Influence of the Target Spatial Filter on the Ranges of Weapon’s Optical Direction System

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Abstract: - The article deals with analysis ranges of weapon’s optical direction system with an optic-mechanical spatial filter (so-called modulator, or modulating disk [4, 5, 7]) of target’s radiance that used to at air defence weapons systems VSHORAD and SHORAD e.g. [1, 6, 7]. On the one hand method of analyses of maximum distance and on the other hand minimum distance detection of target by direction system are showed. Both of distances are influenced by parameters of target, atmosphere and the very optical direction system. Especially at the article the attention is focused on analysis of influence of optic-mechanical spatial filter (next modulator only) to the range of seeker.

Key-Words: - Range, Optical, Direction, System, Spatial, Filter, Modulator, Convolution

1 Introduction

In the most cases a range of optical (optoelectronic – OE) equipments (e.g. a seeker, a direction finder) in atmosphere of a various weapon systems is solved on the based known “energetic” criteria (1 and 2), alternatively by Johnson’s “geometric” criteria (3):

- range of OE direction system $l_{max}$ [m] in atmosphere for target on the no radiated background [1, 4, 11]

$$l_{max} = \sqrt{\frac{\tau \cdot e \cdot S_{obj} \cdot S \cdot \cos \alpha \cdot \cos \varphi \cdot \lambda_{k} \cdot M(\lambda, T) \cdot d\lambda}{\pi \cdot m \cdot NEP}}.$$  

- range of an OE seeker $l_{max,p}$ [m] in atmosphere for target with radiated background [1, 4, 11]

$$l_{max,p} = \sqrt{\frac{\tau \cdot S_{obj} \cdot S \cdot \cos \alpha \cdot \cos \varphi \cdot \left(\frac{e \cdot K \cdot M(\lambda, T) \cdot d\lambda}{r(\lambda)}\right)}{\pi \cdot m \cdot NEP}}.$$  

- range of an OE seeker for each individual survey tasks [3, 4, 10]

$$l_{max,\text{det}} = \frac{h}{n_{p} \cdot d_{\text{det}}} \cdot \omega_{\text{det}}.$$  

where $h$ is altitude of target, $n_{p}$ is number of cycle of equivalent test [1, 10], $\omega_{\text{det}} = \arctan(2 \cdot d_{\text{det}} / f’)$ is basic field of view of one cycle in detector plane, $d_{\text{det}}$ is size of one detector element, $f’$ image focus length of optic system.

From formulas (1 and 2) is known that activity and range of OE equipment is influenced by three parameters group:

1) Characteristics of target and background – square of target $S$, emissivity of target $\varepsilon$, temperature of target $T$; emissivity of background $\varepsilon_{p}$, temperature of background $T_{p}$; $K$ is a common characteristic of radiant flux, spectral transmissivity of atmosphere and spectral sensitivity of photodetector [2, 5, 6].

2) Construction and parameters OE equipment – square of optic system $S_{obj}$, transmisivity of optic system $\tau_{\sigma}$, spectral sensitivity of photodetector $r(\lambda)$ [4, 10].

3) Environment between target and OE seeker – transmisivity of atmosphere $\tau_{\sigma}$ and their mutual arrangement in the space – angles $\alpha, \varphi$ [2, 3, 9].

Specificity of OE seekers is using an optical-mechanical (OM) modulator of radiation compare to general weapon’s optical equipments [2]. OM modulator input radiant flux to get information about position, angular velocity of interest target and space filtration of size differently targets located into field of view of seekers [1, 5, 12].

If the sizes of radiated targets are different, these OM modulator of radiation allows separating signal of “small” radiated target from signal of “huge” target, e.g. natural background of target. It means that OM modulator is spatial filter of object. Principle of this filtration is based on the different a modulation depth of
“small” and “huge” targets. This modulation depth is possible define by formula [1]

\[ m_{HM} = \frac{\Phi_{max} - \Phi_{min}}{\Phi_{max} + \Phi_{min}} \quad [\text{–}], \quad (4) \]

where \( \Phi_{max} \) and \( \Phi_{min} \) are maximum and minimum level of radiant flux behind modulator.

It is possible to show [1, 11, 12] that a modulation depth of modulator is function of a defined transmittance of modulator surface (“image” of modulator), a size of target image in the modulator plane and a position of target image on the surface of modulator (a deviation of target image from centre of modulator. It is explained with help Fig.1 and Fig.2. These curves in the Fig. 2 were solved in work [1].

Fig.1 Example of figure of spatial filter (modulator) with images targets and backgrounds different sizes \( \delta \) [\text{–}] and distances from modulator’s centre \( \rho \) [\text{–}]

Fig.2 Shapes of curves of modulated radiant flux behind OM modulator for Targets 1 to 4 display in the Fig.1; number of lightproof elements are \( n \) [1]

From these curves in the Fig.2 follow these conclusions:
1) If parameters of OM modulator are constant, target image deviation $\rho_c$ from centre of modulator are constant and size of target image $\delta$ increases, then modulation depth $m_{HM}$ decreases.

2) If target image deviation $\rho_c$ increases and the all remain parameters are constant, the modulation depth $m_{HM}$ increases. (It is necessary condition for activity OE seeker).

3) Range of OE seeker with OM modulator is influenced by own OM modulator (is decreased), because the part of optical radiation is absorb by itself modulator.

The practical value of OE seeker range with OM modulator is several times ($\sqrt{\eta_m}$ -times) smaller than value of calculated by formulas (1 and 2). Coefficient $\eta_m$ of OM modulator is determined by the type of OM modulator in a main degree (an integer transmitance of modulator surface). The values of this coefficient are approximately in interval $\eta_m = 0.1225 \pm 0.287$ for chosen type of modulator with amplitude and frequency modulation [5].

On a conclusion basis number 1 is possible to induce the fact that there is a minimum distance between target and optical system of seeker $l_{min}$ for this case the size of target image in modulator plane has such a value (for given deviation $\rho_c$) when the modulation depth is not sufficient. This situation might bring some seeker measurement error or non-utility of seeker and guidance system of missiles too [2]. For this minimum distance is valid

$$l_{min} \geq l(m_{HM \_min}).$$  \hspace{1cm} (5)

2  Problem Formulation

On an information basis showed previous chapter is possible to define that seeker is full function if optical radiation are modulate “sufficiently”, i.e. if the modulation depth is $m_{HM} \geq 0.20$ [1]. At the same time is true that modulation depth is function of image target size $\delta$ in modulator plane and this image target size is function of target distance $l$ from optical system of seeker (parameters of modulator and deviation $\rho_c$ are constant)

$$m_{HM} = m_{HM}(\delta, l).$$  \hspace{1cm} (6)

“Intuitively” and with help formula (4) it is possible say that (without functional dependency (6)) the modulation depth is depended on spatial distribution of a radiant sterance of target image on the modulator plane $L_e(y', z')$ (radiant sterance of target is $L_e(y, z)$)

$$m_{HM} = m_{HM}\left[L_e(y', z')\right],$$  \hspace{1cm} (7)

where $y, z$ a $y', z'$ are spatial coordinates in target plane and target image plane.

For analysis minimum mutual distance of target form optical system of seeker $l_{min}$ is necessary to suggest technique and means of analysis of modulation depth in dependence on parameters showed in formula (6 and 7). In next chapter solution of this problem according to formula (6) is described.

3  Problem Solution

Calculation technique of modulation depth (modulated radiant flux behind OM modulator) suggested in work [1] emerged from theory of Fourier description of Fraunhofer diffraction for linear optical system [8]. For example this technique uses “optical” calculation method of optical transfer function (OTF) of optical system. The philosophy calculation is based on a convolution an input function (e.g. target image) with complex conjugate function (e.g. complex conjugate target image)

$$Target\ Image(x, y) \otimes Target\ Image(x, y)^\ast.$$  \hspace{1cm} (8)

For analysis of modulated radiant flux (or spectrum of spatial frequencies of target image behind the modulator) uses a convolution between target image and this part of surface of modulator image at what target image lies currently (results see the Fig.3 and Fig.5)

$$Target\ Image(x, y) \otimes \ Modulator(x, y)^\ast = \ Modulated\ Radiant\ Flux.$$  \hspace{1cm} (9)

For this purpose a circle OM modulator was transformed (“unroll”) to a linear OM modulator and target image was transformed to angular coordinates (Fig.3).
For analysis was used model of target image with angular (normalize) size $\delta = 0,01 \div 0,18$, that has "step" ("crossing") responds between value of radianc sterance 0 (black) and 1 (white) (see Fig.4). In this way defined target image modulates “ideal” target image without influence of atmosphere and optical system too.

![Fig.4](image)

Fig.4 Models of a “ideal” target image with angular (normalize) size $\delta = 0,01 \div 0,18$ for analysis of OM modulator action

For this analysis was drawn a simulation models in interface Matlab [1] what able to compute modulated radiant flux for chosen parameter of OM modulator and target image. Were chosen parameters: a normalize radius of modulator $R = 1$; a number of lightproof segments of modulator $n = 10$ a 15; a normalize angular size of target image in interval $\delta = 0,01 \div 0,18$ by 0,01; normalize distance of target image from centre of modulator $\rho_c = 0,1 \div 1,0$ by step 0,1.

From analysis results [1] action of modulator with ideal model of target image (without atmosphere and optical system [13]) it is possible to generalize some its characteristics (see Fig.5, Table 1 a 2):

1) OM modulator with small number of segments has a big action region with sufficient value of modulation radiant flux. Other way say this modulator has small not-sensitive region, where some displacement of target image from centre of modulator does not cause a change of modulation depth. The same conclusion is valid for size of image target. But OM modulator with small number of segments is sensitive less on change of size of target image. It means this modulator shows worse spatial filtration big-size image target from small-size image target.

2) In light grey region of modulation depth’s values (in the Table 1 and 2) the modulation depth equals one, otherwise a change of value of target image size $\delta$ or a change of value of distance of target image from centre of modulator $\rho_c$ does not cause a change of modulation depth $m$. Spatial resolution (or differentiation) of targets is only possible provided that targets with parameters $\delta$ and $\rho_c$ lay into grey or dark grey region of modulation depth’s values in Table 1 and 2.

3) In grey region of modulation depth’s values (Table 1 and 2) is the region of action OE seeker with optimal suggested parameters of OM modulator and optical systems for given parameters of image target from this region. It is important that a change of value $\delta$ or $\rho_c$ to cause some change of value $m$.

4) In dark grey region of modulation depth’s values (Table 1 and 2) OE seeker with OM modulator “does not carry out” his purpose, i.e. it does not provide information about angular position of target. This region determines a minimum mutual distance between target and seeker and minimum angle between aiming line of target and optical axis of seeker [1]. These minimum values are defined by size of target image in image plane and distance of target image from centre of modulator.

![Fig.5](image)

Fig.5 Courses from analysis OM modulator for $n=15$ with ideal model of target image with parameters: $\delta = 0,02$, $\rho_c = 0,9$ – (A) and $\delta = 0,05$, $\rho_c = 0,7$ – (B); modulation depth is $m_A = 1$ and $m_B = 1$; (next part of Fig.5 is on the next page)
34 Part of Fig.5 Courses from analysis OM modulator for n = 15 with ideal model of target image with parameters: \( \delta = 0.14 \), \( \rho_r = 0.7 \) – (C); modulation depth is \( m_c = 0.24 \).

Number in Fig.5 marks:
1 – model of image target,
2 – image of linear (“unroll”) OM modulator,
3 – course modulated radiant flux,
4 – 1D course modulated radiant flux.

Table 1 – Values of modulation depth \( m \) for OM modulator with \( n = 15 \) lightproof segments and ideal target image

<table>
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<tr>
<th>( \delta )</th>
<th>( \rho_r = 0.1 )</th>
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<tr>
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In the Table 1 (for OM modulator with \( n = 15 \)) is number of values with sufficient modulation depth, that signed light grey (55) and grey (53) colour, 55 + 53 = 108. Number of values with insufficient modulation depth, signed dark grey colour, is 72.

Table 2 – Values of modulation depth \( m \) for OM modulator with \( n = 10 \) lightproof segments and ideal target image

<table>
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<th>( \delta )</th>
<th>( m_c = 0.7 )</th>
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In the Table 2 (for OM modulator with \( n = 10 \)) is number of values with sufficient modulation depth, signed light grey (82) and grey (53) colour, 82 + 53 = 135. Number of values with insufficient modulation depth, signed dark grey colour, is 45.

4 Conclusion

At the article was showed philosophy of calculation of modulated radiant flux for defined OM spatial filter (modulator) and model of image target with help convolution. Next are showed results obtained by simulation of activity of OM modulator in Matlab [1]. Modulation depth’s values – signed dark grey colour in Table 1 and Table 2 – then defined the minimum distance of a mutual approaching of target and seeker and minimum angle between aiming line of target and optical axis of seeker.

Suggested technique and mathematical model are able to calculation for specific case only – OM modulator with \( 2 \cdot n \) transparent and lightproof segments and circle target with size \( \delta \) and contrast equals one.

Next task that will be solved is generalized mathematical model for general cases of parameters of OM modulator and real shape of target. It will be the target that has not regular shape and it has a various
radiant sterance in his surface. For example some target image influenced by atmosphere (i.e. turbulence and attenuation of atmosphere [3, 9, 10]).

References:


