Using of Clustering Techniques in Optimal Placement of Phasor Measurements Units

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Abstract: - The phasor measurement unit (PMU) is considered to be one of the most important measuring devices in the future of power systems. The distinction comes from its unique ability to provide synchronized phasor measurements of voltages and currents from widely dispersed locations in an electric power grid. This paper proposes an algorithm based on the clustering techniques for solving the optimal PMU placement problem in power system state estimation. The data of IEEE 14-bus, IEEE 30-bus and, IEEE 57-bus have yielded the optimal results.

Key-Words: - optimal placement, phasor measurements unit, clustering techniques.

1 Introduction

The core idea of state estimation is to calculate the immeasurable states from available measurement sets based on physical relationships between them or enhance the accuracy of the observable states utilizing mathematics [3].

State estimators provide optimal estimates of bus voltage phasors based on the available measurements and knowledge about the network topology. By now, available measurement sets did not contain phase angle measurements due to the technical difficulties associated with the synchronization of measurements at remote locations. Global Positioning Satellite (GPS) technology alleviated these difficulties and lead to the development of Phasor Measurement Units (PMU) [1, 3, 4]. One of the applications, which will be significantly affected by the introduction of PMUs, is the state estimator.

The literature cites two important methods for the optimal placement of PMUs: topology based methods and numerical methods. Topology methods use the decoupled measurement model and graph theory. In these methods decision is based on logical operations. Thus, they require only information about network connectivity, measurement types and their locations.

Numerical methods, on the other hand, use either fully coupled or decoupled measurement models. These methods are based on numerical factorization of the Jacobian or information gain matrix. Numerical methods are not suitable for large systems because they are involved with huge matrix manipulation and have their own computational complexity. Therefore, topology based methods are more used [6].

Topology method finds a scheme with minimal PMUs and their installation locations such that the entire system becomes observable. The most used observability rules are as follows [6]:

- For buses with PMUs, voltage and current phasors for all incident branches are known. These are called as direct measurements.
- If voltage and current phasors at one end of a branch are known then voltage phasor at the other end of the branch can be obtained. These are called pseudo measurements
- If voltage phasors of both ends of a branch are known then the current phasor of this branch can be obtained directly. These measurements are also called pseudo measurements
- For a zero-injection bus *i* in a *N*-bus system we have:

$$\sum_{j=1}^{N} Y_{ij} V_j = 0 \qquad (1)$$

  where, Y_{ij} is the ***ij***-th element of admittance matrix of the system and V_j is the voltage phasor of ***j***-th bus.
- If a zero-injection bus with unknown voltage phasor and voltage phasors of its adjacent buses are all known, then the voltage phasor of the zero-injection bus can be found using the nodal equation (1).
- If a group of adjacent zero-injection buses exists, whose voltage phasors are unknown but the voltage phasors of all adjacent buses to the group are known, then the voltage phasors of zero-injection buses can be obtained using the nodal equation (1).

The measurements obtained from rules 4-6 are called extended measurements.

The Branch and Bound method is based on the following optimization model [1, 4, 6]:

$$\min \sum_{i=1}^{N} w_i x_i \qquad (2)$$
$$subject\ \ to\ \ F(X) \geq 1$$

where, N is the total number of system buses, w_i is the weight factor accounting to the cost of installed PMU at bus i, X is a binary variable vector whose entries are defined as (3) and $F(X)$ is a vector function whose entries are non-zero if the corresponding bus voltage is observable using the given measurement set and according to observability rules mentioned earlier; otherwise its entries are zero.

$$x_i = \begin{cases} 1 & if\ a\ PMU\ is\ needed\ \ at\ bus\ \ i \\ 0 & otherrwise \end{cases} \qquad (3)$$

The objective of this paper is to find the minimum number of PMUs in order to make the system fully observable using clustering techniques. The validity of the proposed method was assessed through tests on the IEEE 14-bus, IEEE 30-bus, and IEEE 57-bus power systems.

2 Power System State Estimation

Measurements that are telemetered from the substations are processed at the control centers by the state estimator. State estimator provides the optimal estimate of the system state based on the received measurements and the knowledge of the network model [4]. Measurements may include the following:

- Power injections (real/reactive);
- Power flows (real/reactive);
- Bus voltage magnitude;
- Line current magnitude;
- Current injection magnitude.

Finding the best locations for PMUs, can be formulated as an optimization problem and solved using an appropriate numerical method. A common choice for the objective function is the weighted sum of the measurement residual squares, which leads to the well known Weighted Least Squares (WLS) state estimation solution [1, 3, 4]. The WLS state estimator equations relating to the measurements and the state vector are:

$$z = h(x) + w \qquad (4)$$

where:

z - measurement vector, dimension m;
h - non-linear function vector, dimension m;
x - system state vector, dimension $2N$;
w –measurement error vector, dimension m;
m - number of measurements;
N - number of buses.

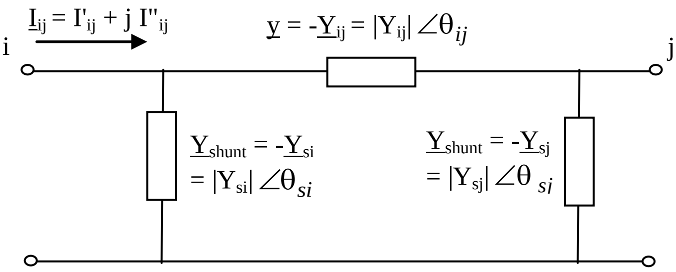

Fig. 1 – Transmission Line Model

The WLS state estimation problem is solved as an iterative calculus:

$$G(x^k)\left[\Delta x^k\right] = H^t\left(x^k\right)R^{-1}\left[\Delta z^k\right] \qquad (5)$$

where:

$$\Delta x^k = x^{k+1} - x^k;$$
$$\Delta z^k = z - h(x^k); \qquad (6)$$
$$G(x^k) = H^t(x^k)R^{-1}H\left(x^k\right)$$

The significances of variables in equations (2) and (3) are the following: R is the measurement covariance matrix, G is the gain matrix and $H(x) = \partial h(x) / \partial x$ is the Jacobian matrix of $h(x)$. The state estimator needs a set of analog measurements and system topology to estimate the system state. In fact, the minimal measurements number is equal to $(2N\ \text{-}1)$ state variables. Therefore, the critical number of real and reactive measurement pair is $(N\text{-}1)$ [3], with additional one voltage magnitude measurement.

There are three most commonly used measurement types in power system state estimation. They are the bus power injections, the line power flows and bus voltage magnitudes. But, one PMU can measure not only the voltage phasor, but also the current phasors. So, if we define y as the series admittance and y_{shunt} as the shunt admittance, current phasor measurements can be written in rectangular coordinates as shown in Fig. 1 [7]. The expressions for currents I'_{ij} and I''_{ij} are:

$$I'_{ij} = \left|V_i Y_{si}\right|\cos\left(\delta_i + \theta_{si}\right) + \left|V_j Y_{ij}\right|\cos\left(\delta_j + \theta_{ij}\right) - \left|V_i Y_{ij}\right|\cos\left(\delta_i + \theta_{ij}\right) \qquad (7)$$

$$I''_{ij} = \left|V_i Y_{si}\right|\sin\left(\delta_i + \theta_{si}\right) + \left|V_j Y_{ij}\right|\sin\left(\delta_j + \theta_{ij}\right) - \left|V_i Y_{ij}\right|\sin\left(\delta_i + \theta_{ij}\right) \qquad (8)$$

where, the state vector is given as

$$x = [V_1\ V_2\ \ldots\ V_n\ \delta_2\ \delta_3\ \ldots\ \delta_n]^t \qquad (9)$$

The measurement vector z contains, δ, I'_{ij}, I''_{ij} as well as the power injections, power flows and voltage magnitude measurements.

$$z = \left[P_{inj}^t\ , Q_{inj}^t\ , P_{flow}^t\ , Q_{flow}^t\ , \left|V\right|^t\ , \delta^t\ , I'^{\ t}_{ij}\ , I''^{\ t}_{ij}\ \right]^t \qquad (10)$$

Generally, those measurements received from PMUs are more accurate with small variances compared to the variances of conventional measurements. Therefore, including PMU measurements is expected to produce more accurate estimates. Multi-area state estimation also benefits from this technology.

3 Clustering Techniques

Clustering techniques is the name of a group of multivariate techniques whose primary purpose is to identify similar entities from the characteristics they possess. The essence of clustering approaches is classification according to natural relationships [5].

Clustering methods have as input a set of elements. An element (or feature vector, observation, or datum) is a single data item used by the clustering algorithm. These methods build a tree which approximates the similarities between elements. These similarities are given in two ways:

- directly as a similarity or distance matrix between elements;
- indirectly: elements are described by some characteristics (attributes) and similarity between two elements is defined according to the similarities between the element characteristics.

The output of these methods is a tree where each item is an element and intermediate nodes represent groups of elements. The main task of the user when performing a clustering of elements is to build a partition of the set of elements into some disjointed classes. This aspect can be illustrated by a bivariate graphic presentation, Fig. 2 and 3. Each point presents one element that has one value for characteristic X and one value for characteristic Y.

For instance, if the set of elements is a set of customers, classes may represent types of customers and aggregate may be the typical load profile for each class of customers.

There are some clustering methods which directly build a partition of elements (K-means method). K-means clustering is one of the simplest unsupervised learning algorithms that solve the well known clustering problem. The main idea is to define K centroids, one for each cluster so minimize an objective function. The function is a squared error function. In this case the user has to choose in advance the number of cluster and different trials are necessary to find the right number of cluster.

Other clustering methods (hierarchical methods) provide different number of clusters depending on the user. So, the result of the hierarchical clustering interpretation can be defined as follows:

- A partition of the elements into classes. Each element is described by a new characteristic which correspond to the class (an element belongs to a single class).

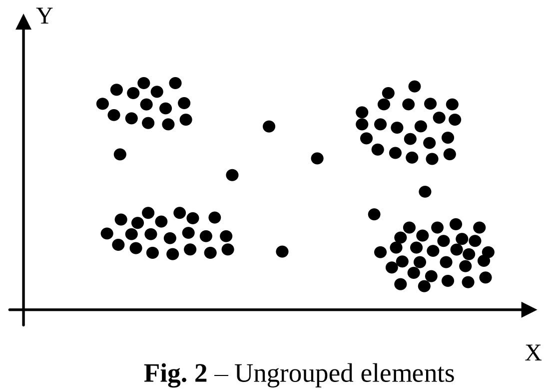

Fig. 2 – Ungrouped elements

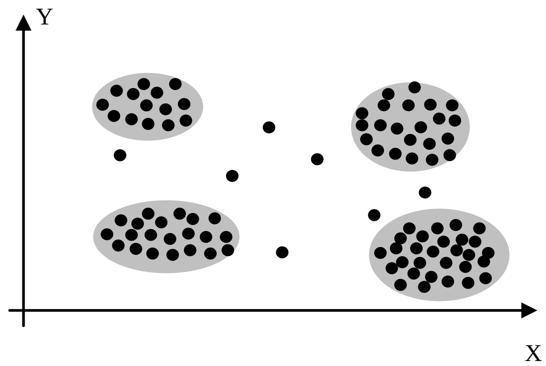

Fig. 3 – Grouped elements

- Each class is summarized by a label given by the user after a careful interpretation of the characteristics of the elements belonging to the class.

4 Optimal PMU Placement Method with Clustering Techniques

In this work, the task of PMU's area identification is formulated as a problem of bus connectivity. Thus, it indicates that the time variation of all phasors in a coherent group (zone) can be satisfactorily approximated by observing a single phasor appropriately selected from the group.

An approach to determine the optimal PMUs placement, which takes into consideration all links of every node, is proposed. In consequence, a hierarchical clustering method is used, which well overcomes problems concerning formation of coherent and representative groups (named below zones).

The main purpose is to compare units (that represent the links of every node with other) from binary connectivity matrix A, and to gather them progressively in coherent groups (zones) in a way that the nodes in the same group belong to the same zone.

The elements of the binary connectivity matrix A are defined below:

$$A_{kxm} = \begin{cases} 1 & if \quad k = m \\ 1 & if \quad k \text{ and } m \text{ are connected} \\ 0 & otherwise \end{cases} \qquad (4)$$

where k and m are two nodes from the analyzed power system.

Then, a distance between pairs of rows "k" and "m" from matrix A is computed. Common used distance is the Euclidean distance. These distances are used to determine proximity of nodes to each other. After that, they are linked together into the new group (zone) Z. The newly formed zones are grouped into larger zones until a hierarchical tree is formed. The process is repeated until there is only one zone if all nodes meet the required criterion.

In the next step, the hierarchical tree is divided into coherent zones by cutting off the hierarchical tree at an arbitrary point, α. The number of zones will depend on the value of α and the characteristics of the system. However, the threshold of inconsistency coefficient strongly influences the final number of zones. Thus, it is defined so that every zone to contain a number of nodes smaller than the maximum number of links of a node from the system.

The PMUs number which can be installed in the system is equal with the number of obtained zones. These PMUs will be installed in nodes that have the maximum number of links with other nodes from the same zone. If the links number among nodes is equal, then the PMU is placed in the first node in the zone from the dendogram of the clustering process.

5 Study Case

To evaluate the proposed approach, the IEEE 14-bus, IEEE 30-bus and IEEE 57-bus systems, are considered. Detailed results will be given only for the IEEE 14-bus system. For the IEEE 30-bus and IEEE 57-bus systems only the final results are presented. The topology of the IEEE 14-bus system is presented in Fig. 4.

In function of information included in the binary connectivity matrix A, the nodes are divided in representative groups (zones), using a statistical clustering method (centroid method).

$$A = \begin{vmatrix}
1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1
\end{vmatrix}$$

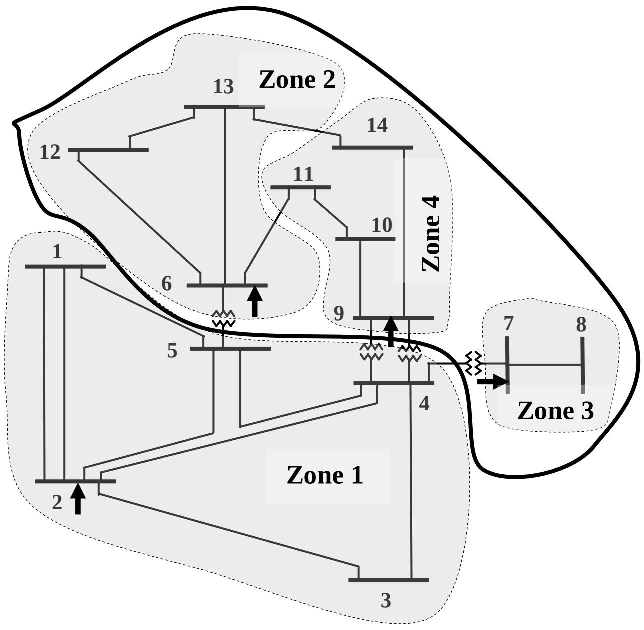

Fig. 4 – The zones of the IEEE 14-bus test system.

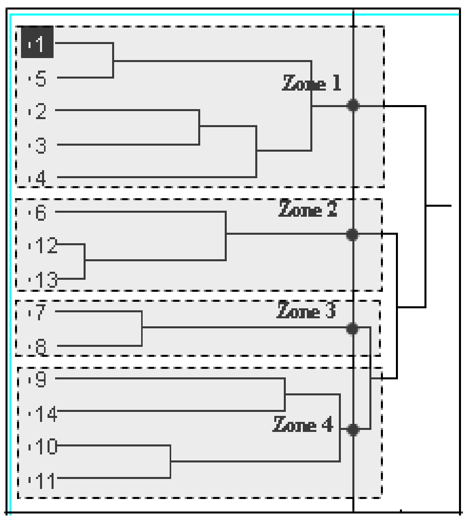

Fig. 5 – Dendogram of the clustering process.

The dendogram of clustering process is presented in Fig. 5. In the next step, the dendogram is divided into coherent zones; every zone contains a number of nodes smaller than the maximum number of links of a node from the system (which in this case is 6).

The approach using clustering techniques has the advantage of partitioning the power system efficiently and in finding the cut – set. For example, when seeking the necessary for splitting the system into two, three and four zones, the results obtained from the method is show in Fig. 4. The figure shows two partition of a single area delimited by a solid line. For the four zone partition, the zone within a solid line is further divided into three, as shown by the dashed lines. Thus, for area partitioning, the method does not require any additional steps.

Table 1 – Results of IEEE 14 – bus test system.

Zones	Bus number	Buses	PMU
1	5	1, 2, 3, 4, 5	2
2	3	6, 12, 13	6
3	2	7, 8	7
4	4	9, 10, 11, 14	9

Table 2 – Synthesis of results for different test systems.

Test Cases	PMU number	Location of PMUs
IEEE 14	4	2, 6, 7, 9
IEEE 30	10	1, 6, 7, 9, 10, 12, 15, 18, 25, 27
IEEE 57	17	1, 5, 7, 9, 15, 19, 23, 27, 30, 32, 36, 38, 39, 41, 46, 50, 53

Table 3 – Optimum number and location of PMUs using the Branch and Bound approach.

Test Cases	PMU number	Location of PMUs
IEEE 14	4	2, 6, 7, 9
IEEE 30	10	1, 5, 6, 9, 10, 12, 15, 19, 25, 29
IEEE 57	16	1, 6, 9, 15, 19, 22, 25, 28, 32, 36, 38, 41, 47, 51, 53, 57

Then, the PMUs were installed in nodes that have the maximum number of links with other nodes from that zone or if the number of links among nodes is equal, then the PMU is placed in the first node in the zone from the dendogram of the clustering process, as suggested in Fig. 4 and Table 1.

Using the proposed method, the final results obtained for all analyzed test systems are given in Table 2. A comparison between the proposed method and the Branch and Bound approach, described by equations (2) and (3), can be made using results shown in Table 3.

It can be observed that results obtained using the proposed method are comparable with those from Branch and Bound approach.

6 Conclusions

In this paper a new approach, based on the clustering technique is proposed for determination of the minimum number of PMUs in order to make the system fully observable. The results obtained using different test systems demonstrate that the methodology can be successfully used in solving optimal measurement placement problems in power system state estimation.

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