Numerical computation of temperature field distribution in a DC machine

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Abstract: - In order to design correctly an electrical machine it is necessary that the machine cooling to be efficient and to know the values and the temperature distribution in this (in the rotor). To determine the temperature distribution, a thermal equivalent circuit is used for the heat transfer of machine. The thermal equivalent circuit is analogous to electric circuit, in which heat is generated by “current sources”, temperature being in analogy with voltage. In this paper is presented an analysis method of electrical circuits, more exactly nodal method, in order to compute the temperature field distribution necessary to the heating and ventilation systems of electrical machine. In the software application called NAPACI, authors analyse the temperature field distribution and the rate of heat transfer which are necessary for a complete study to range of the temperature in steady state. This analysis supposes the knowledge and localization of the temperature losses values, determination of the rate values of heat transfer and their distribution in different sides of electrical machine.

Key-Words: DC machine, steady-state, temperature field, rate of heat transfer, thermal resistance, generated power, consumed power, NAPACI programme

1 Introduction

In electric machines, the design of heat transfers has a high relevance, since the thermal rise of the machine eventually affects the output power of the machine. The problem of temperature rise is based on two aspects: in most motors, adequate with the phenomenon of convection of air, conduction through the fastening of the machines of high power density, also direct cooling methods can be applied. The winding of the machine is made of copper pipe through which the cooling fluid flows during the operation of the machine. The thermal field within electric machines can be analyzed with equations based on the classical theory from the heat transfer.

At the second aspect is that the distribution of heat in different parts of the machine has to be considered. This is a problem of heat diffusion, which is a complicated three dimensional problem with several difficult details such as the question of heat transfer from the copper conductor over the insulation to the stator frame. When the distribution of losses in different parts of the machine and the heat removal power are known, then the distribution of heat in machine can be computed.

Also, the control problems to an electric machine as electrical as thermal should be regarded in a different way to research how to make control systems specialist. In the presence of variations (due to temperature and/or saturation) the response can disrupts in a different operation point [1].

2 Computing methods for heating system of electric machines

Modeling of heating systems of electric machines leads to obtaining some complex equivalent circuits of large sizes. Analysis of these circuits in steady state regime and in transient conditions requires efficient methods for their solution. In order to write the equations of the electric circuits for the purpose their simulation, the modified nodal method is one of the most used methods due to its flexibility. Elaboration of an efficient method in order to analyze the nonlinear resistive electric circuits has a relevant importance which results from the substitution of dynamic circuit elements with discrete resistive models associated to an implicit integration algorithm [2].
2.1 Nodal method

In the nodal method, nodal points are defined at locations in the circuit where there are unknown voltages. The known independent voltage sources and current sources are the independent variables. At these points Kirchhoff’s first law of circuit analysis is applied, namely, the sum of the currents at a nodal point is zero. The currents are then formulated in terms of the dependent voltages and the independent voltages/currents [3]. The linear equations for the voltages are solved and the currents in the various branches are derived from these voltages.

The structure of circuits where the nodal method is applied has the following circuit elements:
- Resistors, magnetically noncoupled coils and linear capacitors;
- Nonlinear resistors controlled in voltage – \( R_0 \);
- Nonlinear capacitors;
- Independent current sources;
- Independent voltage sources which do not form alone a branch;
- Current sources driven in voltage or nonlinear – \( j \epsilon (e) \).

Since the resistive models of nonlinear coils contain current-driven nonlinear resistors, their presence in the circuit makes it impossible to apply the nodal method. Coils and capacitors contribution in order to formulate the nodal equations in transient conditions is essentially different in comparison with continuous current regime. After the resistive model is built the nodal method can be applied by writing the system of nodal equations which is solved by means of Newton-Raphson algorithm [4].

2.1.1 Modified nodal method

The modified nodal method does not require restrictions at the circuit structure which is analyzed. The difference between the nodal method and the modified nodal method is that the modified nodal method includes the currents of current-driven nonlinear coils and the currents of magnetically coupled linear coils as additional variables [2]. Non-coupled linear coils do not introduce additional variables.

The general formulation of the modified nodal equation at time moment \( t + 1 \) is:

\[
\begin{bmatrix}
C^{(k+1)}_{m,n-1} & P^{(k+1)}_{m,n-1} \\
-J^{(k+1)}_{n,m} & R^{(k+1)}_{m,m}
\end{bmatrix}
\begin{bmatrix}
q^{(k+1)}_{n-1,m} \\
q^{(k+1)}_{m,m}
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

(1)

where:
- Superscript \( k+1 \) represents the time moment at which is related to the system of equations.
- \( B^{(k+1)}_{m,n-1} \) has dimensionless elements; this includes elements -1, 0, +1 and transfer factors in current of current-driven sources;
- \( A^{(k+1)}_{m,n-1} \) has dimensionless elements; this includes elements -1, 0, +1 and transfer factors in voltage of voltage-driven sources driven:
- \( R_{m,n} \) has elements with resistance dimension; this includes transfer resistances of current-driven voltage sources and resistances of current-driven resistors;
- \( e_0 \) has e.m.f. of ideal sources independent of voltage and off-load voltages resulted from linearity of resistor characteristics driven in current.

The modified nodal equation is rewritten at each time moment. We assume that the circuit has a number of nonlinear coils driven in flux, \( X^{L_p} \) and a number of nonlinear capacitors driven in charge, \( X^{C_q} \). Additional variables are the vectors \( \varphi \) with \( X^{L_p} \) components and \( q \) with \( X^{C_q} \) components which are taken together with independent variables of these elements.

General form of the system matrix, as in Eq.(1) will be written with \( X^{L_p} + X^{C_q} \) lines and as much as columns. These types of elements have a low interest from applications point of view [3],[5].

2.2 Thermal-electrical analogy

As found in the referenced literature the thermal-electrical analogy is useful in analysis of several steady heat transfer problems from property measurement to modeling. The analogy has some limitations including the non-linearities which there are between voltage and current at extremely high and low values. In the literature, there are some special cases where the analogy doesn’t quite fit the actual physics of a problem. These special cases suppose a range of large or small values of current and voltage as well as heat transfer problems at the nano-scale level. Therefore, modifications can be made to still find the relationship helpful in the analysis [6], [7], [8]. Temperature is the driving force or potential for heat flow. The flow of heat over a “heat flow path” should then be governed by thermal potential difference across the path and the resistance of it. This suggests that heat flow is analogous to electric current flow [8]. In the development of his law for electrical circuits, Ohm performed experiments that modeled Fourier’s law of heat conduction. An analogy between Ohm’s law for electric circuits and Fourier’s law of heat conduction can be observed. Considering steady flows, the electric current can be written:

\[
I = \frac{\Delta V}{R_e} = \frac{V_1 - V_2}{R_e}
\]

(2)

where
- \( R_e = \frac{L}{\sigma_e \cdot A} \) is the electric resistance, [\( \Omega \)]
- \( V_1 - V_2 \) is the voltage difference across the resistance, [\( V \)]
σ_e is the electrical conductivity, [S/m].

Fourier’s law can be written as:

\[ Q_{\text{cond}} = \frac{\Delta T}{R_{t,\text{cond}}} \]  

(3)

where:

- \( Q_{\text{cond}} \) is the rate of conduction heat transfer, [W].
- \( \Delta T \) is the temperature difference between the surfaces of a slab, [K].
- \( R_{t,\text{cond}} = \frac{L}{k \cdot A} \) is the thermal resistance called conductive or internal resistance of a plane wall, [K/W].
- \( k \) is the thermal conductivity, [W/(mK)].
- \( L \) is the wall thickness, [m].
- \( A \) is the wall surface, [m²].

Thus, the rate of heat transfer through a layer is analogue to the electric current, the thermal resistance is analogue to electrical resistance, and the temperature difference is analogue to voltage difference across the layer (Fig.2), [6].

The thermal resistance of a medium depends on the geometry and the thermal properties of the medium. Since heat is understood to transfer through lattice vibrations between adjacent atoms, and electricity is conducted by free valence electrons of atoms, it may seem intuitive that heat and electrical conduction are analogous [7].

Other expressions for the thermal resistance concept are presented below.

Consider convection heat transfer from a solid surface area \( A \) and temperature \( T_s \) to a fluid whose temperature far from a surface is \( T_\infty \).

Newton’s law for thermal convection is:

\[ Q_{\text{conv}} = \frac{T_1 - T_\infty}{R_{\text{conv}}} \]  

(4)

where \( R_{\text{conv}} = \frac{1}{h \cdot A} \) is the thermal resistance of the surface against heat convection or the convective resistance of the surface (Fig.2) [K/W], and \( h \) is the convection heat transfer coefficient, [W/(m² K)].

Then wall is surrounded by a gas, the radiation effects which were ignored, can be significant and may need to be considered. The rate of radiation heat transfer between a surface of emissivity \( \varepsilon \) and area \( A \) at temperature \( T_s \) and the surrounding surfaces at some average temperature \( T_{\text{surr}} \) can be expressed as:

\[ Q_{\text{rad}} = \varepsilon \cdot \sigma \cdot A \cdot (T_s^4 - T_{\text{surr}}^4) = h_{\text{rad}} \cdot A \cdot (T_s - T_{\text{surr}}) \Rightarrow Q_{\text{rad}} = \frac{T_s - T_{\text{surr}}}{R_{\text{rad}}} \]  

(5)

where \( R_{\text{rad}} = \frac{1}{h_{\text{rad}} \cdot A} \) is the thermal resistance of a surface against radiation or radiation resistance, [K/W], and \( h_{\text{rad}} = \varepsilon \cdot \sigma (T_s^2 + T_{\text{surr}}^2) \cdot (T_s + T_{\text{surr}}) \) is the radiation heat transfer coefficient.
3 DC machine description

In Fig.4 is shown the transversal section by a core pole of a direct current machine for which is considered a constant temperature in whole coil mass, taking into account the medium value of temperature. The temperature drop is practically zero as in pole mass as in machine core, for core side which contributes to the cooling of magnetizing coil and corresponds to the angle \( \alpha = \frac{\pi}{2p} \), where \( \alpha \) is the angle between the symmetry axes between a main pole and neighbor auxiliary poles [2].

Real thermal scheme of heat transfer to a coil from a pole of the electrical machine is shown in Fig.5 where are indicated:
- Joule losses from one coil, \( P_{\text{cub}} \);
- thermal resistance from coil to core, \( R_1 \) corresponding to field 1;
- thermal resistance from coil to pole by pole piece, \( R_2 \) which corresponds the field 2;
- thermal resistance from coil to pole by pole body, \( R_3 \) corresponding the field 3;
- thermal resistance from core to pole, \( R_4 \) taking into account that core-pole junction is about 0.1 mm corresponding to the field 4;
- thermal resistance from pole to air through air gap, \( R_5 \) corresponding the field 5;
- thermal resistance from pole to the air between poles, \( R_6 \) corresponding the field 6;
- thermal resistance from core to the air inside the electrical machine, \( R_7 \) corresponding the field 7;
- thermal resistance from core to environment, \( R_8 \) corresponding the field 8;
- thermal resistance from coil to the air, \( R_9 \) corresponding the field 9 [2], [4], [9]-[11].

3.1 Computation of thermal resistances

In order to find the equivalent total thermal resistance, we need the intermediary thermal schemes as shown in Fig.6.
Thermal resistances for determination of the magnetizing coil heating from Fig.5 correspond to the following equations:

\[
R_A = \frac{R_3 R_5}{R_3 + R_5}, \quad R_B = R_1, \quad R_C = R_4
\]

\[
R_D = \frac{R_2 R_6}{R_3 + R_5}
\]

\[
R_E = \frac{R_3 R_8}{R_7 + R_8}
\]

\[
R = R_A + R_B + R_C + R_D + R_E
\]

After transformation of the triangle \( R_C, R_D \) and \( R_E \) from Fig.6a in star \( R_x, R_y \) and \( R_z \), equations for thermal resistances are:

\[
R_x = \frac{R_C R_D}{R_C + R_D + R_E}
\]

\[
R_y = \frac{R_C R_E}{R_C + R_D + R_E}
\]

\[
R_z = \frac{R_D R_E}{R_C + R_D + R_E}
\]

\[
R = R_A + R_x, \quad R_B = R_y + R_y
\]

According to the equations (8) and the equivalent scheme as shown in Fig.6b the total equivalent resistance can be determined as follows:

\[
R_T = \frac{R_1 R_{11} + R_z}{R_1 + R_{11} + R_z}
\]

After the total equivalent resistance \( R_T \) and by means of Eq.3 the temperature drop is:

\[
\Delta T_{cob} = Q \cdot R_T
\]

Therefore, it can be computed the heating of magnetizing coil in regard to the cooling air from machine.

The intermediary thermal schemes in order to find the heating of magnetizing coil are shown in Fig.6.

### 3.2 Processing of data by means of NAPACI

Authors have used the NAPACI Programme (Nodal Analysis Programme of Analogue Circuits) which is written in C++ and uses for the numerical integration of ordinary differential equations, the first order generalized regressive method.

Dynamic circuit elements are substituted with discrete resistive circuits associated to this numerical method, and for this analysis of electric circuits, the NAPACI programme uses the modified nodal method.

Circuit description is made by a file of netlist type with extension .nln having the following structure:

- Number of branches;
- Number of nodes.

It follows a set of \( l \) lines, where \( l \) is the number of circuit branches, which describe the circuit branches.

NAPACI creates two output files:

- In the first file, the voltages and the currents of branches are written, as well as the input power at the terminals of branches for the first ten time steps and the last nine time steps. All these data are necessary for verifying if the integrating process is right, more exactly, at each time step the total power received to circuit at the terminals of branches has to be zero.
- The second output file called result.dat gives the values of currents and voltages of circuit branches at each integrating step.

Therefore, in Fig.7 is shown the equivalent thermal scheme for Fig.6 in order to find the heating of magnetizing coil. In this software application, it is analyzed the temperature field distribution and the rate of heat transfer which are necessary for a complete study to range of the temperature in steady state.
It was presented the systematic computing procedure of heat transfer in steady state, by means of equivalent electric schemes with concentrated losses, for a coil on a pole of the direct-current machine. Therefore, equivalent electric schemes with concentrated losses, for a coil on a pole of the direct-current machine. Therefore, the heating systems of electric machines.

3.3 Discussions
Solving of thermal schemes was made in a similar way with respect to solving the electrical schemes taking into account the connections between the thermal resistances (series, parallel or mixed). Results obtained by simulation with NAPACI were compared with other results obtained with SPICE programme, observing that the differences between these results are negligible (practical the results are identical). This validates the NAPACI use in computation of the heating systems of electric machines.

It was presented the systematic computing procedure of heat transfer in steady state, by means of equivalent electric schemes with concentrated losses, for a coil on a pole of the direct-current machine and for the distributed windings of the same machine. It was computed: thermal resistances for all heat transfer processes as well as the structure of equivalent thermal network.

4 Conclusions
The modified nodal method does not require restrictions of structure of the analyzed, and can be used as the linear circuit analysis as nonlinear circuit. At the modified nodal method, the additional variables are not necessary, and the vector of the constant terms contains the load at the last time moment which represents the initial condition for the actual time moment. The sizes are modified during the inner iterative cycle due to the pass from a segment to another of the voltage-load characteristic.

In this paper, it is presented a computing systematic procedure of heat transfer in steady state by means of equivalent electric schemes with concentrated losses, for a coil on a pole of the direct – current machine. Therefore, the thermal resistances and the rates of heat transfer were computed the structure of equivalent thermal network being presented.

References: