# Workspace Analysis and Design of a 6-DOF Parallel Robot 

LAPUSAN CIPRIAN, MATIES VISTRIAN, HANCU OLIMPIU<br>Department of mechanism, Fine Mechanisc and Mechatronics<br>Technical University of Cluj- Napoca<br>Cluj-Napcoa, Str. Daicoviciu, Nr. 15<br>ROMANIA<br>lapusanciprian@yahoo.com


#### Abstract

In this paper a workspace analysis for a six degree of freedom parallel robot is outlined. We propose a numerical approach for determining and evaluating the workspace of the 6-UPS robot. The algorithm uses inverse kinematic and a set of two constraints, covering struts length and joint angles, for computing the workspace. The analysis and algorithm are used as a design tool to select the dimensions and actuators for the robot. A computer application was implemented using Matlab, the program allow to change parameters and to visualize the results of the analysis. A case study is presented, of a six-degree of freedom virtual reality simulation platform design by the author.


Key-Words: - parallel robot, Matlab, kinematic, 6-UPS, workspace

## 1 Introduction

Today Virtual Reality is used in many different fields with a very wide range of applications, from applications in medicine for surgical planning, visualisation of products and maps, to applications in social sciences to study the human interaction.

Applications using virtual reality can be developed to aid people for developing skills for work or in everyday life. The aim of this work is to develop a virtual reality platform for simulating the behaviour of an auto vehicle. The platform is intended to be used for training purposes and to test the driver reaction for different safety systems implemented in the car. Such system comprise from: a virtual reality cave, a 6 -dof parallel robot platform and the real time simulation hardware.

In this paper is presented the analysis of the workspace of the 6 -dof parallel robot platform. The purpose of this analysis is to define the dimensions of the mechanical structure and chose the actuators with a convenient stroke that fulfils the requirements for the workspace dimensions.
The workspace of a robot is defined as a set of all end effector configurations which can be reached by some choice of joints coordinates. In the literature, various methods to determine workspace of a parallel robot have been proposed using geometric or numerical approaches. Early investigations of robot workspace were reported by Gosselin [1], Merlet [2], Kumar and Waldron [3], Tsai and Soni [4], Gupta and Roth [5], Sugimoto and Duffy [6], Gupta [7], and Davidson and Hunt [8]. The consideration of joint limits in the study of the robot workspaces was presented by Delmas and Bidard (1995). Other works that have dealt with robot workspace are reported by Agrawal [9], Gosselin and Angeles [10], Cecarelli [11].

In this paper the authors propose a numerical approach to determine the workspace of the 6-UPS parallel robot. The workspace is discretizated as a uniform grid of nodes in Cartesian coordinate system. Each node is then examined in order to determine whether it belongs to the workspace or not. The accuracy of the workspace boundary in this case depends on the sampling step, used to create the grid. However, the computation time grows exponentially with the sampling step, therefore limiting the accuracy. Furthermore, various problems may occur in case of singular configurations of the workspace.

## 2 The parallel robot

The 6-UPS robot, presented in the figure 1 , is a six degree of freedom parallel mechanism; the mobile platform can perform three translations and three rotations.


Fig.1. The 6-UPS parallel robot

The robot structure is made from six kinematic chains; each chain is connected to the base through a universal joint and to the mobile platform by a spherical joint.

The actuation of the robot is made using six linear DC actuators with ball-screw transmission. The actuators provide to the mobile platform enough force to lift a load of $120[\mathrm{Kg}]$ with speed of $75[\mathrm{~mm} / \mathrm{s}]$. The stroke of the actuators influences the dimension of the workspace and is defined after the workspace analyses.

## 3 Kinematic analysis

The equations of inverse kinematic are used to determine the workspace of the robot. For the inverse kinematic problem it is known the position of the end effector, position of each joint for the frame and the dimensions for the frame and the moving platform and have to determine the length for all 6 actuators $q_{i}$ $\{i=1 . .6\}$.


Fig.2. Kinematic scheme

The position and orientation of the mobile platform is represented by a position vector $\mathrm{P}\left[\mathrm{P}_{\mathrm{x}}, \mathrm{P}_{\mathrm{y}}, \mathrm{P}_{\mathrm{z}}\right]$ and a set of three angles $[\alpha, \beta, \gamma]$ in the base coordinate system, where: $\alpha$ - roll, $\beta$ - pitch and $\gamma$ - yaw. In order to determine de length for each translation actuator first the position for all platform joints must be found. The position vector for each joint $\mathrm{P}_{\mathrm{i}}$ must be expressed in the based coordinate system, for that each vector must be rotated and translated:

$$
\begin{equation*}
P_{B i}=T+R_{\alpha \beta \gamma} P_{i} \tag{1}
\end{equation*}
$$

The matrix $\mathrm{R}_{\alpha \beta \gamma}$ represents the rotation matrix, and it is computed by multiplication of the three individual rotation matrices.

$$
R_{\alpha \beta \gamma}=\left[\begin{array}{ccc}
a_{1} & a_{2} & a_{3}  \tag{2}\\
a_{4} & a_{5} & a_{6} \\
a_{7} & a_{8} & a_{9}
\end{array}\right]
$$

Where:

$$
\begin{align*}
& a_{1}=\cos \beta \cos \alpha \\
& a_{2}=-\cos \alpha \sin \gamma+\sin \alpha \sin \beta \cos \gamma \\
& a_{3}=\sin \alpha \sin \gamma+\cos \alpha \sin \beta \cos \gamma \\
& a_{4}=\cos \beta \sin \gamma \\
& a_{5}=\cos \alpha \cos \gamma+\sin \alpha \sin \beta \sin \gamma \\
& a_{6}=-\sin \alpha \sin \gamma+\cos \alpha \sin \beta \sin \gamma \\
& a_{7}=-\sin \beta \\
& a_{8}=\sin \alpha \cos \beta \\
& a_{9}=\cos \alpha \cos \beta \tag{3}
\end{align*}
$$

Knowing the position of each joint of the mobile platform $\mathrm{P}_{\mathrm{Bi}}$, and the position for the frame joints $\mathrm{B}_{\mathrm{i}}$, the problem is reduce to find the distance between two points in space. The equation used is presented next:

$$
\begin{equation*}
q_{i}=\sqrt{\left(P_{B i x}-B_{i x}\right)^{2}+\left(P_{B i y}-B_{i y}\right)^{2}+\left(P_{B i z}-B_{i z}\right)^{2}} \tag{4}
\end{equation*}
$$

The obtained equations were implemented in Matlab. Figure 8 presents the developed GUI; the user can set different position and orientation for the mobile platform and the application computes and displays the position and length for each kinematic element.


Fig.3. The GUI for solving IKP

The program allows also changing the $\min$ and $\max$ values for the actuators and the dimension for the mobile platform and fixed frame.

## 4 Workspace analysis

The numerical numeric evaluation of the workspace is done by defining a binary matrix $P_{i j k}$ in the cross-section plane for a cross-section of the workspace as follows: if the $(i, j, k)$ grid pixel includes a reachable point, then $P_{i j k}$ $=1$; otherwise $P_{i j k}=0$, as shown in Fig. 4.


Fig.4. The general scheme for binary representation and evaluation of robot workspace

The volume of the workspace is compute by summing all the discretizated volumes, the equation that describe this is:

$$
\begin{equation*}
V=\sum_{i=1}^{i_{\text {max }}} \sum_{j=1}^{j_{\text {max }}} \sum_{k=1}^{k_{\text {max }}}(\Delta x \Delta y \Delta k) \tag{5}
\end{equation*}
$$

In order to determine if the point $P_{i j k}$ belongs to the workspace two constraints are defined. First constraint defines de minimal and maximal struts length of all six actuators. All struts should adhere to the following physical limits:

$$
\begin{equation*}
\mathrm{q}_{\min }<\mathrm{q}_{\mathrm{i}}<\mathrm{q}_{\max } \tag{6}
\end{equation*}
$$

The second constraint is the maximum value of the angles for spherical and universal joints.

## 5 Experimental results

As the reachable locations of an end-effector are dependent on its orientation, a complete representation of the workspace should be embedded in a 6-
dimensional workspace for which there is no possible graphical illustration; only subsets of the workspace may therefore be represented. There are different types of workspaces namely constant orientation workspace, maximal workspace or reachable workspace, inclusive orientation workspace, total orientation workspace, and dextrous workspace.

In the paper is analyzed the constant orientation workspace, in this case the orientations of the platform is fixed, the algorithm compute all the locations of the moving platform that may be reached. The algorithm that computes the workspace was implemented in Matlab.


Fig.5. Workspace of the 6-UPS robot
Figure 5 presents the workspace of the designed robot, the actuator stroke is 300 [ mm ]. The obtained workspace volume is $0.66\left[\mathrm{~m}^{3}\right]$, all three orientation angles are zero. If the orientation of the platform is changed the volume of the workspace decrease, for example if the orientation angles of the platform are set to $15^{\circ}$ the volume of the workspace decrease to 0.26 $\left[\mathrm{m}^{3}\right]$.


Fig.6. Cross-section of the workspace

In figure 6 a cross-section of the workspace is presented. The section is made along the plane defined by the axes $O y$ and $O z$.

## 6 Conclusions

In this paper a workspace analysis for a six-degree of freedom parallel robot was outlined by using a numerical approach. The workspace of the 6-UPS robot was determined and analyzed using Matlab. Based on this analyze the dimension of the mechanical elements and the stroke of the actuators was defined.

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