IDENTIFICATION FIRST ORDER STOCHASTIC SYSTEM WITH ESTIMATION PARAMETERS: RECURSIVE DESCRIPTION

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Abstract: This paper presents identification analysis problem respect to first-order stochastic invariant time with unknown parameter evolution, expressed it in finite difference within limit velocity time. Requiring in this case the dynamical interaction between two filters (estimation and identification structures), giving both a good enough convergence rate respect to the real observable signal system. The identification filter gain $\mathbf{K}$ as second probability moment error, resulted as difference between the signal model system and its identification results, achieving greater convergence without lost the natural time interval interaction, minimizing the error trajectories respect to proposed model considering in it, dynamical estimation respect to unknown parameters. Emphasizing the dynamics filter interaction, using the stochastic gradient filter method as estimator into first order identification filter. The identification filter model based in the black box scheme didn’t use the transition matrix because didn’t know the dynamical parameter condition; although in ideal description require it. Therefore, in the real sense is better to use dynamical estimator inside it as to see in the concepts developed below. This dynamical interaction could be used in tracking paths, navigation systems and gravimetric measures, description, reconstruction and information systems, requiring its operations to describe internal parameters and states, respectively. The natural stochastic system evolution shines without lost its internal properties.

Keywords: Kalman Filter, Error Functional, Stochastic gradient, Linear Dynamic System.

1 Introduction

The identification filter advantages [1], appeared at the beginning of the sixties in the near last century; compared with the Nyquist [2] and Bode [3] works, in the twenties, and Wiener [4] in the thirties, all of them in the in the near last century. Nevertheless, the Wiener filter commonly applied to linear systems; with stationary conditions: the first probability moments bounded respect to signal reference system.

The identification Kalman filter structure has become a fundamental tool for analyzing and solving a broad class of identification problems. The first application known, was at NASA (National Aeronautics and Space Administration), subdivision ARC Ames Research Center in the early 60s during feasibility studies for navigation and control of the Apollo 11 space capsule. The first step was the creation of a trajectory analysis program capable of simulating a trajectory to the moon and back, were investigated linear perturbation methods for implementing small changes in speed and simulate corrections [5], to find a about the best path to follow for this ship land on the moon and return to earth.

Within the inertial navigation, Kalman filter allowed to use redundant information to identification system state describing positions and velocities derived from observations GNSS (Global Navigation Satellite System) not only helps to correct mistakes identifying their attitude internal states.

The position and velocity determination depend of attitude, equivalently; the error resulted in the identification of the position and speed, as an error function in attitude determination. The quantification covariance matrix vector of the state system allows the filter correction among errors orientation information [6]. In [7] described the Kalman filter as a tool for land navigation with GPS focused on analyzing the errors in inertial sensors in its inertial systems, considering two error types: a) Initialization (INS) and b) alignment errors system. INS was integrated by acceleration IDs register, propagating initial position and velocity. So that, any error selecting the initial
conditions causes consistent position and velocity errors. Solving the acceleration recorded components in the navigation course; into INS allowing the accelerometers orientation relative to the navigation. Usually the GPS / INS Kalman filter model predict the sensors error and try to correct them. In addition, the INS error model would be also used for to identify the error sources. The identification systems need correct answer limiting the answer mistakes as well as possible.

In the last thirty years, the Scientists has implemented several applications of this kind of filter, operating in dynamic structures.

The science impacted by the filtering theory considering the recursive digital filters as a decision tools. For example in automatic control, the prediction states affect the control system regulatory action. In digital systems, the dynamics noise elimination using the windowing method, understanding the information signal. In economy the filter prediction also allow to limit the makes decisions, eliminating risk caused by perturbation variables.

2. Problem Description

The discrete identification filter algorithm requires knowing the transition matrix evolution, respect to dynamics system internally speaking. But as an observer, respect to black box consideration, the system in direct form don’t observe the transition matrix evolution; i.e., in the black box sense the interaction signals is only between the input and output signals. Then, in the identification filter structure don’t describe the internal parameter evolution describing the transition function neither.

How the changes or integrations behind are to transition matrix into identification filter?

Considering, the systems in the first stochastic order, expressed in discrete form seeing into blocks diagrams as:

3. Problem Solution

In this paper consider the gradient stochastic technique as a tool into parameter estimation in recursive sense. So that the parameter dynamical description used into identification filter behind to transition invariant matrix and the identification parameter adjusted as a functional derived respect to identification error allows observe a high-level convergence between the filter description and the real signal.

The stochastic gradient is an optimal parameter estimation tools; and also the identification filter. Both are optimal in local conditions convergence.

Fig 2, illustrates the model block scheme between both filters techniques interacting.

Figure 2. Block scheme between estimator and identification filters.

Theorem 1 (Internal states identification for first order stochastic system): A system described by first order stochastic model expressed in finite differences:

$$x_{k+1} = A_k x_k + B_k v_k \cdot y_k = C_k x_k + w_k.$$  \hspace{1cm} (1)

With time evolution bounded \( \tau_k < \infty \) (for each interval time (Where \( k \) index representing the evolution according to the Nyquist time interval of time that the system moves from one state to another.), accomplishing \( \tau_k = \frac{1}{\sum_{n=1}^{\infty}} \) ( \( f_{max} \) is the system frequency and limited \( f_{max} < \infty \) ) with probability properties:

$$w_k \in N(\mu = k_w, \sigma^2 < \infty), \quad k_w \in \mathbb{R}^+,$$

$$v_k \in N(\mu = k_v, \sigma^2 < \infty), \quad k_v \in \mathbb{R}^+.$$  

$$w_k, v_k \subseteq \{[0,1), k\} x_k, x_{k+1}, y_k \mathbb{R}^{[0,1]}_k;$$

$$A_k, B_k, C_k \in \mathbb{R}^{[0,1]}_k.$$  

The filter identification has the basic form:

$$\hat{x}_{k+1} = \hat{A}_k \hat{x}_k + \hat{B}_k \hat{w}_k. \hspace{1cm} (2)$$

With gain internal parameter and perturbation, respectively:
\[
\begin{align*}
K_k &= \hat{A}_k C_k^T [c_k J_k C_k^T + R_k],
\end{align*}
\]

\[
\hat{w}_k \in N(c_k, \hat{\sigma}_k^2, \infty), \quad k, \in \mathbb{R}_+;
\]

\[
\hat{A}_k, \quad C_k, \quad J_k, \quad R_k \in \mathbb{R}^{(n\times n)}.
\]

Where the error functional, has the form:

\[
J_{k+1, \text{min}} = \hat{A}_k J_k \hat{A}_k^T - \hat{K}_k C_k J_k \hat{A}_k^T + B_k Q_k B_k^T,
\]

\[
Q_k := E\{v_k v_k^T\}.
\]

**Proof:** Considering that the identification error:

\[
e_k := x_k - \hat{x}_k, \quad e_k \in \mathbb{R}_+^{(n\times 1)}, \quad e_k \subseteq (0,1, k),
\]

\[
, \quad e_k \in N(\mu_{e_k} = k, \sigma_{e_k}^2, \infty), \quad k, \in \mathbb{R}_+;
\]

With error functional, and noises variances respectively as:

\[
J_k := E\{e_k e_k^T\}, \quad R_k := E\{w_k w_k^T\},
\]

\[
Q_k := E\{v_k v_k^T\}.
\]

In agreement to (4) one-step time after; and using in it the expressions (1) and (2):

\[
e_{k+1} = A_k x_k + B_k v_k - (\hat{A}_k \hat{x}_k + \hat{K}_k \hat{w}_k).
\]

So that in the identification, the noise accomplish with the following properties:

\[
\lim_{i \rightarrow k} \hat{w}_i \rightarrow w_i \quad \text{with} \quad E\{w_i - \hat{w}_i\} \rightarrow 0
\]

\[
\text{and} \quad \hat{w}_i := y_i - C_i \hat{x}_i.
\]

Substituting (8) in (7):

\[
e_{k+1} = A_k x_k + B_k v_k - (\hat{A}_k \hat{x}_k + \hat{K}_k (y_k - C_k \hat{x}_k))
\]

Grouping respect to internal and its identification state, (9) rewrite as:

\[
e_{k+1} = \left(\hat{A}_k - \hat{K}_k C_k\right) y_k + B_k v_k - \left(\hat{A}_k - \hat{K}_k C_k\right) \hat{x}_k
\]

\[
\text{And that the parameter estimation converges to real gain}
\]

\[
\lim_{i \rightarrow k} \hat{A}_i \rightarrow A_k, \quad \text{with its eigenvalues bounded into interval as} \quad \left\{\lambda_i (\hat{A}_k)\right\} \subseteq (0,1), k : \text{fulfilling the stability discrete conditions}.
\]

\[
e_{k+1} = \left(\hat{A}_k - \hat{K}_k C_k\right) x_k + B_k v_k - \hat{K}_k w_k
\]

The condition described in (5) now in (11), the expression (6) has the new recursive form:

\[
e_{k+1} = \left(\hat{A}_k - \hat{K}_k C_k\right) x_k + B_k v_k - \hat{K}_k w_k
\]

With parameters differences as a gain:

\[
\hat{A}_k = \left(\hat{A}_k - \hat{K}_k C_k\right)
\]

The error developed in (11), has a symbolic result:

\[
e_{k+1} = \hat{A}_k e_k + B_k v_k - \hat{K}_k w_k
\]

According to (6) one-step after:

\[
J_{k+1} = E\{e_{k+1} e_{k+1}^T\}
\]

In (15) use (14) in the second probability moment:

\[
J_{k+1} = E\left\{\hat{A}_k e_k + B_k v_k - \hat{K}_k w_k \right\}
\]

The inner product:

\[
\begin{pmatrix}
\hat{A}_k e_k e_k^T \hat{A}_k^T + \hat{A}_k e_k v_k B_k^T - \hat{K}_k v_k v_k^T \hat{K}_k^T - \hat{K}_k w_k w_k^T B_k^T + \hat{K}_k w_k v_k^T B_k^T \\
\end{pmatrix}
\]

In addition, considering the mathematical expectation properties in the sense that don't have correlation, the expression has the form:

\[
J_{k+1} = \hat{A}_k J_k \hat{A}_k^T + B_k Q_k B_k^T + \hat{K}_k R_k \hat{K}_k^T
\]

With (14) in (18) and develop all inner products:

\[
J_{k+1} = \hat{A}_k J_k \hat{A}_k^T - \hat{K}_k C_k J_k \hat{A}_k^T - \hat{A}_k J_k C_k^T \hat{K}_k^T + \hat{K}_k C_k J_k C_k^T \hat{K}_k^T + B_k Q_k B_k^T + \hat{K}_k R_k \hat{K}_k^T
\]

The stochastic gradient error functional respect to \(\hat{K}_k\), considering that \(\hat{K}_k = R_k^T\), \(\hat{A}_k J_k C_k^T = C_k J_k \hat{A}_k^T, \quad \hat{K}_k R_k = R_k \hat{K}_k^T: i.e.

\[
\nabla J_{k+1} |_{K_k} = -\hat{A}_k J_k C_k^T + \hat{K}_k C_k J_k C_k^T + R_k
\]

And observing that it is minimum in according to hessian expression

\[
\nabla (\nabla J_{k+1} |_{K_k})_{K_k} = \left[C_k J_k C_k^T + R_k \right]
\]

Been necessary that \(\hat{A}_k J_k C_k^T \geq \left[C_k J_k C_k^T + R_k \right];

it’s mean:
\[ T^\ast (x) = \hat{A}_k \cdot x + B_i v_i \]
\[ \ln m_T = \ln m_0 + \ln \left[ e^{(A(T-t_0) + B)(T-t_0)} \right] \] 
\[ \ln m_T = \left[ \ln m_0 + A(T-t_0)x_0 + B \nu(T-t_0) \right] - \left[ \hat{A}(T-t_0) \right] \varepsilon_{t_0} \]
\[ + \hat{K}(T-t_0) \left[ y(T-t_0) - C(T-t_0) \hat{x}_{t_0} \right] \] 
\[ \text{With } m_T = \ln m_T \]
\[ m_T = m_0 + A(T-t_0)x_0 + B \nu(T-t_0) \]
\[ - \left( \hat{A}(T-t_0) \right) \varepsilon_{t_0} \]
\[ + \hat{K}(T-t_0) \left[ y(T-t_0) - C(T-t_0) \hat{x}_{t_0} \right] \] 
\[ \text{Considering } \lim_{t_{\alpha} \to 0} A_{t_{\alpha}} \text{ and } \]
\[ \left\{ \varepsilon(A_{T-t_0}) \right\}_{(T-t_0)} \subseteq \left[ 0,1 \right] (T-t_0) \text{ the error} \]
\[ \text{expressed as: } \]
\[ \bar{m}_T = \bar{m}_0 + \left[ \hat{A}(T-t_0) \right] - \hat{K}(T-t_0) C(T-t_0) m_0 \]
\[ + B \nu(T-t_0) \]
\[ - \hat{K}(T-t_0) w(T-t_0). \] 

4. CONCLUSION

This article presented Kalman an analysis filter using a first order stochastic model identifying its internal state into dynamic invariant. The identification technique was described in finite differences. The optimal filter estimation was required as the Kalman gain K into the filter identification; the last of its based in the error functional between the system and an identification discrete model system proposed to achieve greater convergence between both, seeking in the integrated filter to minimize the tracking error trajectory. Emphasizing into the identifier the estimation of parameters gradient techniques, guarantee the best parameter value respect to model proposed. The integration filter results generate a realistic (theoretically speaking) the greater convergence rate.

References: