Seismic Wave Scattering Analysis using a 3D Boundary Element Method on Topographic Irregularities

REZA TARINEJAD* and MOHAMMAD T. AHMADI
*Faculty of Civil Engineering
University of Tabriz
East Azerbaijan Province, Tabriz
IRAN
r_tarinejad@tabrizu.ac.ir

Abstract: - Amplification of seismic waves in the presence of topographic irregularities is often advocated as one of the possible causes of intensification of damage during earthquakes. Furthermore, topographic site effects may play a significant role in the activation of landslides and rockslides during earthquakes. In this research, topography effects of canyon sites are analyzed using a three-dimensional boundary element procedure. Effects of modeling parameters (free-field and canyon length) are investigated in order to obtain accurate results. It is shown that the free-field length has little effect in comparison with the canyon length to obtain accurate results. Some general rules for three-dimensional modeling of seismic wave scattering with boundary element method are achieved. Also effects of different wave parameters (frequency and direction), material properties (damping ratio and Poisson's ratio) are investigated. It is demonstrated that material properties, wave incident angle and frequency have considerable effects on the wave scattering problems. The system with damping may induce large displacements in the side of the canyon located in the wave arriving direction in comparison to system without damping. Wave parameters (direction and frequency) affect on the pattern of the displacement variation across the canyon on the other hand material properties (damping and Poisson’s ratio) don’t affect on the pattern of the displacement variation. As the wavelength of the input wave becomes comparable to the characteristic length of the canyon, more complicated wave pattern of the scattered field is expected.

Key-Words: - Scattering, Seismic Wave, Boundary Element Method, Topographic Amplification, Parametric Study, 3D-model

1 Introduction
Topographic conditions play an important role on the modification of seismic ground motion. Therefore, their effects may become crucial in the selection or simulation of ground motion for use in structural seismic response analysis. The effects of surface topography can greatly enlarge the site response exerting an important influence on the distribution of damage observed during earthquakes [1]. Some seismic codes are concern about the importance of topographic effects [2]. In the last decades the evaluation of topographic effects was done via different approaches. Different analytical and numerical techniques were adopted to deal with topographic effects on the seismic wave scattering problems [(e.g., Trifunac [3], Sanchez-Sesma [4], Paolocci [5], Chopra [6], Dravinski [7], Luco [8], Assimaki [9] and Kamalian [10]). On the other hand the recorded ground motions after recent destructive earthquakes have made possible empirical estimation of topographical effect (e.g., the 3 March 1985 Chile Earthquake [1] and the 1994 Northridge Earthquake in California [11]). While a great deal of work has been done on the two-dimensional elastic response of an isotropic medium, very little has been published on three-dimensional analyses. In this research the three dimensional boundary element method proposed in Ref. [12] is used to study the amplification of elastic waves by a three dimensional canyon. Incident plane harmonic seismic waves are considered. The accuracy of the method is tested through comparison with results of other studies, and effects of different parameters are investigated.

2 Topography
An arbitrarily shaped canyon of finite length is considered as illustrated in Figure 1. Seismic body waves arrive from an arbitrary direction with angles of $\theta_x$ and $\theta_y$ respect to the horizontal x- and the vertical z-axes, respectively. The half-space is
characterized by the P and S wave velocities $c_p$ and $c_s$, respectively.

Figure 1. The topographic system considered is an arbitrarily shaped canyon of finite length with the incident waves arriving from an arbitrary direction.

3 Background Theory and Boundary Element Method

The governing wave equation for an elastic, isotropic and homogeneous body is:

$$c_1^2 \nabla (\nabla \cdot u) - c_2^2 \nabla \times \nabla \times u - \frac{\partial^2 u}{\partial t^2} = -b$$  \hspace{1cm} (1)

in which $u$ denotes the displacement vector, $b$ denotes the body force vector, and $c_1$ and $c_2$ are the propagation velocities of compression (P) and shear (S) waves, respectively. The velocities are related to the properties of the medium through:

$$c_1 = (\lambda + 2\mu / \rho)^{0.5}, \quad c_2 = (\mu / \rho)^{0.5}$$  \hspace{1cm} (2)

where $\lambda$ and $\mu$ are the Lame constants and $\rho$ is the mass density [10-11].

The boundary element method (BEM) is very effective when dealing with wave propagation problems in infinite media with geometrical irregularities. The main advantage of this method is that discretization is only applied at the boundaries of the physical domain, thus reducing the number of unknown variables significantly in comparison to other methods such as finite element and finite difference techniques. Since the fundamental solutions automatically satisfy the far-field condition, the BEM is especially well-suited to problems involving infinite domains such as a canyon located on a half-space [13,14].

The corresponding governing boundary equation for an elastic, isotropic, homogenous body can be obtained using the well-known dynamic reciprocal theorem as:

$$c' u' + \int_{\Gamma} p^* u d\Gamma = \int_{\Gamma} u^* p d\Gamma$$  \hspace{1cm} (3)

where $p^*$ and $u^*$ are the fundamental solution for traction and displacement respectively, at a point $x$ when a unit Dirac Delta load is applied at point $i$. In the BEM the variables $u$ and $p$ are discretized into the values at the so-called collocation nodes.

The displacement and traction fields are interpolated over each element using a set of shape functions. The same shape functions are also used to approximate the geometry, i.e. the elements are isoparametric. Discretization of Equation (3) yields:

$$c' u' + \sum_{j=1}^{ne} \left[ \int_{\Gamma_j} p^* \Phi d\Gamma \right] u' = \sum_{i=1}^{ni} \left[ \int_{\Gamma_i} u^* \Phi d\Gamma \right] p'$$  \hspace{1cm} (4-a)

The expressions inside the braces can be replaced with the more familiar abbreviations:

$$\int_{\Gamma_j} u^* \Phi d\Gamma = \sum_{i=1}^{ni} \int_{\Gamma_i} u^* \Phi d\Gamma = \Phi_j^i$$

$$\int_{\Gamma_j} p^* \Phi d\Gamma = \sum_{j=1}^{ne} \int_{\Gamma_j} p^* \Phi d\Gamma = \Phi_j^i$$

According to Equation (4) the surface integral is exchanged with a sum of integrals over $ne$ elements. It should be noted that the BEM allows the use of constant elements, where the displacements and tractions are assumed to be constant over the entire element leading to discontinuities at the element edges. However, this kind of element is inadequate for most wave propagation problems as the convergence is very slow compared to that of higher-order elements. In this research quadratic 9-node elements are used. After assembling all equations the following set of equations is obtained:

$$H\mathbf{U} = \mathbf{GP}$$  \hspace{1cm} (5)

in which $H$ is a matrix, $U$ is a displacement vector, $G$ is a traction vector, $P$ is a load matrix, and $\rho$ is the product of the number of elements and number of nodes. The problem has $3n$ unknowns that should be obtained by solving Equation (5).

4 Numerical Results and Discussion

A special-purpose 3-D computer program (TDASC) was developed to implement the boundary element procedures for an incident plane wave with circular frequency $\omega$. The propagation direction of the wave
was defined by angles $\theta_h$ and $\theta_v$ corresponding to the angle of the ray (normal to wave front) from the horizontal $x$- and vertical $z$-axes (Fig.1). The program can be used for canyons of arbitrary shape. The numerical examples of this section are designed to demonstrate the accuracy and efficiency of the method for different cases [15].

4.1 Validation study
To establish the numerical accuracy of the method, problems involving the scattering of a harmonic plane wave, for which results are available from previous works, are solved using the proposed method. In all cases a semi-circular cross section of radius $R$ cut in a homogenous half space is considered. The semi-cylindrical canyon and a length $3R$ of the free field on both side of the canyon are discretized as shown in Fig.2.

![Fig. 2. A schematic picture of the descretized semi-cylindrical canyon](image)

The model has 240 nine-node boundary elements with 1029 nodes along the main boundary. The results obtained by the boundary element (BE) method for a 2-D SH wave of unit amplitude impinging normal to the canyon axis is compared first with the exact solution by Trifunac [3]. The results presented in Fig.3 show the displacement amplitudes around the canyon for the horizontal angle of $\theta_h = 45^\circ$ the vertical angles of $\theta_v = 1^\circ$ and $45^\circ$ for unit dimensionless frequency ($\Omega = \omega R / \pi c = 1$). A similar comparison is presented in Figure 4 for $\theta_v = \theta_h = 0$ , which corresponds to a vertically incident SH wave with particle motion perpendicular to the axis of the canyon. For this test case the numerical result obtained by Zhang and Chopra [8] is used for the comparison. There is good agreement between results obtained by the BE method and the previous results. The finite length of the canyon and free-field has only a small effect on the computed displacements [15].

In order to investigate the effect of a free-field length ($L_h$) that should be modeled in the finite 3D model, two different free-field lengths ($L_h= 1.5R$ and $3R$) with the same canyon length ($L= 5R$) are considered and compared with each other in Fig.5. The results obtained show that this parameter has a little effect on the accuracy of obtained results. Also in order to investigate the effect of canyon length ($L$), different models with the same free-field length are analyzed. The results of these analyses are presented in Fig.6. These results show that the canyon length is more effective parameter than free-field length in obtaining accurate results. The long canyon gives more accurate results than the short one. Analysis with different lengths of canyon show that to obtain accurate results one does not need to model beyond $10R$ length of the canyon. The same analysis with different lengths of free-field is done and shows that in order to obtain accurate results the

![Fig. 3. Displacement amplitudes obtained by the BE method and Trifunac for incident SH-Wave with $\Omega = 1$.](image)

![Fig. 4. Displacement amplitudes obtained by the BE method and Zhang and Chopra for incident SH-Wave with $\theta_h = 0 , \theta_v = 0$ and $\Omega = 1$.](image)
Fig. 5. Comparison of results obtained for two different free-field lengths by the BE method and Sanchez-Sesma for incident P-Wave with \( \theta_s = \theta_c = 45^\circ \), and \( \Omega = 1 \).

free-field surface should be discretized on both sides of the canyon over a distance of at least three times of the canyon radius [15].

Fig. 6. Comparison of results obtained for two different canyon lengths by the BE method and Sanchez-Sesma for incident P-Wave with \( \theta_s = \theta_c = 45^\circ \), and \( \Omega = 1 \).

4.2 Effects of wave characteristics
The results obtained for analysis with different dimensionless frequencies are shown in Fig. 9. As Fig. 7 shows the variation of displacements across the canyon is more complicated for high dimensionless frequency rather than low dimensionless frequency. At unit dimensionless frequency the wavelength of the incident field is equal to the diameter of the valley; on the other hand for dimensionless frequency equal to 0.5 the wavelength of the incident field is equal to two times of the diameter of the valley. Hence for the dimensionless frequency equal to 0.5 the input wave does not detect the presence of the canyon as the scatterer as well as the unit dimensionless frequency case. As the wavelength of the input wave becomes comparable to the characteristic length of the canyon, more complicated wave pattern of the scattered field is expected.

Analyses for different wave incident angles with unit dimensionless frequency are done and results are presented in Fig. 8. Different patterns of the displacement variation across the canyon are achieved for different wave incident angles. Therefore wave incident angle is one of the most important parameter on the pattern of the displacement variation across the canyon.

Fig. 7. Comparison of results obtained for two different dimensionless frequency (0.5 and 1) for incident SH-Wave with \( \theta_s = \theta_c = 45^\circ \).
4.3 Effects of material parameters
The effect of material properties of half space such as damping ratio and Poisson's ratio are investigated by different analysis and illustrated in Figures 9 and 10, respectively. The results show that these parameters don't affect on the pattern of the displacement variation across the canyon but in order to get accurate results of topographic phenomenon, these parameters should be evaluated precisely. The system with damping may induce large displacements in the side of the canyon located in the wave arriving direction in comparison to system without damping. In general difference of maximum displacements obtained in both sides of the canyon is increasing with damping [15].

5 Conclusion
The boundary element method proposed by Ahmad and Banerjee [12] is employed to three dimension problems. Results from this method are compared to previous results obtained by Trifunac [3] and Zhang and Chopra [6]. This method yields accurate results and can be used for real-world problems with complex topographies.
Effects of different parameters are investigated and the following results obtained:
The free-field length has little effect than the canyon length to obtain accurate results. Analysis with different lengths of canyon and free-field show that in order to obtain accurate results one does not need to go beyond 10R and 3R, respectively.
It is shown that material properties, wave incident angle and frequency have considerable effects on the wave scattering problems. The system with damping may induce large displacements in the side of the canyon located in the wave arriving direction in comparison to system without damping. Wave parameters (direction and frequency) affect on the pattern of the displacement variation across the canyon on the other hand material properties (damping and Poisson’s ratio) don’t affect on the pattern of the displacement variation.

Fig. 8. The results obtained for different wave incident angles with unit dimensionless frequency for incident SH-Wave.

Fig. 9. Comparison of results obtained for different damping ratios for incident SH-Wave with $\theta_0 = \theta_c = 45^\circ$ and $\Omega = 1$. 
Fig. 10. Comparison of results obtained for different Poisson ratio for incident SH-Wave with $\theta_e = \theta_t = 45^\circ$ and $\Omega = 1$.

References:


