# Image Restoration via Wiener Filtering with Improved Noise Estimation

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*Abstract:* In this paper, first, the performance of the Wiener filter in the frequency domain for image restoration is compared with that in the time domain for images degraded by white noise. After finding that the Wiener filter in the frequency domain is better than that in the time domain, a noise estimation method for the Wiener filter in the frequency domain is proposed. The frequency band division processing addressed recently is deployed and modified to improve the performance of the Wiener filter. The conventional noise estimation method does not estimate the noise spectrum in a low frequency region, and as a result it is not expected to restore a degraded image accurately. From this point of view, a method to estimate the noise spectrum in both low and high frequency regions is derived. The performance of the Wiener filter with the proposed noise estimation method is investigated through computer simulation experiments. Also, the optimal parameter values for the noise estimation method are searched. The proposed noise estimation method is compared with the conventional one based on Wiener filtering, resulting in the former providing a performance improvement.

*Key–Words:* Image restoration, White noise, Noise estimation, Wiener filter

## 1 Introduction

It is known that in the case where an image is degraded by white noise, the Wiener filter is more suitable for restoration than a variety of smoothing filters such as the Gaussian, median, Kuwahara and morphological filters [1]. The Wiener filter has two types of implementation; one is the frequency domain Wiener filtering [1] and the other is the space domain one [2]. These are commonly derived in the sense to minimize the mean square error (MSE) between the noisy image and desired (original) image. However, both implementations are obviously different, resulting in different performances even for an ideal case where both the original and noisy images are known.

In this paper, we first examine which is better for the frequency domain and space domain Wiener filters for the purpose of image restoration. Simulation results in an ideal case where the original image and additive noise are known a priori suggest that the Wiener filter implemented in the frequency domain is better than that in the time domain for restoration. Thus, based on this result, we set out to improve the performance of the Wiener filter implemented in the frequency domain.

It is necessary for the frequency domain Wiener filter to estimate the noise spectrum from the observed image. The performance of the Wiener filter obviously depends on the noise estimation accuracy. There are several types of methods to estimate the noise spectrum. The frequency band division processing (FBDP) is one of them, which was addressed recently in [3]. The FBDP was derived for implementing a two-dimensional (2-D) spectral subtraction method. It, however, can be applied to the Wiener filter as well.

Although the FBDP provides accurate noise estimation, it leaves room for improvement. To improve the performance of the frequency domain Wiener filter furthermore, we derive a modified version of the FBDP. By investigating a parameter set for the FBDP, we compare the performances the conventional and proposed FBDP techniques provide by implementing the frequency domain Wiener filter.

## 2 Two Types of Wiener Filter and Performance Comparison

In this section, we describe the principle of 2-D Wiener filters in the frequency domain and in the time domain for the purpose of restoration of an image degraded by white noise. Through this paper, let us assume that $d(i,j)$ and $n(i,j)$ represent the original image and additive noise, respectively. The observed degraded image $x(i,j)$ is given by

$$x(i,j) = d(i,j) + n(i,j). \quad (1)$$

The goal is to obtain a restored image $y(i,j)$ from $x(i,j)$, which should be very similar with the original image $d(i,j)$.

### 2.1 Wiener Filter in the Space Domain

If the output of the Wiener filter is $y(i,j)$, it is represented by

$$y(i,j) = \sum_{m=-N}^{N} \sum_{n=-N}^{N} w(m,n)x(i+m,j+n). \quad (2)$$

When the size of the input image is $M \times M$, the weights of the Wiener filter, $w(m,n)$, are found by minimizing

$$J = \frac{1}{M^2} \sum_{i=0}^{M-1} \sum_{j=0}^{M-1} \{d(i,j) - y(i,j)\}^2. \quad (3)$$

The solution for $w(m,n)$ is obtained in a vector form as

$$\boldsymbol{w} = \boldsymbol{R}^{-1}\boldsymbol{p} \quad (4)$$

is found where

$$\begin{aligned}
\boldsymbol{w} &= \big[w(-N,-N),...,w(-N,N),\\
&\quad w(-N+1,-N),...,w(-N+1,N),...,\\
&\quad w(0,0),...,w(N,N)\big]^T\\
\boldsymbol{p} &= \big[p(-N,-N),...,p(-N,N),p(-N+1,-N),\\
&\quad ...,p(-N+1,N),...,p(0,0),...,p(N,N)\big]^T
\end{aligned}$$

$$\boldsymbol{R} = \begin{bmatrix}
R(0,0) & \dots & R(0,2N) & R(1,0) & \dots & R(2N,2N)\\
\vdots & \ddots & \vdots & \vdots & & \vdots\\
R(0,-2N) & \dots & R(0,0) & R(1,-2N) & \dots & R(2N,0)\\
R(-1,0) & \dots & R(-1,2N) & R(0,0) & \dots & R(2N-1,2N)\\
\vdots & & \vdots & \vdots & \ddots & \vdots\\
R(-2N,-2N) & \dots & R(-2N,0) & R(-2N+1,-2N) & \dots & R(0,0)
\end{bmatrix} \quad (5)$$

In (5), $R(m,n)$, the autocorrelation function, and $p(m,n)$, the cross-correlation function, are calculated as

$$\begin{aligned}
p(m,n) &= \frac{1}{M^2} \sum_{i=0}^{M-1} \sum_{j=0}^{M-1} \{d(i,j)x(i-m,j-n)\}\\
R(m,n) &= \frac{1}{M^2} \sum_{i=0}^{M-1} \sum_{j=0}^{M-1} \{x(i,j)x(i-m,j-n)\}
\end{aligned} \quad (6)$$

respectively, from $M \times M$ images of $d(i,j)$ and $x(i,j)$.

The output of the Wiener filter is obtained by (13) with the solution vector in (4).

### 2.2 Wiener Filter in the Frequency Domain

The Wiener filter in the frequency domain is given by

$$H(u,v) = \frac{P_d(u,v)}{P_d(u,v) + P_n(u,v)} \quad (7)$$

where $P_d(u,v)$ and $P_n(u,v)$ represent the power spectra of $d(i,j)$ and $n(i,j)$, respectively. These are obtained as

$$\begin{aligned}
P_d(u,v) &= |D(u,v)|^2\\
P_n(u,v) &= |N(u,v)|^2
\end{aligned} \quad (8)$$

where $D(u,v)$ and $N(u,v)$ represent the discrete Fourier transforms (DFTs) of $d(i,j)$ and $n(i,j)$, respectively.

The output of the Wiener filter is given by

$$y(i,j) = \mathrm{IDFT}\big[X(u,v)H(u,v)\big] \quad (9)$$

where $X(u,v)$ represents the DFT of $x(i,j)$. IDFT in (9) means the Inverse DFT.

### 2.3 Restoration Results in Ideal Case

We investigated the performance of the two types of the Wiener filter in an ideal case where the original and noise images are known a priori. We used 10 images in SIDBA, each of which has a size of $256 \times 256$ gray scales. A white noise was generated and added to each image, resulting in the preparation of 0[dB] and 5[dB] and 10[dB] noisy images for each image.

The results obtained by each implementation of the two Wiener filters on the degraded images are shown in Table 1. The window size of the Wiener filter in the time domain was set to $5{\times}5$. This is because this setting for the window size is very often used [4]. The SNR improvement in [dB] is defined as

$$10\log_{10}\frac{NMSE[d(i,j),x(i,j)]}{NMSE[d(i,j),y(i,j)]} \quad (10)$$

where

$$NMSE[d(i,j),x(i,j)] = 100\times\frac{Var[d(i,j)-x(i,j)]}{Var[d(i,j)]} \quad (11)$$

$$NMSE[d(i,j),y(i,j)] = 100\times\frac{Var[d(i,j)-y(i,j)]}{Var[d(i,j)]}. \quad (12)$$

From Tables 1, it is observed that the Wiener filter in the frequency domain is more effective than that in the time domain.

Table 1: SNR improvement in an ideal case.

| | SNR0 | | SNR05 | | SNR10 | |
|---|---|---|---|---|---|---|
| image | frequ. | time | frequ. | time | frequ. | time |
| Airplane | 9.362 | 7.513 | 7.376 | 5.517 | 5.232 | 3.556 |
| Barbara | 9.802 | 7.478 | 7.739 | 5.651 | 5.791 | 3.964 |
| Boat | 10.636 | 7.839 | 8.368 | 5.743 | 6.327 | 4.064 |
| Bridge | 8.198 | 6.723 | 5.976 | 4.320 | 3.818 | 2.213 |
| Building | 9.917 | 8.165 | 8.394 | 6.342 | 6.463 | 4.528 |
| Cameraman | 9.230 | 7.799 | 7.740 | 5.929 | 5.718 | 3.880 |
| Girl | 11.030 | 9.503 | 9.048 | 7.486 | 6.796 | 5.186 |
| Lenna | 10.882 | 9.018 | 8.603 | 6.678 | 6.285 | 4.383 |
| Lighthouse | 8.451 | 6.976 | 6.552 | 4.890 | 4.482 | 2.915 |
| Woman | 10.346 | 8.592 | 7.890 | 6.038 | 5.577 | 3.764 |

# 3 Noise Spectrum Estimation for Wiener Filter

In practice, we cannot obtain the true original image and noise or their power spectrum counterparts that are necessary for the Wiener filter directly. Therefore, we have to estimate them in order to restore the degraded image by the Wiener filter. When we implement the Wiener filter in the frequency domain according to the results in Section 2, it is enough to estimate only the noise power spectrum because in this case the original image power spectrum is calculated by

$$P_d(u,v) = P_x(u,v) - P_n(u,v). \quad (13)$$

based on the assumption in (1) that the noise is additive. Thus, we consider accurate noise estimation only.

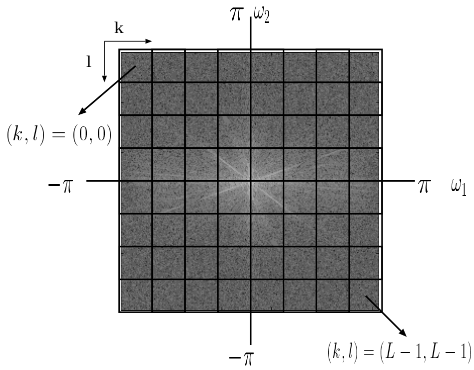



Fig. 1: Blocks in the frequency domain $(L = 8)$.

In this section, we describe first a noise estimation method addressed recently [3], in which the noise power spectrum is estimated from the image degraded by white noise based on the FBDP. Next, we derive its modified version which provides an improvement in the noise estimation accuracy.

## 3.1 FBDP Based Noise Estimation

In general, the power spectrum of an image is concentrated in a low frequency region, while the power spectrum of white noise exists in all frequency regions. Therefore, by restoring a high frequency region of the degraded image, the noise image should be obtained. This is because only the noise components are dominant in a high frequency region of the degraded image. Based on this principle, the following noise estimation method was addressed in [3].

(Step.1) We calculate $X(u,v)$ by the DFT of the input image $x(i,j)$ and obtain its power spectrum and logarithmic power spectrum as $P_x(u,v) = \mid X(u,v)\mid^2$ and $G_x(u,v) = \log P_x(u,v)$, respectively.

(Step.2) Dividing $P_x(u,v)$ and $G_x(u,v)$ into $L\times L$ blocks as shown in Figure 1, we obtain $P_{x,(k,l)}(u,v)$ and $G_{x,(k,l)}(u,v)$, respectively, where $P_{x,(k,l)}(u,v)$ and $G_{x,(k,l)}(u,v)$ correspond to the $(k,l)$-th block of $P_x(u,v)$ and $G_x(u,v)$, respectively. The average of each logarithmic power spectrum $G_{x,(k,l)}(u,v)$, $\bar{G}_{x,(k,l)}(u,v)$, is calculated for each block.

(Step.3) A threshold decision is made for each $\bar{G}_{x,(k,l)}(u,v)$, and $P_x(u,v)$ is divided into its low and high frequency counterparts, $P_{x,low}(u,v)$ and $P_{x,high}(u,v)$, as

$$P_{x,low}(u,v) = \begin{cases} P_{x,(k,l)}(u,v) & \bar{G}_{x,(k,l)}(u,v) \succ TH \\ 0 & otherwise \end{cases} \quad (14)$$

$$P_{x,high}(u,v) = \begin{cases} P_{x,(k,l)}(u,v) & \bar{G}_{(k,l)}(u,v) \le TH \\ 0 & otherwise \end{cases} \quad (15)$$

$$TH = (G_{x,max} - G_{x,min})/100 * p + G_{x,min} \quad (16)$$

where $G_{x,max}$, $G_{x,min}$ and $p$ are the maximum and minimum values of $\hat{G}_x(u,v)$, and the division ratio, respectively.

(Step.4) The noise components in the degraded image are obtained as its noise power spectrum from the high frequency parts as

$$P_n(u,v) = P_{x,high}(u,v) \quad (17)$$

where $P_n(u,v)$ means an estimate of the noise power spectrum.

In [3], the optimal values of parameters $L$ and $p$ were 32 and 12. However, these were the results determined for the spectral subtraction method.

### 3.2 Modified Noise Estimation

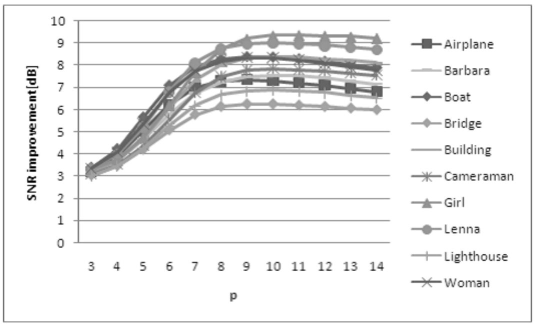
For the noise power spectrum estimation method mentioned above, the noise in a low frequency region is not considered. This results in that we can not remove the noise in a low frequency region if we estimate the noise by using the method and restore the degraded image. Therefore, we propose a method here that estimates the noise power spectrum in both low and high frequency regions so that the Wiener filter provides better performance.

The point of the proposed noise estimation method is compensation for low frequency estimation. Utilizing (15), this is conducted as

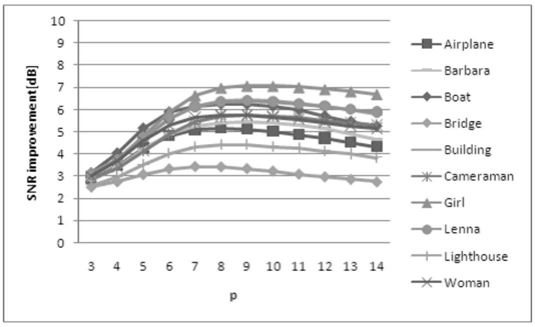
$$P_{x,high}(u,v) = \begin{cases} P_{x,(k,l)}(u,v) & \bar{G}_{x,(k,l)}(u,v) \le TH \\ \text{average}\big[P_{x,(0,0)}(u,v), P_{x,(0,L-1)}(u,v), \\ P_{x,(L-1,0)}(u,v), P_{x,(L-1,L-1)}(u,v)\big] & otherwise \end{cases} \quad (18)$$

where average [] represents an averaging operation.

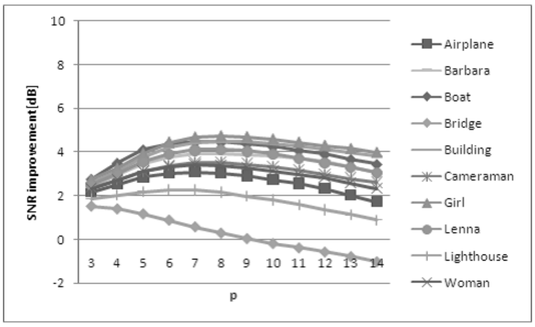
In (18) , the equation for $\bar{G}_{x,(k,l)}(u,v) \le TH$ is the same as that in (15). Additional part is only the averaging one. This addition comes from the following reasons. The original image has its most of the frequency components in a low frequency region. This results in the fact that the high frequency components of the image degraded by white noise are only noise, because the white noise has a flat power spectrum in all the frequency regions. According to this inspection, we set out to cover the low frequency region of




(a) SNR=0[dB]




(b) SNR=5[dB]




(c) SNR=10[dB]

Fig. 2: Relation between $p$ and SNR improvement.

the noise power spectrum with average of blocks of four corners being at the highest frequencies. Thus, the noise power spectrum estimate is obtained by (17) with $P_{x,high}(u,v)$ in (18).

It may be necessary for the proposed noise estimation method to investigate the optimal values of parameters $L$ and $p$ again. $L$ is considered not to be influenced by difference between the conventional and proposed methods. Therefore, we supposed that the optimal value of parameter $L$ is 32 and examined only the parameter $p$ dependency. For images of SNR 0, 5 and 10[dB], the relation between the division rate $p$ and SNR improvement was investigated. The results are shown in Figure 2. From Figure 2, we can observe that for near the optimal value of the division

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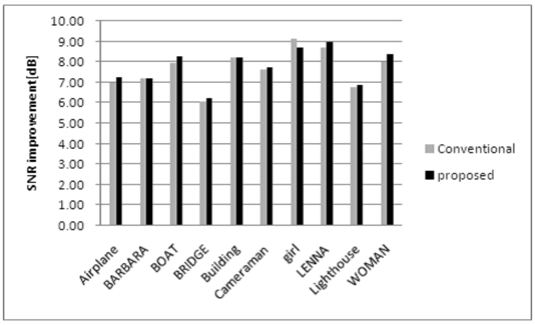
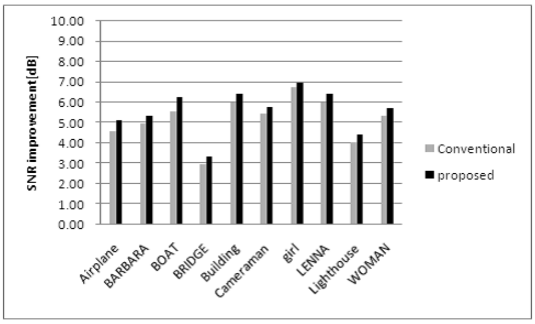
rate $p$, there is no great difference in restoration accuracy. Since the optimal value of $p$ often becomes around 8.5 in many images, we settle the division rate $p$ as 8.5 for the followed simulation experiments.

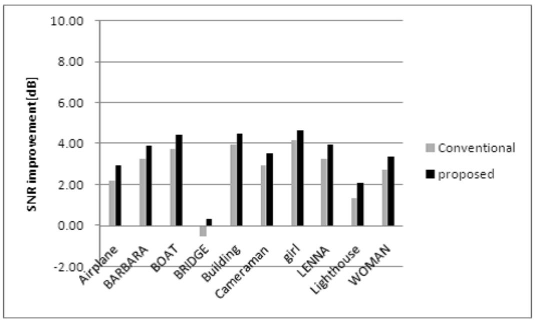
### 3.3 Comparison of Restoration




(a) SNR=0[dB]




(b) SNR=5[dB]




(c) SNR=10[dB]

Fig. 3: SNR improvement for the conventional and proposed methods.

The restoration results obtained by using the conventional and proposed noise estimation methods based on Wiener filtering are shown in Figure 3. It is clear that the performance of the proposed method is better than that of the conventional one for almost all the images. Moreover, as shown in Figure 4, a feature of the proposed method compared with the conventional one is little blur on the processed image. This

(a) original

(b) degraded

(c) conventional

(d) proposed

Fig. 4: Noise estimation comparison for Wiener filtering in case of SNR=0[dB].

is because the parameter setting of division rate $p$ is relatively less sensitive in the proposed method. The results in Figures 3 and 4 obviously suggest that the Wiener filter with more accurate noise power spectrum estimation leads to more noise reduction with less blur.

# 4 Conclusion

In this paper, we have investigated the Wiener filter for restoration from an image degraded by white noise. In an ideal case where both the original and noise images are known, it has been found that the Wiener filter in the frequency domain is more effective than that in the time domain. In order to apply the frequency domain Wiener filter in real cases, we need a noise power spectrum estimation method. To this end, we have proposed a modified noise estimation method based on frequency band division processing. Examining the performance of the frequency domain Wiener filter by using the proposed noise estimation method, in comparison with by using the conventional noise estimation method, we have confirmed that the former brings higher performance with reducing blurring in the processed image.

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