Digital Camera Calibration Analysis Using Perspective Projection Matrix

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*Abstract:*This work presents a novel and simplified technique to estimate the perspective projective matrix elements of calibration matrix for pinhole model in Digital Camera Calibration. The "perspective projective matrix parameters" are variables depending of environmental changes and position and/or orientation camera so we propose a dynamical and stochastic method to model the uncertain in the parameters estimation. It is well suited for use without specialized knowledge of 3D geometry or computer vision. Two morphological matrix operations are introduced: $Central(X_{k,}Y_k)$ and $Column(M_k)$ to generalize the process which obtain the estimated parameters based on pseudo-inverse calculus without consider measures errors, this calibration procedure focus only on perspective projective matrix elements. The mean value respect to 3D points and 2D points used as input information. The theoretical results give a good enough approximation result considering the pseudo-inverse matrix calculus method. In the same sense, the experimental results showed a satisfactory advance in the parameters stochastic estimation theory, respect to velocity change gains.

Keywords: Camera Calibration, Pseudo-inverse, Stochastic process, filtering theory, estimation, Pin-Hole Model.

1 Introduction

Actually, camera calibration on computer vision becomes to be an important problem to Scientific Community because a machine can analyze the environment images automatically and we could use this theory to solve problems in different knowledge areas as medicine, robotics, physics, security and astronomy [1] [2]. We receive useful quantitative information from the scene in the image for example: real object dimensions, distance between objects and textures [3]. Moreover, this information used to control robots, which also take decisions, by itself for controlled motion in planed trajectories. The "Pin Hole" is very successful model used in the camera calibration field also this work based on this. The Fig. 1., shows the geometric scheme to describe the ideal "Pin Hole" model, a closed box with only one small hole and no lenses are used to focus light. The model does not describe geometric distortions or blurring of unfocused objects caused by lenses and finite sized apertures. Any external object emits light rays and those passes through the hole and an inverted object image projected on the image plane. This model is frequently used in robotics and computer sciences even does not consider all the physic variables in the mathematic model [4] [5][6] and[7].

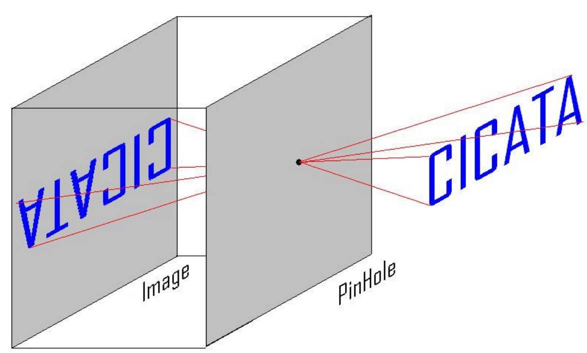

Fig. 1. The "PinHole" model principle consider the external objects image inversion.

2. Basic Equations

The "PinHole" model concept considering the correspondence between the fixed points modeled into finite computer image space, mapping from 3D real image to 2D compute pixel points. Phenomenon studied by Zhang in [13] and [14]; with normalized transformation (1).

$$Y_m = A[R \ \ t]X_n \tag{1}$$

Where:

$X_n = [x, y, z, 1]$: Correspond to homogeneous 3D coordinates point.

$Y_m = [u, v, 1]$: Homogeneous 2D compute pixel points as coordinates point

$[R\ t]$: Extrinsic parameters matrix

A: Intrinsic known parameters matrix

$$A = \begin{bmatrix} f_x & s & u_0 \\ 0 & f_y & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

The vector $\tilde{u} \in R^3$ allows identifying the object coordinates associated to the vector $\tilde{y} \in R^4$ in the image showed in Fig. 1. The normalized transformation (1) describes this correspondence and includes two important matrixes: Extrinsic parameters matrix $[R|\,t]$ models the rotation and translation of the camera respect to external reference and intrinsic parameters matrix models A as the pixel shape and the focal length elements [7] [9] [10] [11] [12] and [13]. Conforming mixed matrix respect to inner product symbolically expressed as M matrix calling as perspective projection matrix (2). Inside of it, considering dynamical elements respect to real coordinate system. Such that, the simplified transformation description has the linear description form:

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \tag{2}$$

The realistic problem consists in estimate the parameters transformation. Commonly could be use the traditional Wienner filter description by this kind of systems; but input and output vectors have different lengths. In imaginary form, respect to the Homogeneous 2D compute pixel points as coordinates point:

$$E\{Y_k Y_k^T\} = M_k E\{X_k Y_k^T\} \tag{3}$$

But $X_k Y_k^T$ *did not a conformal structure and did not express the common solution as:*

$$M_k = E\{y_k y_k^T\}\left(E\{u_k y_k^T\}\right)^{-1} \tag{4}$$

In the "Pin Hole" model, the second probability moment depicted as $E\{X_k Y_k^T\}$ did not mathematical conform structure and the matrix inner product $X_k Y_k^T$ did not realizable.

3. Dynamical model Solution

The "perspective projection matrix parameters" expressed as variables depending of environmental changes and position and/or orientation camera, represented in first instance by basic equation in order to calibrate a digital camera using a reference comparing measurements results and introducing some uncertainty, either using the expectation respect to each of the 3D coordinates points and their projections photography. Such that could be solve the camera calibration problem considering the Pseudo-inverse matrix resolution considering dynamic time-varying internal parameters, as shown in following theorem.

Theorem: Considering the MIMO stochastic system $Y_{k,m\times 1} = M_{m\times n} X_{k,n\times 1} + W_{X_{k,m\times 1}}$ *(3); such that its matrix parameter estimator, respect to fix evolution:*

$$Column\ (M)_{(m\times n)\times 1} = Central\ (X_k, Y_k)^{+}_{(m\times n)\times m} (Y_{k,m\times 1} + W_{x_{k,m\times 1}}) \tag{4}$$

Proof: The stochastic system model conformed symbolically has the expanded form*:*

$$Y_{k,m\times 1} = Central\ (X_k, Y_k)_{m\times(m\times n)} Column\ (M)_{(m\times n)\times 1} + W_{X_{k,m\times 1}}.$$

Where $Central(X_k, Y_k)$ *has the form:*

$$Central(X_k, Y_k)_{m\times(m\times n)} := \begin{bmatrix} [x_i, x_{i+1} \ldots x_m] & \ldots & 0 & y_1[x_i, x_{i+1} \ldots x_m] \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \ldots & [x_i, x_{i+1} \ldots x_m] & y_{i-1}[x_i, x_{i+1} \ldots x_m] \end{bmatrix}$$

$Column(M)$ *Expanded as vector:*

$$Column(M)_{(m\times n)\times 1} := \begin{bmatrix} m_{11} \\ \vdots \\ m_{mn} \end{bmatrix}$$

With W_X as the uncertain added by the environment.

The Central pseudo inverse:

$$Central(X_k)_{(m\times n)\times m}{}^{+} := \left(\left(Central(X_k)_{m\times(m\times n)}\right)^T Central(X_k)_{m\times(m\times n)}\right)^{-1} \left(Central(X_k)_{m\times(m\times n)}\right)^T$$

Such that the parameter set for a fixed evolution is described by (4).■

4. Optimal Solution

The method described in the previous theorem has an optimal solution considering:

$$Y_k = Central\ (X_k)Column\ (M_k) + W_X$$

Where the gradient expressed as:

$$\frac{\partial}{\partial Column\ \ (M_k)} Y_k Y_k^T = 0$$

Allow to obtain:

$$Column(M_k) = -Central(X_k)^+ W_k\ .$$

5. Experimental Results

5.1. Data input

The proposed algorithm tested into Matlab. The closed form solution involves finding a group of points of the same image. Inverse methodology allows to compare the certain using an perspective projective matrix given in Zhang[8][13] and [14]:

M = 1.0e+003 *

0.6573	0.0003	0.3027	2.5257
0	0.6577	0.2423	1.6271
0	0	0.0010	0.0040

where *R:*

$$R = R_\alpha R_\beta R_\theta = \begin{bmatrix} \cos\beta\cos\theta & sen\alpha\, sen\beta\cos\theta + \cos\alpha\, sen\theta & -\cos\alpha\, sen\beta\cos\theta + sen\alpha\, sen\theta \\ -\cos\beta\, sen\theta & sen\alpha\, sen\beta\, sen\theta + \cos\alpha\cos\theta & \cos\alpha\, sen\beta\, sen\theta + sen\alpha\cos\theta \\ sen\beta & -sen\alpha\cos\beta & \cos\alpha\cos\beta \end{bmatrix}$$

with $\alpha = 0$, $,\beta = 0$ *and* $\theta = 0$

and intrinsic matrix A:

A =

657.3025	0.2761	302.7166
0	657.7439	242.3339
0	0	1.0000

The next input homogeneous vectors in fixed evolution used to test the method error.

X =

2	2	50	23	6	2	5
3	34	56	12	23	42	2
4	2	5	7	8	4	4
1	1	1	1	1	1	1

Using the vector X as 3D point input and applied on the equation (1), it obtained the next Y vector output, which represents the correspondences of X points in the images.

Y = 1.0e+004 *

0.5052	0.4455	3.6920	1.9766	0.8898	0.5063	0.7024
0.4570	2.4475	3.9672	1.1216	1.8694	3.0222	0.3912
0.0008	0.0006	0.0009	0.0011	0.0012	0.0008	0.0008

5.2. Central ( ) and Column ( ) operations implementation.

Column() function description using the Matlab function representation code where the % character enclose code comments:

```
% The M matrix is the input data%
% CM is the matrix output%

function [CM] = column(M)

[m,n]=size(M);     %To obtain the M dimensions%
k=1;
for j=1:n
   for i=1:m
      CM(k)=M(i,j)
      k=k+1
   end
end
CM=CM'              %To obtain the CM transpose%
```

Central() function using the Matlab representing code description:

```
%This function obtains the Central Matrix from X and Y vectors%
[m,n]=size(X);
for i=1:n
   if i==1
      CENTRAL=[X(1,i),X(2,i),X(3,i),X(4,i),0,0,0,0,-
(Y(1,i)*X(1,i))/Y(3,i),-(Y(1,i)*X(2,i))/Y(3,i),-
(Y(1,i)*X(3,i))/Y(3,i);0,0,0,0,X(1,i),X(2,i),X(3,i),X(4,i),-
(Y(2,i)*X(1,i))/Y(3,i),-(Y(2,i)*X(2,i))/Y(3,i),-
(Y(2,i)*X(3,i))/Y(3,i)];
      b=[Y(1,i)/Y(3,i);Y(2,i)/Y(3,i)];
   else
      CENTRAL=[CENTRAL;X(1,i),X(2,i),X(3,i),X(4,i),0,0,0,0,-
(Y(1,i)*X(1,i))/Y(3,i),-(Y(1,i)*X(2,i))/Y(3,i),-
(Y(1,i)*X(3,i))/Y(3,i);0,0,0,0,X(1,i),X(2,i),X(3,i),X(4,i),-
(Y(2,i)*X(1,i))/Y(3,i),-(Y(2,i)*X(2,i))/Y(3,i),-
(Y(2,i)*X(3,i))/Y(3,i)];
      b=[b;Y(1,i)/Y(3,i);Y(2,i)/Y(3,i)];
   end
```

The matrix convergence considering A and its estimate described as:

```
EST_M =  1.0e+003 *
   0.4991    0.0002    0.2298    1.9177
  -0.0000    0.4994    0.1840    1.2354
  -0.0000    0.0000    0.0008    0.0027
```

The final illustration depicted in Fig. 2., shows the near matching respect to reference signal.

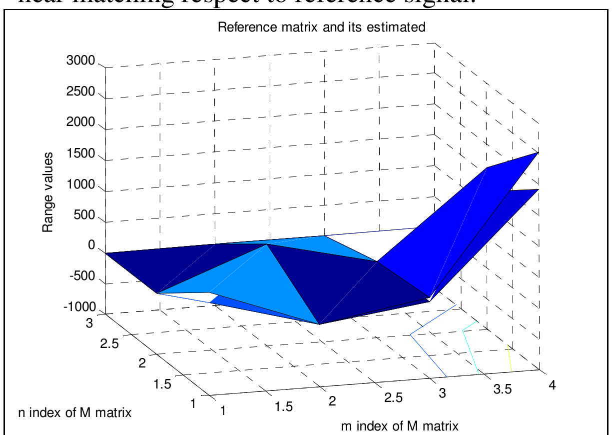

Fig. 2. Reference matrix and its estimation.

The upper surface in Fig. 2., representing the reference projection perspective matrix and the other one its estimation matrix.

6. Conclusion

This work presented a novel and simplified technique to estimate the perspective projective matrix elements of calibration matrix for pinhole model in Digital Camera Calibration. Two morphological matrix operations were introduced: *Central*(X_k, Y_k) and *Column*(M_k) to generalize the process which obtained the estimated parameters based on pseudo-inverse calculus without consider measures errors, this calibration procedure focus only on perspective projective matrix elements. The mean value of 3D points and 2D points used as input information. The theoretical results give a good enough approach considering the pseudo-inverse matrix method. The simulation parameters results showed that the velocity estimation changes had satisfactory evolution respect to the real values.

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