Simplest Chaos Converters: Modeling, Analysis and Future Perspectives

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Abstract: - This paper is focused on the design and analysis of the simplest second-order driven dynamical systems capable to produce complex motion including chaos. It is shown that these nonlinear systems behave like the antifilters, at least for the harmonic input signals with some specific frequency. The corresponding circuitry based on the pair of differential equations is directly synthesized using mixed-mode approach. The proper function of the individual chaos generators is verified by means of the laboratory measurements.

Key-Words: - Analog oscillator, driven systems, chaos, Lyapunov exponents

1 Introduction

Recently, the research of many scientists and engineers are attracted into the field of nonlinear dynamics and theory of chaos. This is understandable since any kind of behavior is associated with a given mathematical model which can describe many different physical events. Such study is useful also from the practical viewpoint. Many building blocks in the radio-communication path can be considered as two-port which is always nonlinear and includes several accumulating elements. This is obvious if the suggested networks are compared with the well-known state-feedback multifunctional filters. Moreover, the probability of chaotic motion increases with growing degrees of freedom. In the case of high-frequency applications the parasitic capacitances plays significant role. Thus the circuit’s order can be increased unwillingly, possibly leading into the system unstability or pushing system into the region of the unpredictable behavior. This is example where chaos is unwanted phenomenon and can be observed in the filters, oscillators, mixers, modulators, i.e. also in the case of the naturally linear networks. It can be distinguished from noise by calculating the state space attractor’s metric dimension, which is a fractal number. It is easy to model [1] the nonlinear dynamical systems by means of the electronic circuit. That is the reason why such chapter is included into this paper. The author believe that also the universal state variable feedback filters or even some portions of them are able to generate chaotic waveform at the output node for the specific amplitudes and frequencies of the input harmonic signal. Of course, this can cause both improper function and destruction of the whole device. Due to this the associated problems are indeed the interesting topics for future study. But there is another area of engineering where bifurcation with the input signal’s parameters can be useful, for example modulation, keying and coding.

2 Mathematical Models

It seems that the simplest driven dynamical systems with possible chaotic behavior are members of the general class of dynamical system which can be expressed as

\[ \dot{x} = A x + f(x) + u, \]  

where \( x \in \mathbb{R}^2 \), \( A \in \mathbb{R}^{2 \times 2} \) and input vector is

\[ u_1 = 0 \quad u_2 = \alpha \cdot \sin(\beta t). \]  

In further text \( \alpha \) and \( \beta \) represents the natural bifurcation parameters. The list of the dynamical systems belonging to this extensive class can be found in the fundamental publication [2]. Here the following systems can be found

\[ A_1 = \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}, \quad A_2 = A_4 = \begin{pmatrix} 0 & 1 \\ 0 & 0.2 \end{pmatrix}, \quad A_3 = \begin{pmatrix} 0 & 1 \\ -1 & 3 \end{pmatrix}, \quad A_5 = \begin{pmatrix} 0 & 1 \\ 1 & -0.25 \end{pmatrix}, \quad A_6 = \begin{pmatrix} 0 & 1 \\ 0 & -0.05 \end{pmatrix}. \]  

Each vector field has polynomial nonlinearity, in detail

\[ f_1 = \begin{pmatrix} -xy^2 \\ 0 \end{pmatrix}, \quad f_2 = \begin{pmatrix} 0 \\ -x^3 - y^3 \end{pmatrix}, \quad f_3 = \begin{pmatrix} 0 \\ -3x^2y \end{pmatrix}, \quad f_4 = \begin{pmatrix} 0 \\ -x^2(1.6y + x) \end{pmatrix}, \quad f_5 = f_6 = \begin{pmatrix} 0 \\ -x^3 \end{pmatrix}. \]  

There are many other dynamical systems with \( f(x) \) being piecewise-linear scalar function. In practice, saturation-type three-segment piecewise-linear curves are typical approximations for the transfer functions of the standard voltage-feedback amplifiers (\( V_{out} \) vs. \( V_{in} \)), transconductance amplifiers (\( I_{out} \) vs. \( V_{in} \)) or similar active blocks.
3 Numerical Investigation
For numerical integration itself or as a part of procedure fourth-order Runge-Kutta method has been utilized with the zero initial conditions, time step $\Delta = 0.1$ and number of steps $N=10000$. The obtained chaotic attractor for first and last dynamical system in the sense of (3) and (4) are provided in Fig. 1. Fig. 2 visualize the two-dimensional contour-surface plot of the largest Lyapunov exponent (LE) for the modified Ueda oscillator which corresponds in (3) and (4) to the index 6 with zero coefficient $a_{22}$. These topographically scaled color plots have parameter step $\Delta_A=0.1$ and $\Delta_\omega=0.01$ resulting into horizontal range $LE_{\text{max}} \in (-0.009, 0.4)$. Note that there are quite large areas where the two neighborhood orbits in some direction separate exponentially. For particular practical purposes the parameter step for calculation can be smaller.

Fig. 1: The state space trajectories for first and last (slightly simplified) dynamical system.

Searching for the positive LE can be used to diagnose of the dynamical system given as mathematical model.

Fig. 2: Largest LE as the function of input signal’s main parameters (see text).

Fig. 2: Largest LE as the function of input signal’s main parameters (see text), continued.
3 Circuitry Implementation

Assuming the state variables are voltages across linear capacitors it is evident that two independent nodes are sufficient for the final circuit. The concrete network can be derived from the general structure (Fig. 3) which consists of the passive grounded resistors $G_g=1/R_g$, floating resistors $G_f=1/R_f$, negative resistors $g_n$ realized by means of the operational transconductance amplifiers (OTA), differential-input single-output OTA (DISO) device $g_o$ and differential-input OTA with balanced-output (DIDO) denoted as $g_{bo}$. For example if each mentioned element is connected to the first node the corresponding admittance matrix becomes

$$
\mathbf{Y} = \begin{pmatrix}
-G_{gl} - G_f + g_o \pm g_h G_f \mp g_o \\
G_f \mp g_h
\end{pmatrix},
$$

(5)

The active building blocks marked OPA660 known also as diamond or ideal transistors have been used as OTA. This device [4] has low input and output capacitances about 2.1 pF and 4.2 pF respectively, input resistance better than 1 MΩ, output resistance greater than 25 kΩ and signal bandwidth hundreds of MHz. Moreover, its transconductance $g_m$ can be slightly adjusted by an external source of dc current. The initial value of $g_m$ is about 2.1 pF and 4.2 pF respectively, input resistance greater than 25 kΩ and signal bandwidth hundreds of MHz. Moreover, its transconductance $g_m$ can be slightly adjusted by an external source of dc current. The initial value of $g_m$ is about 2.1 pF and 4.2 pF respectively, input resistance greater than 25 kΩ and signal bandwidth hundreds of MHz.

$$
Y_{m1} = \begin{pmatrix}
g_m & -g_m \\
0 & 0
\end{pmatrix} + \begin{pmatrix}
0 & 0 \\
0 & g_{m2}
\end{pmatrix}
$$

$$
Y_{m2} = \begin{pmatrix}
0 & 0 \\
0 & 0
\end{pmatrix} + \begin{pmatrix}
0 & 0 \\
0 & g_{m2}
\end{pmatrix}
$$

$$
Y_{m3} = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix} + \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
$$

$$
Y_{m4} = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix} + \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
$$

and consist of up to three DIDO and DISO blocks. Any dynamical system can be constructed analogically. From the circuit synthesis point of view the biggest problem is how to implement the nonlinear part of the vector field by using minimum active devices. Due to the fact that there are much more commercially available devices working in the so-called voltage-mode (high-impedance input nodes) this approach will be adopted. The final oscillator complexity depends strongly on the number and power of the nonlinear terms. Assume the integrated four-quadrant multiplier AD633 with transfer function $I_{out}=K(U_{X1}-U_{X2})(U_{Y1}-U_{Y2})+U_Z$ where $K=0.1$ is constant.

For example, the nonlinear function $f_i$ can be realized by circuit shown in Fig. 4. The resistor $R_r$ is affected by constant $K$ such that its value must be much smaller than values of the usual working resistances.

$$
U_W=K(U_{X1}-U_{X2})(U_{Y1}-U_{Y2})+U_Z
$$

Fig. 4: Implementation of the nonlinear two-port with transfer function $I_{out}=-K^2V_m^3G_i$.

The nonideal properties of the building blocks should be also taken into account. Finite input and output impedances of DTs brings the error terms into the original set of differential equations. To be more specific, there is always a small dissipation although there are no passive resistors directly connected to the particular node. Fortunately the parasitic values if compared to the working ones have negligible effects on the global dynamics.

Fig. 3: Circuitry realization of the oscillator, only linear part of the vector field.
4 Experimental Verification
The entire circuit representing sixth system without term \( a_{22} \) were realized on the solderless board and fed by \( \pm 5V \) symmetrical supply voltage. It follows from (7) that only single OPA660, two AD633, two linear capacitors and the same amount of resistors are needed for the circuit. The approximate values of the circuit components were \( C_1 = C_2 = 2.2 \text{ nF}, R_d = 1 \text{ k}\Omega, R_r = 100 \text{ \Omega} \) and \( g_m = f(I_{set}) \) was trimmed by resistor \( R_{set} = 760 \text{ \Omega} \). Few selected waveforms of both state-variable signals measured by digital oscilloscope Agilent Infinium are given in Fig. 5. Of course, this gallery is not complete since there are many periodical windows and routing sequences in the explored interval. This can be viewed simply by calculating two-dimensional bifurcation diagram.

5 Future Study Motivation
The study of the nonautonomous deterministic dynamical systems deserves at least the same attention as it is for autonomous counterparts. Also the challenge mentioned in [5] about searching for the algebraically simplest forced system capable to generate chaos will be useful. Theory of the driven systems is not restricted only to the analogue networks. Such events can also occur in the case of power electronics, fully digital or hybrid circuits, microwave circuits and optics. Thus the practical importance is indisputable.

6 Conclusion
It has been demonstrated that even two-port circuits with very simple topology can exhibit complex behavior. It turns out that discovered chaotic behavior is robust and structurally stable to the certain level. This contribution can also serve as a tutorial answering the question how to analyze a given two-port from the viewpoint of the complex dynamics.

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