Modeling Ferroresonance in Single-Phase Transformer Cores with Hysteresis

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Abstract—Ferroresonance is a highly dynamic and nonlinear power quality phenomenon caused by nonlinear inductances in ferromagnetic materials and power system capacitances. It is notorious for causing severe damage to power systems. This paper carries out an investigation into single-phase transformer ferroresonance initiated by switching transients. A hysteretic core model is implemented which includes major and minor hysteresis loops for the study of dynamic nonlinear phenomena. Time-domain waveforms of transformer flux, voltage and magnetizing current are computed from the model. Poinca`re and phase-plane portraits are used to examine the stability domain of observed ferroresonance modes.

Index Terms—Ferroresonance, hysteresis, nonlinear models, power quality.

I. INTRODUCTION

Ferroresonance in power networks involving nonlinear transformers and capacitors has been well researched for nearly a century. However, it is only in recent years that nonlinear transformer modeling techniques have begun to approach the level of sophistication required for accurate ferroresonance studies. To that end, the importance of hysteresis nonlinearities in dynamic and transient simulation studies and its impact on the stability domain of ferroresonance modes has recently been demonstrated [1]–[3].

Ferroresonance can be understood as a complex oscillatory energy exchange between magnetic field energy of nonlinear transformer/reactor cores and electric field energy of nearby capacitances (e.g., series compensated lines or circuit breaker grading capacitors). Without adequate dissipation through normal loads and losses, a substantial amount of energy sloshes back and forth within a power system and manifests as over-voltages and currents exhibiting high levels of distortion. This has caused significant equipment damage in several cases and continues to be a large safety hazard [4]–[6].

Over the years, research in ferroresonance has concentrated into three main areas: (1) improving analytical methods and transformer models, (2) development of transformer protection and mitigation strategies and (3) case studies of system level impacts [7], [8]. Despite the extensive literature available in this area, ferroresonance continues to be a challenging problem to analyze, predict and understand due to its highly nonlinear and dynamic behavior. Researchers must adopt complex mathematical notions such as chaos theory to gain insight into this phenomenon [9]–[11].

The four generally accepted ferroresonance modes which can occur are (1) fundamental ferroresonance (period-1), (2) subharmonic ferroresonance (e.g., period-3), (3) quasi-periodic ferroresonance and (4) chaotic ferroresonance. The last two are non-periodic modes. There is also the possibility of mixed modes or unstable modes where gradual system variations or perturbations cause sudden jumps (known as bifurcations) from one mode to another [12], [13]. From a purely mathematical standpoint, these modes are due to multiple competing solutions (known as attractors) to a system of nonlinear differential equations of an electromagnetic circuit. The nonlinearity is due to the magnetic properties of ferromagnetic material.

In this paper, a single-phase transformer model including hysteresis nonlinearity is implemented to study possible ferroresonance behavior (e.g., subharmonic modes). The model is used to generate and plot time-domain waveforms including flux, voltages and magnetizing currents, as well as Poincaré and phase-plane diagrams for selected values of series and shunt capacitors initiating ferroresonance conditions.

II. HYSTERESIS CORE MODELING

The modeling of hysteresis has evolved significantly since the 1970s when digital nonlinear core models were first being developed. References [14], [15] provide a thorough historical review of progress in hysteresis modeling. Early modeling attempts indirectly incorporated hysteresis by the use of single-value nonlinear inductors in parallel with a resistor representing eddy-current and hysteretic losses. Models evolved to use families of ascending and descending curve functions for the inclusion of major and minor loop effects of ferromagnetic material. Some authors ignore minor loops and focus only on major loops or make use of scaling factors on major hysteresis loops to derive minor loops. These nonlinear approximations are typically based on piece-wise, hyperbolic, trigonometric or differential equations.

Today, there is still a tendency for transformer power quality studies to ignore hysteresis and use anhysteretic approximations of the $B - H$ characteristic because of the modeling complexity and computational burden associated with hysteresis nonlinearities. This simplification can be justified for some studies because transformer design has improved over the years and hysteresis loop widths have narrowed significantly to appear almost anhysteretic. Therefore, for steady-state power quality studies such as harmonic power flow in nonlinear transformers, single-valued anhysteretic functions are considered acceptable.
On the other hand, for the study of dynamic, transient and nonsinusoidal power system behavior, the representation of minor hysteresis loop trajectories becomes important as additional operating points are created by the dynamic excitation of a nonlinear hysteretic core. This is especially true in ferroresonance where major and minor loop trajectories can potentially generate more ferroresonant operating points [3]. For this paper, a suitable nonlinear hysteretic core model of a single-phase transformer is implemented to study ferroresonance.

III. SINGLE-PHASE TRANSFORMER CORE MODEL INCLUDING HYSTERESIS NONLINEARITY

A PSPICE computer model was developed for a single-phase transformer with hysteresis nonlinearity (Fig. 2). A scalar hysteresis model based on [16] is implemented for this work due to its simplicity in implementation for circuit simulation. The magnetic flux density \( b \) and field intensity \( h \) are the main parameters for this model. However, for this paper, the hysteresis equations are modified such that flux linkages and magnetizing currents are computed instead of \( b \) and \( h \) (i.e., \( b \rightarrow \lambda \) and \( h \rightarrow i_m \)). These parameters are more accessible in PSPICE and easily measured from laboratory tests. The modified hysteresis equations are as follows:

\[
\frac{di_m}{dt} = \frac{d\lambda}{dt} \cdot \frac{1}{L_0 + \frac{\lambda_-(i_m)-\lambda_+(i_m)}{\lambda_-(i_m)-\lambda_+(i_m)} \left( \frac{d\lambda_+(i_m)}{di_m} - L_0 \right)} \quad \text{for } d\lambda \geq 0 \tag{1a}
\]

\[
\frac{di_m}{dt} = \frac{d\lambda}{dt} \cdot \frac{1}{L_0 + \frac{\lambda_+(i_m)-\lambda_-(i_m)}{\lambda_-(i_m)-\lambda_+(i_m)} \left( \frac{d\lambda_-(i_m)}{di_m} - L_0 \right)} \quad \text{for } d\lambda < 0 \tag{1b}
\]

where \( \lambda_+ \) and \( \lambda_- \) are the limiting ascending and descending curve functions dependent on magnetizing current \( i_m \). \( L_0 \) is the inductance or slope in the saturated region along the limiting hysteresis curves. As noted from (1), the basis for this model is the use of slope functions for \( \lambda_+ \) and \( \lambda_- \) to compute magnetization processes. This is derived from the assumption that domain wall motion density (Barkhausen jumps) are proportional to the growth of domain regions which increase with field strength [16].

The flux linkage and current relationships for the hysteresis model are

\[
e(t) = \frac{d\lambda}{dt} \tag{2}
\]

\[
i(t) = i_m(t) + i_c(t) \tag{3}
\]

Before equation (1) can be computed, the ascending and descending limiting hysteresis loop segments must be specified. This paper proposes nonlinear analytical expressions to describe the ascending (\( \lambda_+ \)) and descending (\( \lambda_- \)) limiting hysteresis loop segments. The proposed nonlinear function (4) can accurately be fitted to measured \( \lambda - i \) characteristics of a real nonlinear transformer.

\[
f(i_m) = \text{sgn}(i_m) \cdot \alpha \log_c(\beta |i_m| + 1) \tag{4}
\]

The fitting parameters \( \alpha \) and \( \beta \) control the vertical and horizontal scaling. The ascending limiting loop segment is derived from (4) by shifting the function \( f(i_m) \) to the right by an increment of \( \sigma \) and similarly the descending loop function is obtained by shifting the function to the left by \( \sigma \) (equis. (5)-(6), Fig. 1). Effectively, \( \sigma \) controls the width of the limiting hysteresis loop.

\[
\lambda_+(i_m) = -f(i_m - \sigma) \tag{5}
\]

\[
\lambda_-(i_m) = f(i_m + \sigma) \tag{6}
\]

The slope of the above ascending and descending functions must be computed before the modified hysteresis equations (1) can be used. The ascending and descending slope functions \( \frac{d\lambda_+}{di_m} \) and \( \frac{d\lambda_-}{di_m} \) are derived through differentiation of (5)-(6) with respect to magnetizing current.

\[
\frac{d\lambda_+}{di_m} = \frac{\alpha \beta}{\beta |i_m + \sigma| + 1} \tag{7}
\]

Equation (1) is then computed from substitution of (2), (5), (6) and (7). The next step is for the program to select one of the two equations (1) for \( \frac{di_m}{dt} \) based on whether the magnetization is increasing or decreasing. An IF statement in PSPICE is used to select one of the two equations in (1) based on the sign of the induced voltage (2). The magnetizing current \( i_m \) is evaluated by integrating (1).

There are two approaches to realize the above expressions in a PSPICE circuit. The first method is to implement a controlled voltage source for the induced voltage (2) based on (1). The second approach is more abstract but more numerically stable and is thus the favored approach. Equation (1) is realized by implementing a circuit loop consisting of an arbitrary capacitor with inherent current and voltage relationship together with a controlled current source.

\[
i = C \frac{dV}{dt} = 1 \cdot \frac{d\lambda}{dt} \tag{8}
\]
The capacitor voltage is in fact the flux linkage and its current is governed by the controlled current source based on (1). A large resistance \((10)^{12} \, \Omega\) is placed in parallel with the capacitor to suppress numerical convergence problems.

The developed hysteresis model can form major and minor hysteresis loops based on specified \(\lambda_+\) and \(\lambda_-\) functions. This is demonstrated in Fig. 1 for linearly increasing sinusoidal excitation.

IV. SIMULATION RESULTS

The scenario under investigation is an unloaded single-phase transformer operating under steady-state conditions interrupted by a switch which is opened at \(t = 0.1\)s. This case is representative of single-phase fuse or circuit breaker action resulting in a series ferroresonance circuit as shown in Fig 2. This can also occur for three-phase transformer banks (i.e., 3 single-phase transformers) where one of the phases has developed a fault. The series capacitance could be grading capacitors in circuit breakers, stray capacitances in bus-bars and feeders, or possibly series reactive power compensation capacitor banks. The impedance values and \(\lambda - i\) hysteresis loop data of the circuit are based on a real single-phase 440 V 50 Hz transformer \((R_s = 9.4 \, \Omega, L_s = 6.34 \, \text{mH})\). The parameters for the hysteresis model are listed in the appendix. In the following simulations, selected values of \(C_{series}\) and \(C_{shunt}\) were chosen to generate different ferroresonance modes.

Fig. 3 demonstrates period-3 type ferroresonance initiated by the opening of the switch at \(t = 1.0\)s when \(C_{series}\) and \(C_{shunt}\) are set to 10 and \(38 \, \mu\text{F}\), respectively. The flux and voltage waveforms show sustained harmonic distortions with excessive magnetizing currents. The phase-plane and Poincaré diagrams indicate the system settling to a stable attracting limit cycle.

For \(C_{series}\) and \(C_{shunt}\) set to 10 and \(22 \, \mu\text{F}\), harmonic distortions have increased for flux and voltage waveforms (Fig. 4). Furthermore, the transient period from normal operation to ferroresonance has a longer duration compared to the previous case. The group of five dots in the Poincaré map indicate this is period-5 type ferroresonance. The phase-plane diagram shows the presence of competing attractors in the stability domain.

Similarly, when \(C_{series}\) and \(C_{shunt}\) are set to 29 and \(38 \, \mu\text{F}\), respectively, the transformer exhibits extremely distorted period-7 ferroresonance voltages (Fig. 5). The phase-plane portrait indicates the presence of multiple competing attractors in the stability domain.

Finally, to emphasize the sensitivity of this phenomenon to a small change in a system parameter, \(C_{series}\) is varied by a small amount from the previous case to \(28 \, \mu\text{F}\) and the simulation is repeated (Fig. 6). The resulting waveforms show unstable ferroresonance modes which appear to be period-3 type lasting for only a few cycles before dampening out. It is interesting to observe that due to the flux and voltage relationship (2), the flux waveform takes a long time to stabilize compared to the voltage waveform.

In some simulations, the developed PSPICE model had difficulties converging to a solution. This is most likely due to the discontinuities presented by (1) at the point of magnetization reversal when the model switches between equations. Problems can be circumvented somewhat by reducing the time-step in the solver but at the expense of increasing processing time.
V. Conclusion

This paper demonstrates a single-phase transformer model including hysteresis nonlinearity applied to the study of ferroresonance. The model is used to compute a number of useful outputs such as phase-plane trajectories and bifurcation response, as well as, fluxes, voltages and current waveforms. It is observed that the seemingly innocuous switching action can lead to many different ferroresonance modes causing havoc in a power system. A method for defining and modeling limiting hysteresis loop segments for incorporation into an existing hysteresis model exhibiting major and minor loop effects is proposed and simulated in PSPICE.

Appendix

Hysteresis Model Parameters

\[ \alpha = 0.7, \beta = 1000, \sigma = 0.06, L_0 = 0.07 \]

Fig. 4. Subharmonic ferroresonance (Period-5) occurring at \( C_{series} = 10 \mu F \) and \( C_{shunt} = 22 \mu F \); (a) time-domain waveforms for flux, magnetizing current and voltage, (b) Poincaré and phase-plane diagrams showing resulting steady-state ferroresonance with competing cyclical attractors.

Fig. 5. Subharmonic ferroresonance (Period-7) occurring at \( C_{series} = 29 \mu F \) and \( C_{shunt} = 38 \mu F \); (a) time-domain waveforms indicating extreme voltage and flux distortions, (b) Poincaré and phase-plane diagrams showing resulting steady-state ferroresonance with multiple competing attractors.

Fig. 6. Temporary ferroresonance condition for \( C_{series} = 28 \mu F \) and \( C_{shunt} = 38 \mu F \). The waveforms exhibit Period-3 type ferroresonance for the first few cycles after switch is opened before dampening out.
REFERENCES


