A Method for Obtaining the Electric Arc Model Parameters for SF$_6$ Power Circuit Breakers

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Abstract: In this work, a new method for calculating the parameters of electric arc model for SF$_6$ power circuit breaker is proposed. The methodology consists in the optimization of a theoretical function with respect to a group of model parameters in order to achieve a better approximation to a set of experimental data. The theoretical function is an asymptotic solution of the equation for the electric arc model obtained in a selected period of time. The developed method is applied to calculate the parameters of an arc model previously published in the literature. By using the variation of the cooling power, an improved electric arc model is also obtained including its parameters. The new model has a very compact form and exhibit a good correlation between voltage curves measured and calculated.

Key-Words: Black box modeling, Mayr and Cassie models, Optimization, SF6 power circuit breaker.

1 Introduction

The performance of high-voltage circuit breakers during current interruption has been a topic of interest for many years. A successful operation implies that the circuit breaker is capable of interrupting currents in a short period of time. In order to assess circuit breaker capability during current interruption a test program is required. Based on these tests, many electric arc models have been proposed [1-10].

The developed models provide additional information on the behavior of the electric arc under different power system operating conditions. This reduces test requirements in power breakers with significant time and economic savings.

At present time, black box models are commonly used to describe the interaction of the arc with the electric network during a current interruption process. Usually, black box models are modifications of the models proposed by Cassie and Mayr [11,12], which provide a qualitative description of the phenomena in the low and high current regions respectively. These models include a differential equation that depends on a set of parameters $x_1, x_2, ..., x_k$ which should be obtained from experimental data. By selecting a suitable set of arc parameters it is possible to obtain a better approximation to the arc dynamics described by experimental data. However, increasing the number of parameters can lead to difficulties for calculating their magnitudes. Therefore the number of parameters should be reasonable and sufficient to predict successfully the current interruption capabilities of power circuit breaker.

Recently, new black box models for a SF$_6$ circuit breaker have been proposed (250 kV/50kA/50Hz/single-unit SF6 puffer circuit breaker) [1, 10]. In reference [1], it is considered a linear relation between the cooling power and the entry power, i.e., $P_0 + P_1u_i$. The corresponding differential equation is of the form:

$$\frac{d \ln g}{dt} = \frac{1}{\tau} \left( \frac{u_i}{\max(U_{arc}, P_0 + P_1u_i)} - 1 \right)$$

(1)

where:

- $g$ arc conductance
- $u$ arc voltage
- $i$ arc current
- $P_0$ cooling power constant in W
- $P_1$ cooling constant
- $\tau$ time constant
- $U_{arc}$ constant arc voltage in the high current area.

From Fig. 1, despite the fact that for electrical currents above the critical current $i_{cr} = 10^4A$, i.e. the current for which the sudden voltage drop occurs, the voltage curves calculated by means of equation (1) show a significant difference with experimental oscillograms, equation (1) has been considered in [1] appropriate to describe the arc dynamics before current zero. To obtain this conclusion, in the computer simulations the authors took arbitrarily an initial voltage almost 550V below its real value.
In [10], an improved arc model that successfully describes the dynamic arc behavior in the high and low current regions before current zero is proposed. However, this model is difficult to implement computationally.

In the first part of this work the aim is to develop an improved arc model for SF6 power circuit breakers capable of predicting the voltage behavior with a great accuracy for a wide range of currents. This model should be mathematically simple and contains only the minimum necessary number of arc parameters.

The second part addresses the development of a methodology for the calculation of the parameters for the model taking as a reference experimental data. This is a very important stage in the modeling process because the accuracy of the model is related with the accuracy for determining these parameters.

The first problem is solved by varying one of the parameters in model (1), which is taken as the starting point for the analysis. The second one is solved by obtaining an asymptotic solution for the voltage dynamics in a certain segment of time $[t_s, t_f]$, with the subsequent optimizing of this solution with respect to the parameters of the model. As a result, the developed model describes the electric arc dynamics successfully, giving an excellent correlation between theoretical and experimental voltage curves for a wide range of currents.

### 2 Improved Arc Model

Model (1) can be used to obtain an improved arc model. Thus, for currents $i < i_{cr}$ the cooling power is a linear function of the electric power input and equation (1) takes the form:

$$\frac{d \ln g}{dt} = \frac{1}{\tau} \left( \frac{ui}{P_0 + P_1 ui} - 1 \right)$$  \hspace{1cm} (2)

In this paper we take equation (2) as the starting point to develop an improved arc model.

The improved model should describe the irregular behavior of the electric arc, which consists in the sudden voltage drop as shown in Fig. 1. In reference [1] it was mentioned that the “exact” cooling power curve shows a small deviation from the linear behavior around the electric power input $1.5 \times 10^6 W$. Such a non-linearity can be the cause of the irregular behavior of the arc voltage, i.e., the cooling power should be taken in the form $P_1 + (P_1 + \delta P_1)ui$, where the small variation $\delta P_1$ of the parameter $P_1$ is dependent on the arc voltage and current. As a result, the arc model becomes:

$$\frac{d \ln g}{dt} = \frac{1}{\tau} \left( \frac{ui}{P_0 + (P_1 + \delta P_1) ui} - 1 \right)$$  \hspace{1cm} (3)

The variation of the left-hand side of equation (2), caused by the variation of the parameter $P_1$, has the form:

$$\delta \frac{d \ln g}{dt} = \frac{d \ln g}{dt}_{\text{reg}} - \frac{1}{\delta P_1} \left( \frac{P_0}{ui} + P_1 \right) \frac{1}{ui} \left( \frac{P_0}{ui} + (P_1 + \delta P_1) \right)$$  \hspace{1cm} (4)

where the derivative $\frac{d \ln g}{dt}_{\text{reg}}$ corresponds to the regular electric arc behavior given by model (2). The maximum contribution of the small variation $\delta P_1$ to the voltage dynamics is reached if the right-hand side of equation (4) is the maximum, i.e.,

$$\frac{1}{\left( \frac{P_0}{ui} + P_1 \right)} \frac{1}{ui} \left( \frac{P_0}{ui} + (P_1 + \delta P_1) \right)$$  \hspace{1cm} (5)

Substituting this result into equation (4) yields:

$$\delta P_1 \approx \frac{\delta P_1^2}{1 + \delta P_1^2} \frac{1}{\frac{du}{dt}_{\text{reg}}} \left( \frac{1}{\frac{du}{dt}} - \frac{1}{\frac{du}{dt}_{\text{reg}}} \right)$$  \hspace{1cm} (5)

To obtain the form of the variation $\delta P_1$ as a
function of arc voltage and current we should model the sudden voltage drop. In this case the voltage is considered as a function of the current, i. e., \( u = u(i) \).

Then:

\[
\frac{du}{dt} = \frac{di}{dt} \frac{du}{di}
\]

The derivative \( \frac{du}{dt} \big|_{|i|} \) around the current \( i_{cr} \), where the sudden voltage drop occurs, behaves like the Dirac delta-function \([10,14,15]\). Then, the derivative \( \frac{1}{\sigma} \frac{du}{dt} \big|_{|i|} \) can be approximated by the normal distribution:

\[
\frac{du}{di} \bigg|_{|i|} = \frac{\Delta u}{\sigma \sqrt{2\pi}} e^{-\frac{(i-i_0)^2}{2\sigma^2}}
\] (6)

where \( \Delta u \) is the magnitude of the voltage drop which can be obtained directly from the experimental data as shown in Fig. 2.

The derivative \( \frac{1}{u} \frac{du}{dt} \bigg|_{\text{log}} \) is much smaller then \( \frac{1}{\sigma} \frac{du}{dt} \bigg|_{|i|} \) and therefore can be neglected. Substituting (6) into (5), it can be obtained:

\[
\frac{di}{dt} \bigg|_{|i|} = \Delta u / \sigma \sqrt{2\pi}
\]

Substituting (8) into (7), we finally find the form of the variation \( \delta P_1 \) as a function of the arc voltage and current. As a result the model equation for this SF\(_6\) power circuit breaker takes the form:

\[
d \ln g = \frac{1}{\tau} \left( \frac{iu}{P_0 + \frac{P_1u^2}{P_2e^{P_3(u-v_{i_0})} + u}} - 1 \right)
\] (9)

This mathematical model has three additional parameters: \( P_2 \), \( P_3 \) and \( i_{cr} \). The current \( i_{cr} \) can be obtained directly from Fig. 1. The parameters \( P_2 \) and \( P_3 \) can be calculated according to the formulations:

\[
P_2 = \frac{\tau_1}{\cot \psi}, \quad P_3 = \frac{\pi}{\left( \frac{di}{dt} \Delta u \tan \psi \right)^2}
\] (10).

### 3 Asymptotic Solution

The proposed methodology for parameters calculation consists in the optimization of a theoretical function \( u(i, P_2, P_3, \tau) \) with respect to the set of parameters of the model \( P_0, R_1 \) and \( \tau \), with the purpose of obtaining the best approach to a set of experimental values \( u_1, u_2, \ldots, u_n \) measured in the corresponding moments of time \( t_1, t_2, \ldots, t_n \).

Nevertheless, the theoretical voltage, i. e., an analytic solution of equation (9) with respect to the arc voltage, is required.

Due to the complexity of equation (9), the determination of analytic results has to be carried out for certain ranges of time in which some terms of the equation can be neglected. Firstly, the variation of the cooling power obtained above is significant only for the small region near the current \( i_{cr} \). Outside this region, equation (9) approximately coincides with equation (2). Therefore, equation (9) is solved in a time segment \( [t_a, t_b] \) which does not contain the voltage drop.

Equation (2) can be represented as a nonlinear differential equation with respect to the electric
power input \( w \):

\[
\frac{dw}{dt} + q(t)w = Q(w(t)) \tag{11}
\]

where:

\[
q(t) = \frac{1}{\tau} \left( \frac{1}{P_1} - 1 \right) - \frac{2}{\tau} \frac{di}{dt}
\]

\[
Q(w) = \frac{P_0}{P_1 \tau} \left( 1 - \frac{P_0/P_1 w}{1 + P_0/P_1 w} \right)
\]

and the current \( i(t) \) is a linear function of time, as shown in Fig. 1, i.e., \( di/dt = -\alpha \). By selecting the circuit impedance higher than the arc resistance, it is always possible to reach the arc current behavior being linear or almost linear, Fig. 1. Equation (11) can be transformed into the following integral equation:

\[
u(t) = \frac{u_a}{i_a} i(t) e^{-\alpha t} \left( \frac{1}{\tau} \right) \]

\[
+ i(t) e^{-\alpha t} \left( \frac{1}{\tau} \right) \int_{i_a}^{i(t)} \frac{2 \lambda w(t)}{i(t)} \frac{1}{\tau} \frac{1}{R} \frac{du}{dt} dt_1
\]

Let also the time segment \( [t_a, t_b] \) be such that the inequality

\[
0 < \frac{P_0}{P_1 w(t)} < \varepsilon \ll 1 \tag{13}
\]

is satisfied, i.e., the electrical power input \( w(t) \) is high enough. This assumption should be verified later. Then we can write:

\[
Q(w) = \frac{P_0}{P_1 \tau} \left( 1 + O(\varepsilon) \right)
\]

As a result the following asymptotic solution of equation (12) can be obtained:

\[
u(t) = \frac{u_a}{i_a} i(t) e^{-\alpha t} \left( \frac{1}{\tau} \right) \]

\[
+ i(t) e^{-\alpha t} \left( \frac{1}{\tau} \right) \frac{P_0}{P_1 \tau} \left( 1 + O(\varepsilon) \right) \int_{i_a}^{i(t)} \frac{e^{-\alpha t} \left( \frac{1}{\tau} \right)}{i(t)} dt_1
\]

where \( i_a \) and \( u_a \) are the current and voltage at the moment of time \( t_a \). Equation (13) has been obtained for an arbitrary current \( i(t) \). For the current shown in Fig. 1 it is necessary to substitute \( i_a - \alpha(t-t_a) \) into (14). Finally we obtain the following approximate solution:

\[
u(t, P_0, P_1, \tau) = \left( i_a - \alpha(t-t_a) \right) e^{-\alpha t} \left( \frac{1}{\tau} \right) \]

\[
\times \left[ e^{-\alpha t} \left( \frac{1}{\tau} \right) - e^{-\alpha t} \left( \frac{1}{\tau} \right) \left( \frac{1}{P_1} - 1 \right) e^{-\alpha t} \left( \frac{1}{\tau} \right) \right]^{i(t)}_{i_a - \alpha t}
\]

(15)

\[
E_{i(z)} \text{ is the exponential integral function:}
\]

\[
E_{i(z)} = \int_{-\infty}^{\infty} e^{-z} dx
\]

where the principal value of the integral is taken.

### 4 Optimization

The least squares method can be used for the optimization of function (15). The method consists in the minimization of the function:

\[
L(P_0, P_1, \tau) = \sum_{k=1}^{n} [u(t_k, P_0, P_1, \tau) - u_k]^2
\]

(16)

The minimization of the function (16) can be significantly simplified if one of the unknown parameters is expressed through other parameters. The necessary information to eliminate one of the parameters can be obtained from the experimental results shown in Fig. 1. As a reference point it is possible to take the local maximum of the voltage curve \( (t_m, u_m) \), where \( t_m \) is the time moment in which the voltage arrives to its maximum \( u_m \). Substituting \( du/dt \bigg|_{t_m} = 0 \) into equation (2) we can express the parameter \( P_0 \) through the parameters \( P_1 \) and \( \tau \):

\[
P_0(P_1, \tau) = t_m u_m \left( 1 - P_1 + \frac{d \ln(1)}{dt} \right)_{t=t_m}
\]

\[
= u_m \left( (t_m - \alpha t_m) - \left( \frac{i_0 - \alpha i_m}{i_0 - \alpha(t+1) + P_1} \right) \right)
\]

(17)

By substitution of equation (17) into (15) we obtain
the analytic solution \( u(t, P_1, \tau) \) as function of two parameters, \( P_1 \) and \( \tau \). The solution \( u(t, P_1, \tau) \) should be optimized, i.e., the following function should be minimized:

\[
L(P_1, \tau) = \sum_{k=1}^{n} \left[ u(t_k, P_1, \tau) - u_k \right]^2
\] (18)

The minimization of the function (18) is based on solving the following system of equations:

\[
\begin{align*}
\frac{\partial L(P_1, \tau)}{\partial P_1} &= 0 \\
\frac{\partial L(P_1, \tau)}{\partial \tau} &= 0
\end{align*}
\]

with respect to unknown variables \( P_1 \) and \( \tau \). In Fig. 3 the minimum of the function \( L(P_1, \tau) \) is shown. The parameters involved in the function (15) are shown in Table I.

The optimization is fulfilled with the use of the Mathematica 5 software. The results of the optimization is presented in Table II.

### Table I. Parameters obtained from measurements, Fig. 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_a )</td>
<td>42.7</td>
<td>( \mu s )</td>
</tr>
<tr>
<td>( t_b )</td>
<td>85.0</td>
<td>( \mu s )</td>
</tr>
<tr>
<td>( u_a )</td>
<td>1171.87</td>
<td>( V )</td>
</tr>
<tr>
<td>( i_a )</td>
<td>946.0</td>
<td>( A )</td>
</tr>
<tr>
<td>( u_m )</td>
<td>2328.12</td>
<td>( V )</td>
</tr>
<tr>
<td>( t_m )</td>
<td>88.0</td>
<td>( \mu s )</td>
</tr>
<tr>
<td>( \alpha = -di/dt )</td>
<td>20</td>
<td>( A/\mu s )</td>
</tr>
<tr>
<td>( \Delta u )</td>
<td>500</td>
<td>( V )</td>
</tr>
<tr>
<td>( \psi )</td>
<td>0.00259</td>
<td>( )</td>
</tr>
</tbody>
</table>

### Table II. Parameters for the model (9).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i_{cr} )</td>
<td>10(^i)</td>
<td>( A )</td>
</tr>
<tr>
<td>( P_3 )</td>
<td>26.788</td>
<td>( V )</td>
</tr>
<tr>
<td>( P_4 )</td>
<td>4.6923 \times 10(^4)</td>
<td>( A^2 )</td>
</tr>
<tr>
<td>( P_0 )</td>
<td>3413.35</td>
<td>( W )</td>
</tr>
<tr>
<td>( P_1 )</td>
<td>0.999275</td>
<td>( )</td>
</tr>
<tr>
<td>( \tau )</td>
<td>0.069365</td>
<td>( \mu s )</td>
</tr>
</tbody>
</table>

5 Validation of the Model

The model (9) can be validated by means of two ways. The first one considers the asymptotic solution (15). This solution has been obtained under the assumption of (13) in the segment \([t_a, t_b]\) where \( t_a = 42.7 \mu s \) and \( t_b = 85 \mu s \). Fig. 4 shows two voltage curves, the asymptotic solution (continuous line) and the voltage calculated by means of model (9) under the assumption of a linear behavior of the electric current (dashed line). The parameters used in the model are given in Table II. The comparison of both curves shows clearly that the asymptotic solution (15), the calculated voltage (9) and the measured voltage (Fig. 1) are very well correlated in the time range of interest \([t_a, t_b]\). This is clear evidence that the asymptotic solution (15) is obtained with a very good accuracy, i.e., the assumption (3) is satisfied in the time segment \([t_a, t_b]\).

From Fig. 5, the second validation is made by comparing the results of model (9) with the irregular behavior of the measured voltage shown in Fig. 1 for \( i > i_{cr} \). It can be noticed that model (9) successfully describes the sudden voltage drop at \( t = 40 \mu s \). Moreover, the calculated voltage in Fig. 5 has an excellent correlation with the measured voltage shown in Fig. 1 for all time ranges.

Fig. 6 shows the behavior of the cooling power versus the electrical power input in the region of the electric arc voltage drop. It can be noticed a small deviation of the cooling power from the linear behavior. Such a deviation from the linear behavior can be observed in the corresponding experimental
cooling power curve published in [1]. In this sense, the obtained shape of the cooling power curve is in a good agreement with experimental data.

![Cooling Power Curve](image)

Fig. 4. Arc voltage calculated with the model (9), and the asymptotic solution (13) in the time range \([t_a, t_b]\).

![Arc Voltage and Current](image)

Fig. 5. Arc voltage and current calculated with the model (9).

![Cooling Power vs Electrical Power](image)

Fig. 6. Calculated cooling power versus electrical power input. A small deviation from the linear behavior.

![d ln g/dt Curve](image)

Fig. 7. \(d \ln g/dt\) curve calculated with the model (9).

6 Conclusion
In this paper a new method for obtaining the parameters of electric arc for SF\(_6\) power circuit breakers is presented. The methodology consists in solving analytically the model equation and the optimization of the solution with respect to a group of parameters in order to achieve a better approximation to a set of experimental data. Nevertheless, the solution of the model equation is complicated, especially because the arc behavior is closely related with the electrical circuit. Therefore the solution can be obtained for the regions where one or more terms (or parameters) can be neglected and the equation can be solved asymptotically. Such an asymptotic solution is optimized.

The developed methodology is applied to a SF\(_6\) power circuit breaker. The asymptotic solution (15) of the model equation (9) for the arc voltage is obtained for a selected period of time \([t_a, t_b]\). Then, the asymptotic solution (15) is optimized and the arc parameters calculated.

The model (9) is obtained by means of variation of the constant \(P_1\) of the linear cooling power \(P_0 + P_1iu\) introduced in reference [1]. In the proposed model (9), the cooling power depends on the arc current and voltage, as shown in the following equation:

\[
P(i,u) = P_0 + \frac{P_1iu^2}{P_2e^{-\frac{P_0(i-u)}{P_2}} + u}
\]

It has been shown in Fig. 5 that this approach to the cooling power is in a good agreement with the experimental data published in [1].
The new model has a very compact form and the results show a good correlation between the theoretical voltage curves and the experimental oscillograms. The model given by (9) describes in a very efficient way the irregular behavior of the voltage when the current reaches the value $i_{cr} = 1\text{kA}$. In comparison with the model obtained in [10] the model (9) has only two additional parameters, $P_2$ and $P_3$, equation (9) is quite suitable for simulation.

References: