Investigation of the heat index in Georgia based on the most typical fuzzy expected values

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Abstract: The paper concerns influences of global warming on a climate change in Georgia and health of the population. In particular, one important parameter of a human thermal comfort - a heat index- (that represents a combination of the air temperature and relative humidity) has been investigated. The investigation was carried out on the base of real data collected in the Ministry of Preservation of the Environment of Georgia. Typical characteristics of the heat index -two most typical fuzzy expected values: fuzzy expected value and modified clustering fuzzy expected value - were calculated and on their base the changes of a heat index in Georgia were shown.

Key–Words: heat index, fuzzy expected value, clustering

1 Introduction

The global warming [3] is a well-known problem that leads to the climate change and has a great impact on the human health. Among several parameters that allow to estimate this effect (for example, variation on the intensity of solar radiation, wind spreed, etc.) a heat index (HI) is often used. A heat index is a combination of air temperature and air humidity. HI determines the human-perceived equivalent temperature. A good example of the difference between a heat index and a true temperature would be comparing the climates of Miami and Phoenix. Miami averages around 90°F in summer due to the easterly trade winds coming from the Atlantic Ocean, but it has a high humidity (e.g. 75%). Phoenix averages around 104.5°F in summer, but typically has a low humidity (e.g. 10%). According to the heat index, the relative temperature in Miami will be 109.5°F, but the relative temperature in Phoenix will be lowered due to the lower humidity, to around 98.6°F. Given sunshine, Miami is likely to feel hotter than Phoenix [4].

There are several ways to estimate HI: by special formulas (for example, Steadman [8] that will be discussed further in the paper) or due to the special tables where empirical data are collected. For example, Figure 1 shows dependences, i.e., HI, between relative humidity (RH) and air temperature in °C (modified from [1]). The data in this table are subjective estimations of the human thermal comfort in Celsius.

Notice, that in applications, HI is measured in Celsius or in Fahrenheit.

Investigation of HI are mainly carried out in the following directions:

- influence of HI onto human health;
- HI prognoses in countries under global warming circumstances.

For example, impacts of HI (shade values) based on the investigations done in [4] are presented in the Table 1.

Methods, used for HI prognoses, usually depend on data available. From this point, data collected in Georgia are immense and allow a wide spectrum of experiments.

Observations of the global warming influence on the climate change have been started in Georgia since middle 50th [9]. Since that time empirical data are collected. We chose information about air temperature and air humidity is collected in the four main cites of Georgia: Tbilisi, Poti, Lentexi, Dedoplistskharo during the two periods (1955-1970 and 1990-2007), and only five months are taken into consideration (May, June, July, August, September).

RH (%)	Temperature (°C)															
	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
90	28.04	30.73	33.75	37.08	40.72	44.68	48.95	53.54	58.45	63.67	63.67	75.06	81.22	87.71	94.51	101.62
85	27.85	30.22	32.89	35.87	39.14	42.71	46.58	50.76	55.22	59.99	65.06	70.43	76.1	82.07	88.34	94.91
80	27.67	29.74	32.1	34.74	37.67	40.88	44.37	48.14	52.21	56.54	61.17	66.08	71.28	76.75	82.51	88.56
75	27.48	29.28	31.36	33.69	36.3	39.17	42.31	45.72	49.39	53.33	57.53	62.01	66.75	71.76	77.03	82.57
70	27.29	28.86	30.67	32.73	35.04	37.6	40.41	43.47	46.78	50.34	54.15	58.21	62.52	67.08	71.89	76.95
65	27.11	28.46	30.03	31.84	33.88	38.66	38.66	41.41	44.38	47.58	51.02	54.69	58.59	62.73	67.09	71.69
60	26.93	28.08	29.45	31.03	32.83	34.84	37.07	39.52	42.18	45.05	48.14	51.44	54.96	58.69	62.64	66.81
55	26.74	27.73	28.92	30.31	31.89	33.67	35.64	37.81	40.18	42.75	45.51	48.47	51.63	54.98	58.53	62.28
50	26.56	27.42	28.45	29.66	31.05	32.62	34.36	36.29	38.39	40.68	43.14	45.78	48.59	51.59	54.77	58.12
45	26.38	27.13	28.03	29.09	30.32	31.7	33.24	34.94	36.81	38.83	41.02	43.36	45.86	48.52	51.34	54.33
40	26.21	26.86	27.67	28.61	29.69	30.91	32.28	33.78	35.43	37.22	39.14	41.21	43.42	45.77	48.27	50.9
35	26.02	26.63	27.36	28.2	29.17	30.26	31.47	32.8	34.26	35.83	37.53	39.34	41.28	43.34	45.53	47.83
30	25.84	26.42	27.09	27.87	28.75	29.73	30.82	32	33.28	34.67	36.16	37.75	39.44	41.24	43.13	45.13
Note: Exposure to full sunshine can increase HI values by up to 10°C																

Figure 1: Heat index chart [1] (temperature and relative humidity)

Fahrenheit	Celsius	Notes
80–90 °F	27–32 °C	Caution — fatigue is possible with prolonged exposure and activity
90–105 °F	32–41 °C	Extreme caution — sunstroke, heat cramps, and heat exhaustion are possible
105–130 °F	41–54 °C	Danger — sunstroke, heat cramps, and heat exhaustion are likely; heat stroke is possible
over 130 °F	over 54 °C	Extreme danger — heat stroke or sunstroke are likely with continued exposure

Table 1: Effects of the heat index (shade values)

We have investigated these empirical data and results of our investigation are presented in this paper. Out task was to deduce typical characteristics of the heat index for each this month and on its base to investigate the changes of a heat index in Georgia.

Why the task just mentioned is important? It is not possible to define exactly how hot was this or another month, which criteria to choose: maximum value of heat indexes, or the number of days with a high heat index, or an intensiveness, or a relativity of heat indexes, or may be something else? In this case the methods based on fuzzy set theory are very well suitable. We chose the methods of FEV [6, 7] and CFEV [10], compared them. The last one we have modified due to our practical demands.

The paper is organized as follows. In Section 2 we shortly describe the data available for the investigation, how they are organized and how a heat index is calculated. In Section 3 FEV and CFEV approaches are applied to our problem. Modification of CFEV is described. In Section 4 we compare these two approaches and the final remarks and conclusions are collected in Section 5.

2 Heat Index in Georgia

As was already mentioned in the Introduction, for our investigations we have used information about air temperature and air humidity of the four main cites if

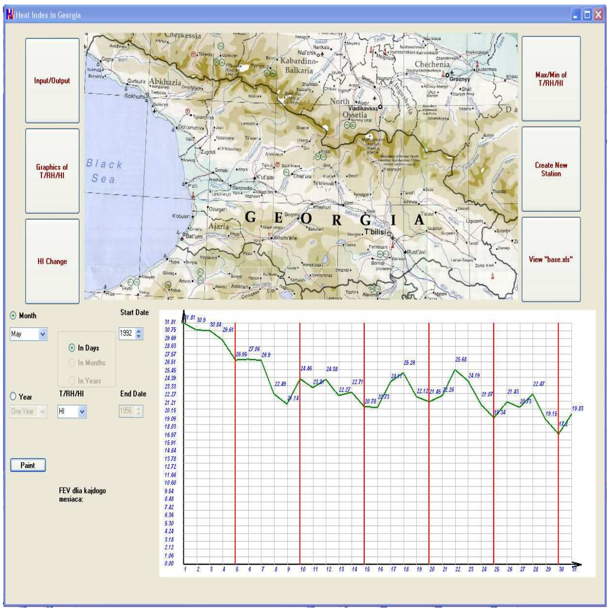


Figure 2: A screening of the front-page of the program with heat indexes on the low part of it.

Georgia: Tbilisi, Poti, Lentexi, Dedoplistskharo during two periods (from 1955 till 1970 and from 1990 till 2007) and only five months are taken into consideration (May, June, July, August, September). These data are a privilege of Ministry of Preservation of the Environment of Georgia and they are actively used in daily work of the Second Communication of Georgia to the United Nations Framework Convention on Climate Change (UNFCCC).

A special program was written to use these data flexibly. It means the data can be visualized graphically or in the form of tables, and simple manipulations - such as automatically calculation of heat index of each month, each year, each period; finding minimal and maximal values of the heat indexes of a period, etc., - are implemented (e. g., see Figure 2).

Notice that a simple dependence between air temperature and air humidity does not exist. In our investigation we have used the calculations based on the work of G. Steadman [8]: the author proposed a formula for approximating the heat index in degrees Fahrenheit. It is useful only when the air temperature is at least 80°F and the relative humidity is at least 40% (this is a case for Georgia):

$$HI = c_1 + c_2T + c_3R + c_4TR + c_5T^2 + c_6R^2 + c_7T^2R + c_8TR^2 + c_9T^2R^2$$

where
HI is a heat index (in degrees Fahrenheit);

T is an ambient temperature (in degrees Fahrenheit);

R is a relative humidity (in percent);

$c_1 = -42.379$; $c_2 = 2.04901523$; $c_3 = 10.14333127$; $c_4 = -0.22475541$; $c_5 = -6.83783 \times 10^{-3}$; $c_6 = -5.481717 \times 10^{-2}$; $c_7 = 1.22874 \times 10^{-3}$; $c_8 = 8.5282 \times 10^{-4}$; $c_9 = -1.99 \times 10^{-6}$.

The results of HI calculation for Tbilisi in August are presented in Figure 2 (down the map).

3 Fuzzy expected value

Having HI at hand, a question is how to characterize a period of time (week, month, etc.) - hot or not hot, normal, very hot and so on - thus, to give characteristics that used by people for estimations. Based on these characteristics we can estimated a tendency of HI developing in Georgia and the next step (do not considered in this paper) to estimate an influence of HI change on the human health.

One can use a maximal value of HI during the period, or a number of days with a high heat index, relative frequencies of heat indexes and some criteria else. The difficulty is that each of these characteristics do not reflect the influence of HI on the health in all complexity: for example, it has been established that the human health is sensitive to the heat waves. To capture the majority of the HI characteristics an approach based on fuzzy sets have been chosen. For estimation we have taken fuzzy expected value and clustering fuzzy expected value approaches.

The fuzzy expected value has been introduced by A. Kandel to indicate the most "typical" grade of the membership of a given fuzzy set and the corresponding element from the universe.

Let us briefly describe FEV [6, 7].

Consider a fuzzy set $A : X \to [0,1]$, $X = \{x_1, x_2, ..., x_n\}$ (discrete case). Let membership degrees be ordered as follows: $A(x_1) \leq A(x_2) \leq \dots \leq A(x_n)$. Let μ be a fuzzy measure defined over the subsets of X. Then the fuzzy expected value FEV, of A is defined as follows

$$FEV =_{def} \max\{A(x_i) \vee \mu(X_i)\} \quad (1)$$

where $X_i \subset \mathscr{P}(X)$, $\mathscr{P}(X)$ is a power set of X.

The $FEV(A)$ may be calculated as the median of the set $\{A_1, \dots, A_m, \mu_1, \dots \mu_{m-1}\}$, where $A_i = A(x)$, $1 \leq i \leq m$, $0 \leq A_1 \leq A_2 \leq \dots A_m \leq 1$ and, $\mu_i = \frac{1}{N}\sum_{j=i+1}^{m} n_j$, $1 \leq i \leq m-1$, $N = \sum_{i=1}^{m} n_j$ and n_i is a finite population.

For more details about FEV a reader is referred to the corresponding sources [6, 7].

Let us consider an example. Let HI (in Celsius) each day a month (see Figure 4(a)) be given. A fuzzy set **hot weather** is given as shown in Figure 3(a). Notice that a user can choose another type of a membership function. Using the fuzzy set **hot weather**, each HI of a day is assigned a membership degree in the fuzzy set **hot weather**. For example, HI of the second day (see Figure 4(a)) is $41°C$. The membership of this value in the fuzzy set **hot weather** due the diagram (Figure 3(a)) is 0.71. Analogically, for the whole month such fuzzification is done and the corresponding fuzzy set (let us call it **high HI**) is shown in Figure 4(b).

Notice, that the fuzzy set **hot weather** is divided into segments, and each segment has a linguistic name (*warm, very warm, hot, very hot*) and is expressed by different colors (see Figure 3(a)).

In the same way the fuzzy set **high HI** (see Figure 4(b)) is divided into different color-segments, that show the ranges of HI.

Results from Figure 4(b) are summarized in the Table 3, where abbreviation “Gr”, “Int”, “Mem.” are for groups, interval and membership, correspondingly.

The first column of the Table 3 is a linguistic name of a segment from the fuzzy set **hot weather**: *warm, very warm, hot, very hot*; the second column shows segments’ interval representation; the third column shows how many days have the HI from the corresponding segment. The last, forth, column represents membership degrees. Let us call the elements from the last column thresholds. As can be seen, we have four different thresholds: 0.22, 0.45, 0.56, 0.64.

The next step in the process of finding FEV for the fuzzy set **high HI** for this month is to check how many days are above each threshold (in percentage terms, see (1)), N is the number of all days, in our case $N = 31$. As can be seen, 31 days HI is above or equal to 0.22; 14 days HI is above or equal to 0.45; 4 days air temperature is above or equal to 0.56; 1 days HI is above or equal to 0.64. Pairing the data and rearranging it by increasing order of the membership grade, due to (1), we obtain the following four pairs:

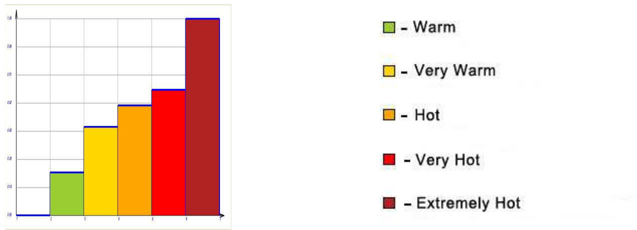
$$(0.22; 1.00)$$
$$(0.45; 0.45)$$
$$(0.56; 0.13)$$
$$(0.64; 0.03)$$

Now, the minimum value of each pair is $\min(0.22; 1.00) = 0.22$; $\min(0.45; 0.45) = 0.45$; $\min(0.56; 0.13) = 0.13$; $\min(0.64; 0.03) = 0.03$ and, therefore, following (1), the FEV, which is the maximum of all these minima, is

$$\max(0.22; 0.45; 0.13; 0.03) = 0.45$$

Gr.	Int.	Days	Mem.
warm	$[4.20°C - 26.60°C)$	17	0.22
very warm	$[26.6°C - 32.2°C)$	10	0.45
hot	$[32.2°C - 40.5°C)$	3	0.56
very hot	$[40.5°C - 54.4°C)$	1	0.64

Table 2



(a) Fuzzy set **hot weather** (b) Legend

Figure 3

Thus, the FEV is 0.45. From this result and the corresponding linguistic descriptions (see Figure 3) we can state that the fuzzy expected month temperature is *very warm.*

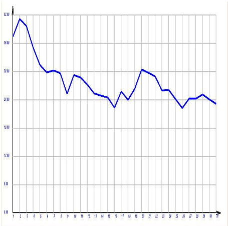
At Figure 5 the FEVs for Tbilisi in August for periods 1955-1970 and 1990-2007 are presented.

3.1 Some remarks concerning FEV

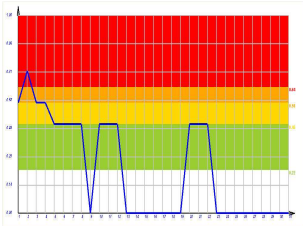
It has been suggested that FEV “may occasionally generate improper results”[10]. This can be explained because the computation of FEV suggests that the “opinions”of people and the “percentages” of people having those opinions are related by the definition of the fuzzy measure to produce a typical opinion. In the paper [10] authors critically view the approaches to the most typical value -FEV and weighted fuzzy expected value (WFEV) - “ to replace the FEV whenever it fails to function” [5]. They consider cluster fuzzy expected value (CFEV) as an alternative to WFEV. We modify CFEV introduced in [10] for our practical task and compare results of FEV and modified CFEV for the problem of heat index investigation in Georgia.

4 Clustering fuzzy expected value (CFEV)

Clustering fuzzy expected value tool is used for computing the most typical fuzzy expected value of a membership function in a fuzzy set [10]. CFEV is based on grouping of individual responses, that meet certain criteria, to clusters. Each cluster is consid-



(a) HI values



(b) Fuzzy set **high HI**

Figure 4

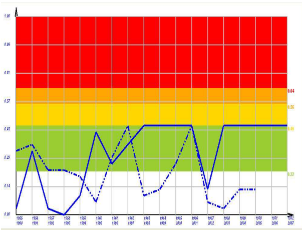


Figure 5: FEVs for Tbilisi in August for periods 1955-1970("-"line) and 1990-2007("-.-"line)

ered as a "super response" and contributes to the result proportionally to its relative size and the difference in opinion from the mean of the entire sample. In so doing, CFEV represents the opinion of the majority of the population, but it also respects the opinion of the minority [2, 10].

4.1 Description of CFEV and its modification

CFEV consists of two parts: forming of clusters and a combination of means of each cluster in a particular way. We propose our own approach for clustering.

Let s be the maximum allowable distance between the first and last elements in a cluster, c_{ij} be j element in cluster i, d is the maximum allowable distance between two consecutive elements within a cluster.

$$|c_{ij-1} - c_{ij}| \leq d \text{ (gap)} \tag{2}$$

$$|c_{i1} - c_{in}| \leq s \text{ (size)} \tag{3}$$

Clustering Algorithm

- Step 1. We fuzzify heat indexes during a month by means of the fuzzy set **hot weather** similar to the procedure described above: given temperature values (see Figure 4(a)) one assigns to temperature value a membership degree calculated from the fuzzy set shown in Figure3(a), and obtains fuzzification values as presented in Figure 4(b)
- Step 2. We calculate a difference between maximum and minimum values of the obtained fuzzy set **high HI** and assign this value to s (see 3). d is a value that is equal to $s : (N : 10)$, where N is equal to 30 or 31.
- Step 3. A cluster is constituted which elements are restricted by d. It means that an element is chosen for the cluster by inequality (2). This process continues till first and the last element do not satisfy inequality (3). In this case a cluster is built and the process is finished if there are no elements more.

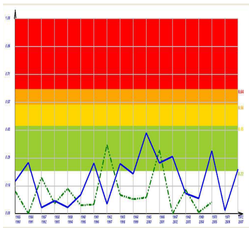
Notice, that by means of the data available and the algorithm described above HI-es in each month of are splited into 5-7 clusters.

To calculate CFEV we use the following formula [10]:

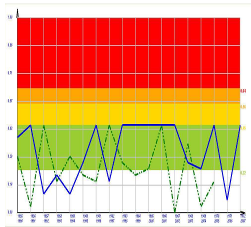
$$CFEV = W_A + \sum_{i=1}^{m} \left(\frac{N_i}{N}\right)^2 (W_{A_i} - W_A) \tag{4}$$

where W_{A_i} the mean of each cluster A_i, $W_A$is the mean of the entire set.

Notice that the fuzzy set is finite and the algorithm terminates. At the Figure 6(a) the CFEV for July in Tbilisi during the periods of time are presented.



(a) CFEV in July, Tbilisi



(b) FEV in July, Tbilisi

Figure 6: CFEV and FEV in July for Tbilisi

5 A comparison of FEV and CFEV

Let us now compare FEV and CFEV.

FEV shows the most typical value for a fuzzy set very well, but it is sensitive to the small change of the number of elements in the population[10]. FEV does not take into consideration the order of elements in the population.

Consider an example. In the case of heat index investigation assume that among 31 days in May, 12 days have heat index that belong to the risk group. Let us consider two cases. The first case is when the days with high heat index are disposed one after another

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	Tbilisi		Poti		Lentekhi		Dedoplis-Tsqaro	
	1955-1970	1990-2007	1955-1970	1990-2007	1955-1970	1990-2007	1955-1970	1990-2007
Warm	2002	2080	2339	2490	2095	2523	2146	2451
Very Warm	427	633	108	232	273	194	258	269
Hot	19	40	1	34	69	36	40	29
Very Hot	0	1	0	0	11	1	4	5
Extremely Hot	0	0	0	0	0	0	0	0
Total number of dangerous days	446	674	109	266	353	231	302	303

Table 3: Comparison of "dangerous" days in Georgia

and the second case is when between these days there the are normal days (i.e. the days with heat index that do not belong to the risk groups).

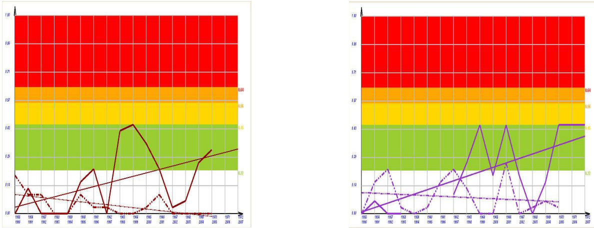
It is clear that both cases are dangerous for the human health because so many days are with high risk factor. But in the first case the density of days with high heat index is higher and, consequently, the negative influence of the case to the human health is bigger. FEV does not make difference between these two cases, it shows the same results for both cases, whereas the CFEV defines the first case as a more dangerous one then the second one.

The two values CFEV and FEV for Tbilisi, during the periods in July are graphically presented at the Figure 6.

6 Results of investigations for four different cities in Georgia

In the Table 3 one can see the HI-es for Tbilisi, Poti, Lentekhi, Dedoplistskharo during the periods classified into five groups. Comparing the behavior of HI for Tbilisi, Poti, Lentekhi, Dedoplistskharo, it has been concluded that most typical values of warm HI increase during the last 15 years, very warm HI increase in Tbilisi, Poti and Dedoplistskharo, but decrease in Lentekhi. Significant increasing of HI is observed in Tbilisi, whereas in Lentekhi the total number of dangerous days is decreased.

As an example, Figures 7(a) and 7(b) show FEVs and their tendencies in Dedoplistskharo and Poti in July during two periods of time (1955-1970 and 1990-2007).



(a) FEV in July, Dedoplistskharo

(b) FEV in July, Poti

Figure 7: FEV in July for Lentexi and Poti with tendencies

7 Conclusions

The paper concerns influences of global warming on a climate of Georgia and health of the population. In particular, one important parameter of a human thermal comfort - a heat index (which represents a combination of the air temperature and relative humidity) - has been investigated.

The research has been done in close contact to the Ministry of Preservation of the Environment of Georgia. The data -temperature and relative humidity-were available. The received information has been classified and ranged, and on its base the software has been created (by means of C #). The software enables to see both numerical and graphical representation of the heat indexes, and also the following operations are possible:

- by means of the radio-buttons, allocated on the map of Georgia, it is possible to select an interesting city in Georgia (Tbilisi, Poti, Lentekhi and Dedoplistskharo);
- during input of daily average temperature and relative humidity the heat index is automatically calculated;
- it is possible to consider a graphical image of a daily average air temperature, a relative humidity and the heat index of each month, each year and each period;
- to define and count up heat indexes for an appropriate interval (a month, or other periods);
- graphically view the tendency of the heat indexes of each month for the first and second periods for the purpose of forecasting;
- to select maximum and minimum value of temperatures, relative humidity and the heat index of the day, month, year and the period;

- some other functions.

On the basis of fuzzy methods, investigations of the available data have been conducted. In particular, two methods of a finding of the most typical value, fuzzy expected value (FEV) and clustering fuzzy expected value (CFEV) are used and further developed.

Both approaches have been updated for the research problem of the heat index and applied to the available data. Matching of results has shown, in which cases it is better to apply FEV, and in which CFEV. (Matching FEV and CFEV is spent, advantages and disadvantages of these methods for heat index investigation are shown).

The software and methods of the estimation of the results are used in daily work of the Second Communication of Georgia to the United Nations Framework Convention on Climate Change.

The results presented in the paper open new possibilities for the future research: for example, on the base of numerical data available to build fuzzy rules describing the influence of the heat index on the human health in Georgia.

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