert knowledge of various properties of researched object. Purposeful construction of model implies to overlook minor details in conformity with the final purpose. It, naturally, gives non-precise model. Therefore, simultaneous consideration of a number of models allows creating the best, more complete representation of a problem. Such an approach is offered in the presented work. Three fuzzy models describing researched object from the different points of view are constructed. Models are based on the following fuzzy-statistical methods: statistical method of fuzzy grades’ analysis [3,8,21], fuzzy discrimination analysis [12,15] and case based reasoning method [11,14]. For the final decision-making, i.e. for a choice of the most optimal decision, an expertons method [4,5] is applied.

For the object of the decision-making containing a fuzzyness in the definition, with known set of activities and in view of the investigated fuzzy methods the model of decision-making support system (DSS) is constructed. Model is entirely based on offered combined method of decision-making.

DSS is tested for a specific target of the forecast decision-making regarding the possibility of earthquake occurrence. For the forecasting factors some geophysical activities of an atmosphere are taken. To solve the problem mentioned above, the software has been created (algorithms of representation the fuzzy information in a computer, algorithms of creation and use of the knowledge base, and also algorithms of all fuzzy-statistical methods used for decision-making are developed and implemented) as forecast making system.

At the end of paper the example of a forecast is given, which is result of DSS work.
2 Fuzzy Decisions Making Methods

Methods of decision-making are characterized by various approaches. For each method the information is received from a general database, which contains primary historical data with exhibited activities and correctly made decisions. Let’s consider each of them.

2.1 Statistical Method of Fuzzy Grades’ Analysis

According to a statistical method of fuzzy grades’ analysis (hereinafter referred to as statistics of fuzzy grades) the decision making (forecasting) object is described by the forecast value. Area of the forecast value is divided into forecasting grades (classes). For each class the numerical interval is put in conformity. Corresponding membership functions are defined. Definition of the membership functions includes a human factor, since an expert has a subjective viewpoint on a degree of belonging of the given forecasting object to the forecasting classes. The mentioned classes are fuzzy, therefore supports of membership functions are intersected.

The forecasting value depends on the certain parameters, or of forecasting factors (activities). Each of activities, in turn, is divided into classes. The numbers of activities, their classes and range of their numerical intervals can be selected arbitrarily. Let’s introduce some designations: Forecasting grades - \( M_1, M_2, \ldots, M_f \); Corresponding membership function - \( \mu_1, \mu_2, \ldots, \mu_f \); Activities - \( A_k, A_2, \ldots, A_m \); Classes of activities - \( A_{k1}, A_{k2}, \ldots, A_{kr}, k = \bar{1}m; A_k = \bigcup_{j=1}^{r} A_{kj} \).

Further, let us deﬁne selective frequencies \( n_{kj} \), that represent the frequencies of \( j \) class of \( A_k \) activity occurring in \( i \) forecasting grade. The values of \( n_{kj} \) calculate from the initial data received as a result of observations and measurements [21]. \( n_{kj} \) and \( \mu_j \) numbers are used to deﬁne the fuzzy selective frequencies, the fuzzy relative frequencies and the weights of each interval of the activity in accordance to the known formulas [3]:

\[
\begin{align*}
\tilde{n}_{kj} &= \sum_i n_{kj} \cdot \mu_j, \quad \tilde{n}_{k} = \frac{\tilde{n}_{kj}}{\sum_i \tilde{n}_{kj}}, \\
\bar{w}_{kj} &= \frac{\sum_i \tilde{n}_{kj}}{\sum_j \sum_i \tilde{n}_{kj}},
\end{align*}
\]

(1)

where \( \mu_j^m \) is average value of membership function when the forecasting value from \( i \) forecasting interval belongs to \( m \) forecasting grade.

After that, it becomes possible to make a decision for the certain sample of forecasting factors. For this purpose, we need to define fuzzy weights of each activity according to its interval, and then to carry out multifactorial linear synthesis of fuzzy weights and fuzzy relative frequencies. As a result of multifactorial linear synthesis we receive the generalized decision (weighed vector of the possible decisions) [21]:

\[
D_a = \tilde{u}_a \cdot \tilde{f}_a. \tag{2}
\]

At last, in order to receive the unique decision it is necessary to use an additional principle. For example, it is possible to use a principle of a maximum of possibilities. Then the final decision will be [13]:

\[
D_{class}^{(a)} = \max \{D_a(i)\}, \tag{3}
\]

where \( D_a(i) \) is \( i \) component of a vector \( D_a \).

2.2 Fuzzy Discrimination Analysis

Discrimination analysis is better approach for modelling intellectual activity of the expert during decision-making [12].

The essence of the discrimination analysis comprises the following: from the information in a general database the frequency distribution table \( \{f_{ij}\} \) is established, where \( f_{ij} \) is the relative frequency of activity \( A_i \), accompanying decision \( D_j \):

\[
f_{ij} = \frac{m_{ij}}{N}, \tag{4}
\]

where \( m_{ij} \) is number of those correctly accepted decisions \( D_j \) for which \( A_i \) activity was exhibited, and \( N \) is the general number of cases.

The frequency distribution table, which is the basis of the numerical-tabular knowledge base, contains the primary information for two further tables called positive discrimination table \( \{p_{ij}\} \) and negative discrimination table \( \{n_{ij}\} \), which are calculated as follows:

\[
p_{ij} = \frac{1}{C_D - 1} \sum_{k \neq j} \chi_{large-ratio} \left( \frac{f_{ij}}{f_{ik}} \right),
\]

\[
n_{ij} = \frac{1}{C_D - 1} \sum_{k \neq j} \chi_{large-ratio} \left( \frac{f_{ik}}{f_{ij}} \right). \tag{5}
\]

Here \( p_{ij}, n_{ij} \in [0;1] \); \( C_D = Card \, D \) denote cardinality of set of decisions; \( \chi_{large-ratio} \) denote the fuzzy subset \( R_0^+ \) with membership function:

\[
\chi_{large-ratio} : R_0^+ \rightarrow [0,1],
\]

which puts relations (real numbers) \( f_{ij}/f_{ik} \) into the interval \([0,1]\).
The heuristic interpretation of positive and negative discrimination is following: \( p_y \) represents an accumulated belief that \( A_j \) is more characteristic for decision \( D_j \) than for other decision, and \( n_y \) represents an accumulated belief that \( A_j \) is more characteristic for decision non- \( D_j \) than for others.

The generalized decision is represented as a fuzzy subset of the set of possible decisions with the following membership function:

\[
\delta(D_j) = \frac{1}{2} \left( \chi_{\text{Large}}(\pi_j) + \chi_{\text{Small}}(v_j) \right), \quad j \in D,
\]

where

\[
\pi_j = \frac{1}{C_A} \sum_{i=1}^{r} p_y^i, \quad v_j = \frac{1}{C_A} \sum_{i=1}^{r} n_y^i.
\]

Here \( \pi_j \) and \( v_j \) represent the averages of positive and negative discrimination measures respectively for decision \( D_j \); The fuzzy sets Large and Small have characteristic membership functions: \( \chi_{\text{Large}} \cdot \chi_{\text{Small}} : [0,1] \rightarrow [0,1] \), where \( \chi_{\text{Large}} \) is monotonic increasing, and \( \chi_{\text{Small}} \) – monotonic decreasing in its argument; \( p_y^i \) and \( n_y^i \) are elements of matrices \( \{ p_y^i \} \) and \( \{ n_y^i \} \) corresponding to a particular set of activities \( A' = \{ A'_1, \ldots, A'_r \} \). These matrices are produced by selecting from \( \{ p_y \} \) and \( \{ n_y \} \) only those rows which correspond to \( A' \);

\[
C_A = \text{Card}A' = \text{Card}\{ A'_1, \ldots, A'_r \} = r.
\]

To make the final “classic” decision an additional defuzzification principle is needed [15]. For example, the decision can be made according to the maximum of function \( \delta(D_j) \):

\[
\delta^{\text{Class}} = \max_{j \in D} \delta(D_j),
\]

i.e. the decision \( j \) with maximum value in \( \{ \delta_j \} \) can be recognized as a most believable decision.

### 2.3 Case Based Reasoning

To receive a correct new result of decision-making or forecasting, among the existing known cases analogues to a newly introduced case are searched and the same decision which was correct for analogues is accepted [11]. Therefore this method is often named a method of “the nearest neighbour”. The method has following advantages:

- It is possible to give the best explanation and a substantiation of the decision on the basis of consideration of the previous cases, etc.

Measure of similarity between a new case and other cases stored in the general base include two stages:

a) The distance between two \( i \) activities of two cases is calculated according to the formula:

\[
DV_i = \min(CB_i, ND_i), \quad i = 1, n,
\]

where \( n \) is number of all activities; \( CB_i \) is value of \( i \) activity of existing case; \( ND_i \) is value of \( i \) activity of new case; \( DV_i \) is the distance value between two \( i \) activities.

b) The similarity measure between two cases is calculated as follows:

Let \( SV_j \) be the similarity value between the new examining case and the existing in general database \( j \) case. \( SV_j \) can be calculated as follows:

\[
SV_j = \frac{1}{n} \sum_{i=1}^{n} w_i \cdot DV_i, \quad j = 1, k, \quad i = 1, n,
\]

where \( k \) is number of precedents; \( \hat{w} \) represents a vector of weights, whose \( w_i \) component indicates the importance of the \( i \) activity for decision making. \( w_i \in [0,1] \) and \( w_i = 0 \) means that \( i \) activity is not important, \( w_i = 1 \) means that \( i \) activity is absolutely important, \( 0 < w_i < 1 \) indicates the importance degree of \( i \) activity. Determination of weights often can be based on experience of experts.

The final decision is accept as follows: find the \( r \) which satisfies to a condition

\[
SV_r = \max(SV_j), \quad j = 1, k.
\]

In advance experts provide some threshold for similarity measure. If the degree of similarity \( SV_r \) is equal or greater than this threshold, conclusion for examined case is the same as that of the precedent. Otherwise conclusion for new case cannot arrive at the same conclusion as that of the precedent. Then it is necessary to infer decision using other precedents or to change approaches for this query case.

### 2.4 Expertons Method

An experton is the generalized notion of a probable random fuzzy event when the probability of a random event of each \( \alpha \)-cut is replaced by confidence intervals. These intervals in their turn are statistically defined by the group of experts. The concept of the expertons theory can be briefly described as follows [4,5].

Let \( E \) be a finite or infinite set of certain objects, factors and so on. The group of \( r \) experts is requested to express their subjective opinion regarding each element from \( E \) in the form of a
confidence interval \( \forall P \in E: [a_j^I(P), a_j^U(P)] \subseteq [0,1] \), where the symbol \( \subseteq \) denotes an inclusion and \( j \) the order number of an expert. We consider the statistics when to each element \( P \in E \) we assign both the lower and the upper bound of confidence intervals. The cumulative distribution law \( F_r(\alpha, P) \) is constructed on the basis of \( a_j^I(P) \), and \( F^*(\alpha, P) \) on the basis of \( a_j^U(P) \). Thus we obtain
\[
\forall P \in E, \forall \alpha \in [0,1]: \tilde{A}(P) = [F_r(\alpha, P), F^*(\alpha, P)],
\]
where \( \tilde{A} \) denotes an experton. The following properties of an experton are obvious:
\[
\forall P \in E, \forall \alpha, \alpha' \in [0,1]: (\alpha < \alpha') \Rightarrow (\left[ F_r(\alpha', P), F^*(\alpha', P) \right] \subseteq \left[ F_r(\alpha, P), F^*(\alpha, P) \right]),
\]
where \( \subseteq \) denotes an interval inclusion, i.e. \( (\alpha < \alpha') \Rightarrow (F_r(\alpha', P) \geq F_r(\alpha, P)) \) and \( F^*(\alpha', P) \leq F^*(\alpha, P) \).

3 Fuzzy-statistical Methods for Concrete Decision Making Task

Fuzzy-statistical methods are applied in concrete forecasting task - decision-making regarding the possibility of earthquake occurrence. As the factors-precursors the some geophysical activities of an atmosphere are taken. The forecasting object - earthquake - is described by great number of activities. Initial data comprises the earthquakes’ statistics in the Dusheti Region of Georgia.

Therefore some modifications of fuzzy-statistical methods became necessary.

a) Statistics of Fuzzy Grades:

The "classical" variant of a method became a subject to modification in order to satisfy the condition of a great number of activities. With this purpose, the author introduced a concept of a measure of possibility which is used to build a generalized decision [9]:
\[
\overline{\text{Poss}}_\alpha = \tilde{D}_\alpha \cdot \max_j(D_\alpha(j)) \tag{11}
\]
where \( D_\alpha(j) \) is \( j \) component of a vector \( \tilde{D}_\alpha \). To receive the unique decision now it is possible to use a principle of a maximum of possibilities measure. Then the final decision will be:
\[
D_\text{Class}^{(\alpha)} = \max_i (\text{Poss}_\alpha(i)) \tag{12}
\]
where \( \text{Poss}_\alpha(i) \) is \( i \) component of a vector \( \overline{\text{Poss}}_\alpha \).

Belonging to a certain class of forecasting is determined through application of a so-called membership functions. Definition of membership functions is based on intellectual activity of experts. Since membership functions are defined with experts’ subjective viewpoint, they can be any kind. "Right" definition of membership functions is the basic guarantee of the method’s success. The present work offers a model of membership function developed for a concrete case of the forecast. It represent a new modification of Zadeh’s model ( see formulas (15)).

b) Discrimination Analysis:

By analogy to a statistics of fuzzy grades each activity divided into classes. Since initial data is only objective, it helps to calculate relative frequencies \( f_{ij} \). The quantity of classes and range of their numerical intervals are selected on the basis of expert data.

\( \chi_{\text{Large-ratio}}, \chi_{\text{Large}} \) and \( \chi_{\text{Small}} \) membership functions was defined according to their properties and to initial data:
\[
\chi_{\text{Large-ratio}}\left( \frac{f_{ij}}{f_{ik}} \right) = k \cdot \left( \frac{f_{ij}}{f_{ik}} \right),
\]
where coefficient \( k \) is found in accordance with the primary data;

\( \chi_{\text{Large}} \) define as \( \chi_{\text{Large}}(\pi_j) = \pi_j \);

\( \chi_{\text{Small}} \) as \( \chi_{\text{Small}}(\nu_j) = -\nu_j + 1 \).

c) Case Based Reasoning:

Here also each activity is divided into classes. It allows to prepare initial data for the method on a stage of fuzzification. The quantity of classes and their numerical intervals coincide with these defined in discrimination analysis. Since activities classes are fuzzy subsets, the distance between two \( i \)-th activities of investigated case and precedent is calculated as follows [14]:
\[
DV_i = 1 - \left| CB_i - ND_i \right|. \tag{13}
\]
As activities are the known values, while calculating the similarity measure between two cases, all activities should be considered as absolutely important for decision making. I.e. all \( w_i = 1 \). Calculations are made under the formulae:
\[
SV_j = \sum_{i=1}^{2n} DV_i \cdot \left| \frac{2n}{i} \right|. \tag{14}
\]
Recall that \( n \) is the number of activities.

d) Expertons Method:

If during decision-making in the fuzzy system more than one method is applied, their composition is necessary to receive of a unique fuzzy subset of values. With this purpose it is possible to apply an expertons method.

Let \( E \) be a fuzzy subset of decisions and include \( k \) elements: \( E = \{\delta_1, \delta_2, \ldots, \delta_k\} \). Using joint interval estimations of experts for each decision, the
expertons method will allow to find the unique most believable decision as
\[ \delta_{\text{Class}} = \max(\delta_j), \ j = 1, k, \]
where \( j \) is number of decision.

4 Decision Making Example

For a specific example of earthquake forecasting we consider the following geophysical atmosphere data to be factors-precursors:

- \( A_1 \) - Value of intensity of the electric fields (volt/m);
- \( A_2 \) - Temperature of air (in degrees of Celsius);
- \( A_3 \) - Temperature of ground (in degrees of Celsius);
- \( A_4 \) - Atmospheric pressure (in mb);
- \( A_5 \) - Absolute humidity (elasticity water pair in mb);
- \( A_6 \) - Relative humidity (in %);
- \( A_7 \) - The general overcast (in points);
- \( A_8 \) - The bottom overcast (in points);
- \( A_9 \) - Speed of a wind (in m/s).

Values of factors were measured during the day in three hour interval.

Decision-making process consists of the following:

The object of forecasting, earthquake, is described by means of a linguistic variable with following values: "noise", "moderate earthquake", "strong earthquake" and is characterized by numerical value of magnitude \( (M) \).

At \( 0 \leq M \leq 3 \) "noise" is observed; at \( 3 < M < 5 \) "moderate earthquake" is observed; at \( 5 \leq M \leq 8 \) "strong earthquake" is observed. Let us designate the defined forecast classes (intervals of earthquake intensity) as \( M_0, M_1 \) and \( M_2 \).

a) Statistics of Fuzzy Grades:

Let's define the corresponding membership functions. The model of membership functions, applied in the given method, is:

\[
\mu_0(M) = \begin{cases} 
\frac{1}{1 + (\alpha_1 M)^2}, & 0 \leq M \leq 3; \\
0, & M > 3 
\end{cases}
\]
\[
\mu_1(M) = \begin{cases} 
\frac{1}{1 + (\alpha_2 (M-4.9))^2}, & 4.4 \leq M \leq 5.4; \\
0, & M > 5.4 
\end{cases}
\]
\[
\mu_2(M) = \begin{cases} 
\frac{1}{1 + (\alpha_3 (M-8))^2}, & 4.4 \leq M \leq 8; \\
1, & M > 8 
\end{cases}
\]

Coefficients \( \alpha_1, \alpha_2 \) and \( \alpha_3 \) are chosen empirically in accordance with the available data and experts’ recommendations. In our case \( \alpha_1 = 0.15 \), \( \alpha_2 = 4.99 \) and \( \alpha_3 = 0.5 \).

Since forecasting classes are presented in the form of intervals, it is necessary to average membership functions on these intervals. Let \( \mu_j^i \) be an average value of \( \mu_j \) considering the intersection of a support of \( i \) forecasting fuzzy class and \( \text{supp } \mu_j \). Then:

\[
\mu_0(M) = \frac{0.93968}{0 < M \leq 3} + \frac{0}{M > 3};
\]
\[
\mu_1(M) = \begin{cases} 
0, & 0 < M < 4.4; \\
0.55192, & 4.4 \leq M \leq 5; \\
0.3641, & 5 < M \leq 5.4; 
\end{cases}
\]
\[
\mu_2(M) = \begin{cases} 
0.26968, & 4.4 \leq M \leq 5; \\
0.6552, & 5 < M \leq 8; \\
1, & M > 8. 
\end{cases}
\]

Each of the factors-precursors is divided into three classes (subfactors). Intervals of the classes are fuzzy sets. Their boundaries are chosen empirically in accordance with the available data and estimations by the experts.

Selective frequencies \( n_{kj}^i \) for each interval of intensity and each class of each activity are calculated. The \( \tilde{n}_{kj}^i \) fuzzy selective frequencies, \( \tilde{f}_{kj}^i \) fuzzy relative frequencies and \( w_{kj}^i \) fuzzy weights of the forecasting factors are calculated on the basis of formulae (1). Now all the data necessary for decision-making exist - the knowledge base is constructed.

Assume, we need to study a new case and values describing its activities are:

\[ 5.125, 9.675, 8.875, 913.98, 8.12, 69, 3.25, 2.625, 6.125. \]

According to initial data and selected classes of activities the following set of the classes of activities corresponds to the above given set of activities:

\[ A_{12}, A_{23}, A_{32}, A_{42}, A_{52}, A_{62}, A_{71}, A_{81}, A_{93}. \]

From knowledge base we select a vector of fuzzy weights

\[ \tilde{w} = (w_{12}, w_{23}, w_{32}, w_{42}, w_{52}, w_{62}, w_{71}, w_{81}, w_{93}) \]

and matrix \( \tilde{f} \) of fuzzy relative frequencies, where to each component \( w_{kj}^i \) of a vector \( \tilde{w} \) corresponds a row \( \tilde{f}_{kj}^i \) of a matrix \( \tilde{f} \).

In our case

\[ \tilde{w} = (0.6619, 0.3762, 0.3381, 0.5130, 0.4749, 0.5816, 0.4346, 0.3279, 0.1429), \]

a matrix of fuzzy relative frequencies is:
According to formula (2) we receive the weighted vector of possible decisions
\[
\tilde{\mathbf{f}} = \begin{pmatrix}
0.3647 & 0.3272 & 0.3081 \\
0.3850 & 0.3256 & 0.2894 \\
0.2856 & 0.3337 & 0.3807 \\
0.2823 & 0.3858 & 0.3319 \\
0.2023 & 0.3964 & 0.4003 \\
0.3320 & 0.3071 & 0.3609 \\
0.4443 & 0.2596 & 0.2961 \\
0.2944 & 0.3735 & 0.3321 \\
0.3379 & 0.3294 & 0.3326
\end{pmatrix}.
\]

\(\tilde{\mathbf{f}}\) is divided into two classes: \(A_1\) and \(A_2\).

As with the corresponding measure of possibility (see (11))
\[
\overline{P_{OSS}} = (0.96819, 1, 0.99835).
\]

And, finally, we receive the forecast (see (12)):
\[
D_{Class} = 1 \quad (\Rightarrow M_1 = "\text{moderate earthquake}").
\]

b) Discrimination Analysis:

In this method each of the activity (factor—precursor) is divided into two classes: \(A_{k1}, A_{k2}\); \(k = 1, 2, 3\). Relative frequencies of each class of each activity are calculated. On the basis of these calculations (made by rules of type “If—Then”) the first table of the knowledge base \(\{f_{ij}\}\) (see (4)) is received. Then under formulas (5) positive \(\{p_{ij}\}\) and negative \(\{n_{ij}\}\) discrimination tables are constructed. Coefficient \(k = 1/3.75\) in \(\chi_{Large—ratio}\) membership function.

Let’s make the forecast for a set of activities: \(5.125, 9.675, 8.875, 913.98, 8.12, 69, 3.25, 2.625, 6.125\).

At the predetermined boundaries of classes to the set above corresponds the following set
\[
\begin{align*}
&1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1
\end{align*}
\]

The distance between two \(i\) activities of investigated case and precedent is calculated according formulae (13). The similarity measure between two cases calculated according formulae (14).

After necessary calculations we receive set of measures of the maximal similarity on forecasting classes:
\[
(0.7778, 0.6667, 0.8889).
\]

The final decision is found according to formula (10). The forecast is "strong earthquake".

d) Expertons Method:

Methods give an ambiguous forecast. This is a precondition to apply expertons method.

Let 3 experts give the interval estimations concerning reliability of acceptance of each of decisions: \(\delta_0\) - possibly "noise" will be observed; \(\delta_1\) - possibly "moderate earthquake" will be observed; \(\delta_2\) - possibly "strong earthquake" will be observed. The interval estimations of experts are given in the table:

\[
\begin{array}{c|c|c|c|c}
\text{Expert Method} & \delta_0 & \delta_1 & \delta_2 \\
\hline
1 & [0.2, 0.3] & [0.4, 0.6] & [0.6, 0.7] \\
2 & [0.5, 0.6] & [0.6, 0.8] & [0.5, 0.6] \\
3 & [0.1, 0.7] & 1 & [0.7, 0.8] \\
\hline
1 & [0.3, 0.4] & 1 & 1 \\
2 & [0.6] & [0.7, 1] & [0.6, 0.8] \\
3 & [0.8, 1] & [0.6, 1] & [0.2, 0.3] \\
\hline
1 & [0.4, 0.8] & 0.6 & [0.2, 0.3] \\
2 & [0.4, 0.5] & [0.3, 0.7] & 1 \\
3 & [0.2, 0.4] & [0.2, 0.9] & [0.2, 0.9]
\end{array}
\]

The unique, classical decision it is received as
\[
\delta_{Class} = \max_j \delta_j.
\]

In our case
\[
\delta_{Class} = 0.51704.
\]

This maximal value corresponds to an interval of intensity "moderate earthquake". That also will be the forecast.

c) Case Based Reasoning:

From a general database activities values which describe known cases as "noise", "moderate" and "strong" (precedents) are selected. All activities for each case are divided into two classes. Then by rules of type “If—Then”, we produce the table of the knowledge base where for each class of each activity we do have values 0 or 1. We have 1, if value of activity of a precedent belongs to a class of activity. Otherwise we have 0.

Let’s make the forecast for a set of activities:
\[
\begin{align*}
5.125, 9.675, 8.875, & 913.98, 8.12, 69, 3.25, \\
2.625, & 6.125.
\end{align*}
\]

The interval estimations of experts are
\[
\begin{align*}
&[0.2, 0.9],
&[0.3, 0.7],
&[0.2, 0.3],
&[0.5, 0.6],
&[0.6, 0.8],
&[0.2, 0.9],
\end{align*}
\]

The similarity measure between two cases calculated according formulae (14).

After necessary calculations we receive set of measures of the maximal similarity on forecasting classes:
\[
(0.7778, 0.6667, 0.8889).
\]

The final decision is found according to formula (10). The forecast is "strong earthquake".
Here I designates method of statistics of fuzzy grades; 2 - discrimination analysis; 3 - case based reasoning.

For each of the possible decisions $\delta_i, i=0,1,2$ calculate two statistics on set of the levels $\{0, 0.1, 0.2, \ldots, 0.9, 1\}$: one for the lower boundary of an interval and the other for the upper boundary. Then we obtain the following table which is an experton:

<table>
<thead>
<tr>
<th>Level</th>
<th>$\delta_0$</th>
<th>$\delta_1$</th>
<th>$\delta_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>[1, 1]</td>
<td>[1, 1]</td>
<td>[1, 1]</td>
</tr>
<tr>
<td>0.1</td>
<td>[1, 1]</td>
<td>[1, 1]</td>
<td>[0.9, 1]</td>
</tr>
<tr>
<td>0.2</td>
<td>[0.9, 1]</td>
<td>[1, 1]</td>
<td>[0.9, 0.9]</td>
</tr>
<tr>
<td>0.3</td>
<td>[0.7, 1]</td>
<td>[0.9, 1]</td>
<td>[0.7, 0.9]</td>
</tr>
<tr>
<td>0.4</td>
<td>[0.6, 0.9]</td>
<td>[0.8, 1]</td>
<td>[0.7, 0.8]</td>
</tr>
<tr>
<td>0.5</td>
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<td>[0.7, 1]</td>
<td>[0.7, 0.8]</td>
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<tr>
<td>0.6</td>
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<td>[0.7, 1]</td>
<td>[0.6, 0.8]</td>
</tr>
<tr>
<td>0.7</td>
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<td>[0.3, 0.8]</td>
<td>[0.3, 0.7]</td>
</tr>
<tr>
<td>0.8</td>
<td>[0.1, 0.2]</td>
<td>[0.2, 0.7]</td>
<td>[0.2, 0.6]</td>
</tr>
<tr>
<td>0.9</td>
<td>[0, 0.1]</td>
<td>[0.2, 0.6]</td>
<td>[0.2, 0.3]</td>
</tr>
<tr>
<td>1</td>
<td>[0, 0.1]</td>
<td>[0.2, 0.4]</td>
<td>[0.2, 0.2]</td>
</tr>
</tbody>
</table>

Further experton we will transform as follows: (a). averaged experton we will calculate by taking a mean arithmetic value of the boundaries of each interval; (b). averaged experton is reduced to a possibility distribution on decisions set $\{\delta_i\}, i=0,1,2$ by taking mean value of all levels; (c). if necessary, we search a nonfuzzy set, the closest to the fuzzy one.

After the transformations (a) - (b) we will obtain the possibility distribution on decisions set:

<table>
<thead>
<tr>
<th>$\delta_0$</th>
<th>$\delta_1$</th>
<th>$\delta_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.536364</td>
<td>0.7500</td>
<td>0.654545</td>
</tr>
</tbody>
</table>

The principle of a maximum is applied to draw of the unique decision:

$$\delta(\delta_i) = \max_i \delta_i, i=0,1,2.$$ 

In our case, in conformity with the common opinion of the experts, the experton gives preference to the decision $\delta_1$, i.e. final forecast is - possibly "moderate earthquake" will be observed.

The described result conforms with the statistical data: values of forecasting factors in the given example correspond to a real data for November, 20-th, 1981 when in 18-th there was an earthquake with magnitude 4.6 (according to our classification - "moderate earthquake").

5 Conclusion

This article proposed decision making method, which is combining three methods of decision-making and with the purpose of their final composition uses the fourth method - an expertons method.

As shown, each of three methods makes a decision independently and has own approach to a problem.

Because during the decision-making three methods are applied at once, their composition is necessary for reception of the unique decision. It carries out an expertons method.

By means of this combined method DSS is constructed, which does the forecast of earthquake. At the end of paper the example of DSS work is given, in which on the basis of an offered method the forecast is done.

In spite of the fact that the purpose of the given paper was not the description of the created DSS, but the description of the combined method which the system applies at decision-making, it would be desirable to tell a bit more about DSS. DSS is based on numerical-tabular knowledge base and is implemented by means of Web-programming and client-server technology. MySQL is chosen for a general database

DSS is a general use system, since it is possible to use the created software in various research fields (for example, medical diagnostics, weather forecast, forecasting of flooding, etc.).

Acknowledgements

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References:


[17]. Sirbiladze G., Sikharulidze A., Weighted Fuzzy Averages in Fuzzy Environment, Parts I,