Design and Implementation of Embedded Fuzzy Controllers Based on Fourier computation of Membership Functions

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Abstract:
In an earlier paper, we derived explicit Fourier series expressions for systematic computation of grade of membership in the overlap and non-overlap regions of triangular fuzzy sets; and by implication, computation of union and intersection of the fuzzy sets. In this paper, we hereby extend the methodology to cover cases of the cosine, exponential and Gaussian fuzzy sets by presenting explicit Fourier series representation for encoding fuzziness in the overlap and non-overlap domains of membership functions of cosine and Gaussian fuzzy sets. The paper further present the development of a corresponding embedded "Fuzzy controller", which incorporates the formal mathematical representation, to measure temperature and pressure and produce output that can serve as input to other sub-systems or systems. In particular, it is established that the technique can indeed be incorporated in engineering systems for dynamic determination of grade of membership of adjoining Fuzzy sets and thus provides a basis for the design and implementation of embedded Fuzzy controller for mission-critical applications.

Key-Words: Fuzzy controller, cosine fuzzy set, Gaussian fuzzy set, Fourier series, Membership functions

1 Introduction

Whereas most *Fuzzy logic* applications are intended for control and analysis purposes, some other applications have been in the area of system state prediction. According to Zadeh [1], the key elements in human thinking are not numbers, but labels of *Fuzzy set*s, that is classes of objects in which the transition from membership to non-membership is gradual rather than abrupt. In various applications, *Fuzzy set*s may stem either from experts or from input-output numeric data. For instance, the work of Mamdani and Assilian [2] used knowledge from an expert in the form of rules from *Fuzzy* antecedents to *Fuzzy* consequents, whereas the work of Tagaki and Sugeno [3] employed input-output data to identify a *Fuzzy* model for a system whose antecedents are conventional *Fuzzy set*s and the consequents are linear input-output relations. In practise, Mamdani ([4],[5]) inference is used for control systems and when a system aims to emulate the intuitive human expert thought process. A computationally cheaper alternative is the Sugeno [6] inference. *Fuzzy logic* has found ample applications for control and analysis purposes, as for example recorded in the work of Bellman and Zadeh [7], Berenji and Khedar [8]. Ruan and Fantoni [9] also reported industrial applications of *Fuzzy logic*. Olunloyo and Ajofoyinbo [10] applied hybrid *Fuzzy-stochastic* methodology for maintenance optimization. In this work, *Fuzzy* methods were used to enhance or improve the parameters of the stochastic process. Araujo, Sandri and Macau [11], Marinke and Araujo [12], and Moura, Rodrigues and Araujo [13] presented some other industrial applications of *Fuzzy* systems/logic most of which are related to thermal-vacuum processes, in particular, in the qualification of space devices. In current literature, researchers generally treat the overlap region as Intersection or Union of two or more *Fuzzy set*s and have invoked the Min and Max Operators, respectively, as needed. Badiru and Arif [14], for example, treated the *Fuzzy* overlap as an intersection of two adjoining *set*s viz: "Low" and "not Low" and invoked the Min Operator to generate output. Olunloyo, Ajofoyinbo and Badiru [15] in an earlier paper, also proposed another algorithm for the treatment of overlap of adjoining

*Fuzzy set*s based on partitioned grids. In view of the importance of this *Fuzzy* overlap region, especially where there is need to monitor and ensure smooth transition between the adjoining *Fuzzy set*s in relation to the design of mission critical applications, Olunloyo and Ajofoyinbo [16] proposed an alternative approach for determination of membership function based on the Fourier series representation of the envelope of the *Fuzzy* patch. This methodology, in conjunction with the Min- and Max- Operators can provide a sound basis for the design and building of fault-tolerant mission critical engineering systems. In the Literature, for example, as in the work of King and Mamdani [17], and Zimmermann [18], most control applications use triangular and trapezoidal profiles for membership functions. However, such triangular or trapezoidal assumptions, in most applications are generally based on approximation from the Gaussian Membership function.

1.1 Membership Function

Membership function essentially embodies all fuzziness for a particular *Fuzzy set*, and its description is the essence of a *Fuzzy* property or operation, Ross [19]. The determination of membership function can be categorized as either being manual or automatic. Watanabe [20] asserts that the statistical techniques for determining membership functions fall into two broad categories viz: use of frequencies and direct estimation. The two methods were analysed by Turksen [21] when he reviewed the various methods and researched into the acquisition of the various methods. The automatic generation of membership function emphasise the use of modern soft computing techniques (in particular Genetic Algorithm and Neural Networks). Meredith, Karr and Krishna [22] applied Genetic Algorithm (GA) to the fine tuning of membership functions in a *Fuzzy logic* controller for a helicopter. Karr [23] applied GA to the design of *Fuzzy logic* controller for the Cart Pole problem. Lee and Takagi [24] also tackled the Cart problem. They took a holistic approach by using GA to design the whole system (determine the optimal number of rules as well as the membership functions). Ross [25] reported on six popular methods for developing membership functions namely: Intuition, Inference, rank ordering, neural networks, genetic algorithm and inductive reason. The manual and automatic techniques for determining membership functions of *Fuzzy set*s are non-systematic and suffer from certain deficiencies. On one hand, the existing automatic techniques are heuristic in nature, which implies that different values can be obtained for same input values presented at different times. On the other hand, the manual techniques suffer from the deficiency that they rely on subjective interpretation of words and the peculiarities of the engaged human expert.

1.2 Basic Properties of Fourier series

By analyzing the nature of the overlap patches defined by the Intersection and Union of a typical grade of membership function for a linguistic variable, it is shown that the resultant signal does fall into the class of functions for which a Fourier series representation can be written. The problem then is to construct such a series and compute the corresponding coefficients. Furthermore, in order to align the results with the properties of membership grade functions, some element of normalization and standardization is introduced. To be more specific, starting with triangular *Fuzzy set*s, Olunloyo, Ajofoyinbo and Badiru [26] formulated explicit Fourier series representation for computing the grade of membership in the overlap and non-overlap regions. We hereby extend that methodology by obtaining explicit Fourier series expressions for computing the Union and Intersection of the Gaussian, cosine and exponential *Fuzzy set*s.

2 Problem Formulation

The trigonometric series of the form

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty}\left(a_k \cos kwx + b_k \sin kwx\right) \qquad (1)$$

where a_k, b_k are coefficients and the period $p = \frac{2\pi}{w}$, are encountered in the treatment of many physical problems. Examples abound in the theory of sound, heat conduction, electromagnetic waves, electric circuits and mechanical vibrations (Sokolnikoff and Redheffer, [27]). This is the Fourier series. One fundamental feature of the series is that it has period 2π. The period is taken as $-\pi \le x \le \pi$ or $0 \le x \le 2\pi$, and outside this interval, $f(x)$ is determined by the periodicity condition $f(x + 2\pi) = f(x)$. In general, this periodicity condition also applies for any interval $a \le x \le a + 2\pi$. Fundamental conditions for Fourier series representation are:

a) Function must be periodic
b) Function must have finite number of discontinuities
c) Function must be bounded

We note that the universe of discourse in a *Fuzzy* plane consists of one or more data points. However, each of the data points in a given universe of discourse has some form of data distribution around it in the form of some shape, whether Gaussian, exponential, triangular or any other. Since all data points in the universe of discourse would have same form of data distribution around every data point, we could therefore derive an explicit Fourier series expression for the envelope of the *Fuzzy* patch since we can be assured of the repetition of the distribution pattern around each data point. Furthermore, in as much as the distribution around the data points has same shape, then appropriate normalisation can be introduced to transform the Union and Intersection of such *Fuzzy set*s into functions that are amenable to Fourier series representation. Although various functional profiles of membership functions could be used, the triangular and trapezoidal also serve as approximations of the others in the first instance. In fact, the trapezoidal form can, further, be approximated by the triangular form since the end-points of the tolerance interval in a trapezoidal distribution have the same grade of membership and could therefore be assigned a point value that represents the peak of the triangular profile. It is important to note that, although triangular approximations of Gaussian and cosine *Fuzzy* membership functions may be acceptable globally, they nonetheless suffer from serious local errors. To emphasise this, we shall demonstrate the nature of the local errors which may have significant impact on the accuracy of the *Fuzzy* controllers that are derived from such approximations, and the performance of *Fuzzy* subsystems that are based on such approximations when deployed in mission-critical applications. The approximation error profile presented in Sections 2.1 and 2.2 are based on data obtained from natural gas company in Lagos, Nigeria (refer to Appendix A)

2.1 Union of Gaussian and Cosine *Fuzzy set*s

Whereas percentage error of membership ranges from -130.58% to 88.14% for triangular approximation of the Gaussian membership function, it ranges from -474.05% to 108.02% for the triangular approximation of cosine membership function as illustrated in Figs. 1 and 2 respectively below.

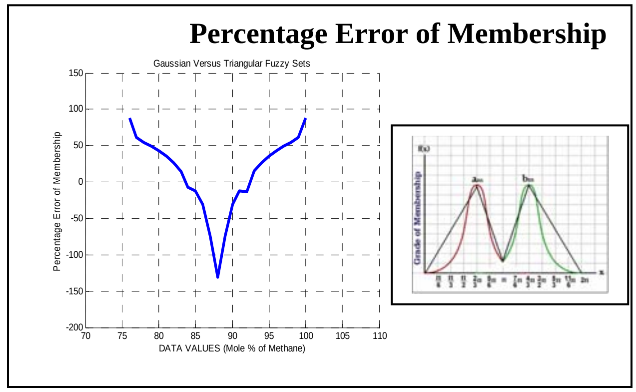

Fig. 1: Gaussian Vs Triangular *Fuzzy set*s

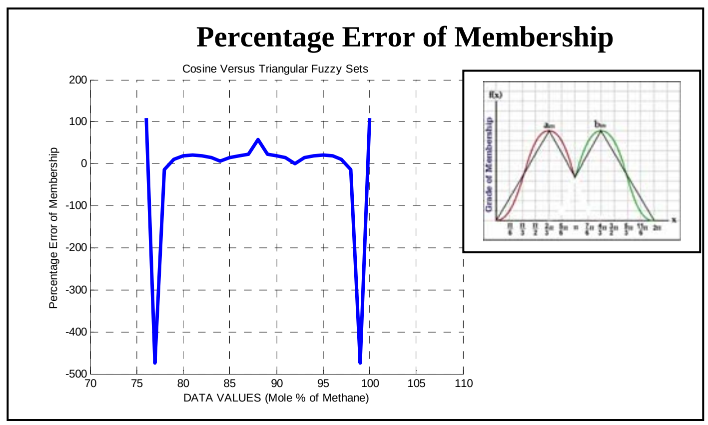

Fig. 2: Cosine Vs Triangular Fuzzy sets

2.2 Intersection of Gaussian and Cosine Fuzzy sets

Similarly, while the percentage error of membership ranges from -569.58% to 4.89% for triangular approximation of the Intersection of adjoining Gaussian *Fuzzy set*s, it ranges from -663.091% to -9.701% for the triangular approximation of the Intersection of adjoining cosine *Fuzzy set*s. These observations are encoded within Figs. 3 and 4 below.

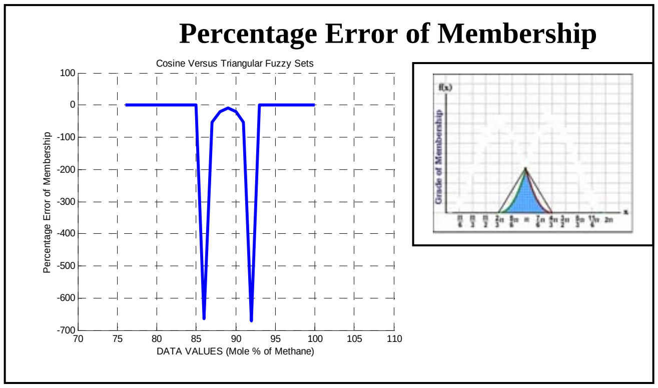

Fig. 3: Cosine Vs Triangular Fuzzy sets

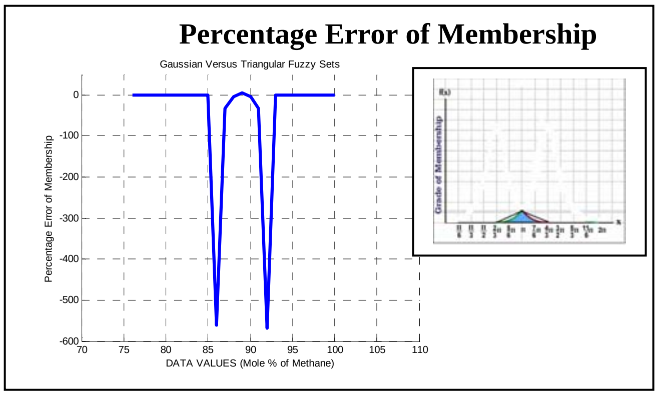

Fig. 4: Gaussian Vs Triangular Fuzzy sets

2.3 Other distributions

We present some of the other distributions that are prevalent in engineering systems as follows:

2.3.1.1 Union of Gaussian Fuzzy sets

The Gaussian membership function is commonly used in engineering problem domain especially for engineering measurements, as it gives actual representation at every point.

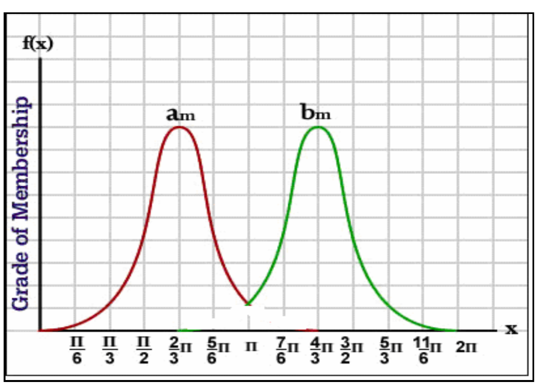

Fig. 5: Union of Gaussian Fuzzy sets

The corresponding normalized Union of the Gaussian *Fuzzy sets* is described by Fig. 5. In particular, the Union of the Gaussian *Fuzzy sets* is described by the function $g(\bar{x})$, where

$$g(\bar{x})=\begin{cases}\dfrac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\bar{x}-\frac{2\pi}{3}\right)^2} & ; \quad 0\le\bar{x}\le\pi \\ \dfrac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\bar{x}-\frac{4\pi}{3}\right)^2} & ; \quad \pi\le\bar{x}\le2\pi\end{cases} \tag{2}$$

The membership function of union of Gaussian *Fuzzy sets* is computed as:

$$f(\bar{x})=\frac{a_0}{2}+\sum_{k=1}^{n}\left(a_k\cos(kw\bar{x}) + b_k\sin(kw\bar{x})\right) \tag{3}$$

where

$$\frac{a_0}{2}=\frac{1}{8\pi^3}\left(\int_0^{\pi}e^{-\left(\frac{1}{2}\bar{x}^2+2\left(\frac{-2\pi}{6}\right)\bar{x}+\frac{4\pi^2}{18}\right)}d\bar{x}+\int_{\pi}^{2\pi}e^{-\left(\frac{1}{2}\bar{x}^2+2\left(\frac{-4\pi}{6}\right)\bar{x}+\frac{16\pi^2}{18}\right)}d\bar{x}\right) \tag{4}$$

Recall from Abramowitz and Stegun (28) that:

$$f(t)=\int e^{-\left(at^2+2bt+c\right)}dt=\frac{1}{2}\sqrt{\frac{\pi}{a}}*e^{\frac{b^2-ac}{a}}*erf\left(\sqrt{a}*t+\frac{b}{\sqrt{a}}\right)+Const \tag{5}$$

Thus

$$\frac{a_0}{2}=\frac{1}{8\pi^3}\left\{\begin{matrix}0.5*\sqrt{2\pi}\left[erf\left(\sqrt{0.5}*\pi+\dfrac{\left(\frac{-2\pi}{6}\right)}{\sqrt{0.5}}\right)-erf\left(\dfrac{\left(\frac{-2\pi}{6}\right)}{\sqrt{0.5}}\right)\right] \\ +0.5*\sqrt{2\pi}\left[erf\left(\sqrt{0.5}*2\pi+\dfrac{\left(\frac{-4\pi}{6}\right)}{\sqrt{0.5}}\right)-erf\left(\sqrt{0.5}*\pi+\dfrac{\left(\frac{-4\pi}{6}\right)}{\sqrt{0.5}}\right)\right]\end{matrix}\right\} \tag{6}$$

From the foregoing we note as follows:

$$I_1=0.5\sqrt{2\pi}*\left(erf\left(\sqrt{0.5}*\pi+\frac{\left(\frac{-2\pi}{6}\right)}{\sqrt{0.5}}\right)-erf\left(\frac{\left(\frac{-2\pi}{6}\right)}{\sqrt{0.5}}\right)\right) \tag{7}$$

Similarly,

$$I_2=0.5\sqrt{2\pi}*\left(erf\left(\sqrt{0.5}*2\pi+\frac{\left(\frac{-4\pi}{6}\right)}{\sqrt{0.5}}\right)-erf\left(\sqrt{0.5}*\pi+\frac{\left(\frac{-4\pi}{6}\right)}{\sqrt{0.5}}\right)\right) \tag{8}$$

$$a_k = \frac{1}{2\pi^3}\left(\begin{array}{l} \displaystyle\int_0^{\pi} e^{-\frac{1}{2}\left(\bar{x}^{-2} - 2\left(\frac{2\pi}{6}\right)\bar{x} + \frac{4\pi^2}{18}\right)} \cos(k\bar{x})\, d\bar{x} \\ \displaystyle + \int_{\pi}^{2\pi} e^{-\frac{1}{2}\left(\bar{x}^{-2} - 2\left(\frac{4\pi}{6}\right)\bar{x} + \frac{16\pi^2}{18}\right)} \cos(k\bar{x})\, d\bar{x} \end{array} \right) \quad (9)$$

We recall I_1 from equations (7) and I_2 from equations (8) above and express equation (9) in terms of I_1 and I_2 as follows:

$$a_k = \frac{1}{2\pi^3}\left(\int_0^{\pi} I_1 \cos(k\bar{x})\, d\bar{x} \quad + \quad \int_{\pi}^{2\pi} I_2 \cos(k\bar{x})\, d\bar{x} \right) \quad (10)$$

$$a_k = \quad 0 \quad (11)$$

and

$$b_k = \frac{1}{2\pi^3}\left(\begin{array}{l} \displaystyle\int_0^{\pi} e^{-\frac{1}{2}\left(\bar{x}^{-2} - 2\left(\frac{2\pi}{6}\right)\bar{x} + \frac{4\pi^2}{18}\right)} \sin(k\bar{x})\, d\bar{x} \\ \displaystyle + \int_{\pi}^{2\pi} e^{-\frac{1}{2}\left(\bar{x}^{-2} - 2\left(\frac{4\pi}{6}\right)\bar{x} + \frac{16\pi^2}{18}\right)} \sin(k\bar{x})\, d\bar{x} \end{array} \right)$$

(12)

Similarly we recall I_1 from equations (7) and I_2 from equations (8) above and express equation (12) in terms of I_1 and I_2 as follows:

$$b_k = \frac{1}{2\pi^3}\left(\frac{I_1}{k}\left(1-(-1)^k\right) + \frac{I_2}{k}\left(\cos(-1)^k - (-1)^{2k}\right) \right) \quad (13)$$

2.3.1.2 Intersection of Gaussian *Fuzzy sets* (Exponential Membership Function)

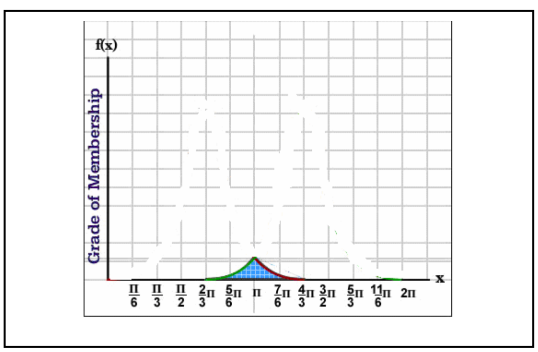

Fig. 6: Intersection of Gaussian *Fuzzy sets*

We obtained the expression for computing membership function in the overlap region of Gaussian *Fuzzy sets* as:

$$f(\bar{x}) = \frac{a_0}{2} + \sum_{k=1}^{n}\left(a_k \cos(kw\bar{x}) + \quad b_k \sin(kw\bar{x})\right) \quad (14)$$

where

$$\frac{a_0}{2} = \frac{1}{2\pi}\left(\begin{array}{l} \displaystyle\int_{\frac{2\pi}{3}}^{\pi} \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{1}{2}\bar{x} + 2\left(\frac{-4\pi}{6}\right)\bar{x} + \frac{16\pi^2}{18}\right)} d\bar{x} \\ \displaystyle + \int_{\pi}^{\frac{4\pi}{3}} \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{1}{2}\bar{x} + 2\left(\frac{-2\pi}{6}\right)\bar{x} + \frac{4\pi^2}{18}\right)} d\bar{x} \end{array} \right)$$

(15)

From equation (5), we re-write equation (15) as follows:

$$\frac{a_0}{2} = \frac{1}{2\pi}\left\{ \begin{array}{l} 0.5*\sqrt{2\pi}\left[erf\left(\sqrt{0.5}*2\pi + \frac{\left(\frac{-4\pi}{6}\right)}{\sqrt{0.5}}\right) - erf\left(\sqrt{0.5}*\frac{2\pi}{3} + \frac{\left(\frac{-4\pi}{6}\right)}{\sqrt{0.5}}\right)\right] \\ + \quad 0.5*\sqrt{2\pi}\left[erf\left(\sqrt{0.5}*\frac{4\pi}{3} + \frac{\left(\frac{-2\pi}{6}\right)}{\sqrt{0.5}}\right) - erf\left(\frac{\left(\frac{-2\pi}{6}\right)}{\sqrt{0.5}}\right)\right] \end{array} \right\}$$

(16)

From equation (16), we define J_1 and J_2 as follow:

$$J_1 = 0.5*\sqrt{2\pi}\left[erf\left(\sqrt{0.5}*2\pi + \frac{\left(\frac{-4\pi}{6}\right)}{\sqrt{0.5}}\right) - erf\left(\sqrt{0.5}*\frac{2\pi}{3} + \frac{\left(\frac{-4\pi}{6}\right)}{\sqrt{0.5}}\right)\right]$$

(17)

$$J_2 = 0.5*\sqrt{2\pi}\left[erf\left(\sqrt{0.5}*\frac{4\pi}{3} + \frac{\left(\frac{-2\pi}{6}\right)}{\sqrt{0.5}}\right) - erf\left(\frac{\left(\frac{-2\pi}{6}\right)}{\sqrt{0.5}}\right)\right]$$

(18)

Thus,

$$a_k = \frac{1}{\pi}\left(\begin{array}{l} \displaystyle\int_{\frac{2\pi}{3}}^{\pi} \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{1}{2}\bar{x}^{-2} + 2\left(\frac{-4\pi}{6}\right)\bar{x} + \frac{16\pi^2}{18}\right)} \cos(k\bar{x})\, d\bar{x} \\ \displaystyle + \int_{\pi}^{\frac{4\pi}{3}} \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{1}{2}\bar{x}^{-2} + 2\left(\frac{-2\pi}{6}\right)\bar{x} + \frac{4\pi^2}{18}\right)} \cos(k\bar{x})\, d\bar{x} \end{array} \right)$$

(19)

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By plugging-in J_1 and J_2 from equations (17) and (18), we can re-write equation (19) as:

$$a_k = \frac{1}{\pi k}\left\{ J_2 \sin\left(\frac{k4\pi}{3}\right) \quad - \quad J_1 \sin\left(\frac{k2\pi}{3}\right)\right\} \tag{20}$$

Similarly, by plugging-in J_1 and J_2 from equations (17) and (18), we obtain:

$$b_k = \frac{1}{\pi k}\left\{ J_1\left(\cos\left(\frac{k2\pi}{3}\right) - \cos(k\pi)\right) \quad + \quad J_2\left(\cos(k\pi) - \cos\left(\frac{k4\pi}{3}\right)\right)\right\} \tag{21}$$

2.3.2 Cosine Membership Function

2.3.2.1 Union of Cosine Fuzzy sets

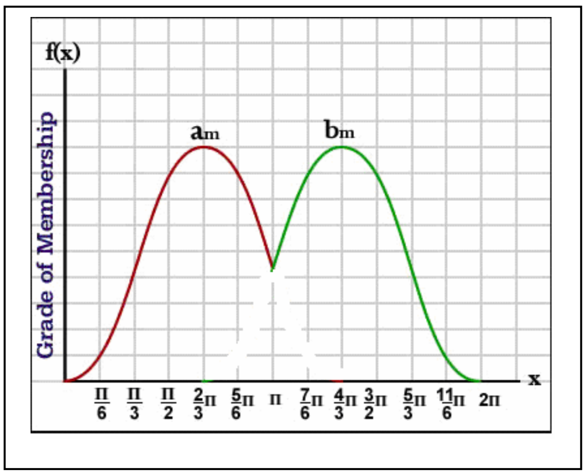

Fig. 7: Union of Cosine *Fuzzy sets*

The corresponding normalized Union of the cosine *Fuzzy sets* is described by Fig. 7. In particular, the Union of cosine *Fuzzy sets* is described by the function $g(\bar{x})$, where

$$g(\bar{x}) = \begin{cases} \frac{1}{2} + \frac{1}{2}\cos\left(\frac{3}{2}\bar{x} - \pi\right) & ; \quad 0 \le \bar{x} \le \pi \\ \frac{1}{2} + \frac{1}{2}\cos\left(\frac{3}{2}\bar{x} - 2\pi\right) & ; \quad \pi \le \bar{x} \le 2\pi \end{cases} \tag{22}$$

The membership function of Union of cosine *Fuzzy sets* is computed as:

$$g(\bar{x}) = \frac{a_0}{2} + \sum_{k=1}^{n}\left(a_k \cos(kw\bar{x}) + b_k \sin(kw\bar{x})\right) \tag{23}$$

where

$$\frac{a_0}{2} = \frac{1}{2\pi}\left(\begin{array}{l} \int_0^{\pi} \frac{1}{2}\, d\bar{x} + \frac{1}{2}\int_0^{\pi}\left(\cos\left(\frac{3\bar{x}\pi}{2\pi}\right)\cos\left(\frac{2\pi^2}{2\pi}\right) + \sin\left(\frac{3\bar{x}\pi}{2\pi}\right)\sin\left(\frac{2\pi^2}{2\pi}\right)\right) d\bar{x} \\ + \int_{\pi}^{2\pi} \frac{1}{2}\, d\bar{x} + \frac{1}{2}\int_0^{2\pi}\left(\cos\left(\frac{3\bar{x}\pi}{2\pi}\right)\cos\left(\frac{4\pi^2}{2\pi}\right) + \sin\left(\frac{3\bar{x}\pi}{2\pi}\right)\sin\left(\frac{4\pi^2}{2\pi}\right)\right) d\bar{x} \end{array} \right) \tag{24}$$

$$= \frac{1}{2\pi}\left(\frac{1}{2}(\pi) + \frac{1}{2}(\pi) \right) \tag{25}$$

$$\frac{a_0}{2} = \frac{1}{2} \tag{26}$$

From equations (24) and (25), we define R_1 and R_2 as follows:

$$R_1 = \int_0^{\pi}\left(\frac{1}{2} + \frac{1}{2}\cos\left(\left(\frac{3\bar{x}\pi}{2\pi}\right) - \left(\frac{2\pi^2}{2\pi}\right)\right)\right)dx \quad = \quad \frac{\pi}{2} \tag{27}$$

$$R_2 = \int_{\pi}^{2\pi}\left(\frac{1}{2} + \frac{1}{2}\cos\left(\left(\frac{3\bar{x}\pi}{2\pi}\right) - \left(\frac{4\pi^2}{2\pi}\right)\right)\right)dx \quad = \quad \frac{\pi}{2} \tag{28}$$

$$a_k \quad = \frac{1}{\pi}\left(\int_0^{\pi} g(\bar{x})\cos(k\bar{x})\, d\bar{x} + \int_{\pi}^{2\pi} g(\bar{x})\cos(k\bar{x})\, d\bar{x}\right) \tag{29}$$

We express equation (29) in terms of R_1 and R_2:

$$a_k = \frac{1}{\pi k}\left(R_1\left[\sin\left(\frac{k4\pi}{3}\right)\right] \quad + \quad R_2\left[-\sin\left(\frac{k2\pi}{3}\right)\right]\right) \tag{30}$$

and

$$b_k = \frac{1}{\pi}\left(\begin{array}{l} \int_0^{\pi}\left(\frac{1}{2} + \frac{1}{2}\cos\left(\frac{3}{2}\bar{x} - \pi\right)\right)\sin(k\bar{x})\, d\bar{x} \\ + \int_{\pi}^{2\pi}\left(\frac{1}{2} + \frac{1}{2}\cos\left(\frac{3}{2}\bar{x} - 2\pi\right)\right)\sin(k\bar{x})\, d\bar{x} \end{array} \right) \tag{31}$$

Similarly, we express equation (31) in terms of R_1 and R_2:

$$b_k = \frac{1}{\pi k}\left(- R_1\left(\cos\left(\frac{k4\pi}{3}\right) - 1\right) - R_2\left(1 - \cos\left(\frac{k2\pi}{3}\right)\right)\right) \tag{32}$$

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2.3.2.2 Intersection of Cosine *Fuzzy sets* (Exponential Membership Function)

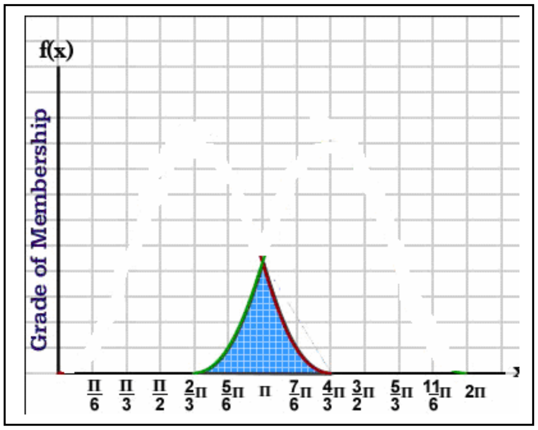

Fig. 8: Intersection of Cosine *Fuzzy sets*

$$h(\bar{x}) = \begin{cases} 0 & ; \quad 0 \le \bar{x} \le \frac{2\pi}{3} \\ \frac{1}{2} + \frac{1}{2}\cos\left(\left(\frac{3\bar{x}\pi}{2\pi}\right) - \left(\frac{4\pi^2}{2\pi}\right)\right) & ; \quad \frac{2\pi}{3} < \bar{x} \le \pi \\ \frac{1}{2} + \frac{1}{2}\cos\left(\left(\frac{3\bar{x}\pi}{2\pi}\right) - \left(\frac{4\pi^2}{2\pi}\right)\right) & ; \quad \pi < \bar{x} \le \frac{4\pi}{3} \\ 0 & ; \quad \frac{4\pi}{3} \le \bar{x} \le 2\pi \end{cases} \tag{33}$$

Similarly, we compute the membership function of the Intersection of cosine *Fuzzy sets* as:

$$f(\bar{x}) = \frac{a_0}{2} + \sum_{k=1}^{n}\left(a_k \cos(kw\bar{x}) + b_k \sin(kw\bar{x})\right) \tag{34}$$

where

$$\frac{a_0}{2} = \frac{1}{2\pi}\left(\int_{\frac{2\pi}{3}}^{\pi} \left(\frac{1}{2} + \frac{1}{2}\cos\left(\left(\frac{3\bar{x}\pi}{2\pi}\right) - \left(\frac{4\pi^2}{2\pi}\right)\right)\right) d\bar{x} + \int_{\pi}^{\frac{4\pi}{3}} \left(\frac{1}{2} + \frac{1}{2}\cos\left(\left(\frac{3\bar{x}\pi}{2\pi}\right) - \left(\frac{2\pi^2}{2\pi}\right)\right)\right) d\bar{x} \right) \tag{35}$$

We define Q_1 and Q_2 as follows:

$$Q_1 = \int_{\frac{2\pi}{3}}^{\pi} \left(\frac{1}{2} + \frac{1}{2}\cos\left(\left(\frac{3\bar{x}\pi}{2\pi}\right) - \left(\frac{4\pi^2}{2\pi}\right)\right)\right) d\bar{x} \tag{36}$$

$$Q_1 = \left[\frac{1}{2}\bar{x}\right]_{\frac{2\pi}{3}}^{2\pi} + \frac{1}{2}\left(\int_{\frac{2\pi}{3}}^{2\pi} \cos\left(\left(\frac{3\bar{x}}{2}\right)\cos(2\pi)\right) d\bar{x} + \int_{\frac{2\pi}{3}}^{2\pi} \sin\left(\left(\frac{3\bar{x}}{2}\right)\sin(2\pi)\right) d\bar{x} \right) \tag{37}$$

(Note: We used 2π as upper integral limit instead of point of intersection (π) to enable us fully capture the curve)

$$= \frac{1}{2}\left(2\pi - \frac{2\pi}{3}\right) + \frac{1}{2}\int_{\frac{2\pi}{3}}^{2\pi} \left(\cos\left(\left(\frac{3\bar{x}}{2}\right)(1)\right) d\bar{x} \right) d\bar{x} \tag{38}$$

$$Q_1 = \frac{2\pi}{3} \tag{39}$$

Similarly,

$$Q_2 = \int_{\pi}^{\frac{4\pi}{3}} \left(\frac{1}{2} + \frac{1}{2}\cos\left(\left(\frac{3\bar{x}\pi}{2\pi}\right) - \left(\frac{2\pi^2}{2\pi}\right)\right)\right) d\bar{x} \tag{40}$$

$$Q_2 = \left[\frac{1}{2}\bar{x}\right]_{0}^{\frac{4\pi}{3}} + \frac{1}{2}\left(\int_{0}^{\frac{4\pi}{3}} \cos\left(\left(\frac{3\bar{x}}{2}\right)\cos(2\pi)\right) d\bar{x} + \int_{0}^{\frac{4\pi}{3}} \sin\left(\left(\frac{3\bar{x}}{2}\right)\sin(2\pi)\right) d\bar{x} \right) \tag{41}$$

(Note: We used 0 as lower integral limit instead of point of intersection (π) to enable us fully capture the curve)

$$Q_2 = \frac{2\pi}{3} \tag{42}$$

Thus

$$\frac{a_o}{2} = \frac{1}{2\pi}\left(\frac{2\pi}{3} + \frac{2\pi}{3}\right) \tag{43}$$

$$a_k = \frac{1}{\pi}\left(\int_{\frac{2\pi}{3}}^{\pi} \left(\frac{1}{2} + \frac{1}{2}\cos\left(\left(\frac{3\bar{x}\pi}{2\pi}\right) - \left(\frac{4\pi^2}{2\pi}\right)\right)\right)\cos(k\bar{x})\, d\bar{x} + \int_{\pi}^{\frac{4\pi}{3}} \left(\frac{1}{2} + \frac{1}{2}\cos\left(\left(\frac{3\bar{x}\pi}{2\pi}\right) - \left(\frac{2\pi^2}{2\pi}\right)\right)\right)\cos(k\bar{x})\, d\bar{x} \right) \tag{44}$$

Plugging-in equations (39) and (42) into equation (44), we obtain:

$$a_k = \frac{1}{\pi}\left(-\frac{Q_1}{k}\left(\sin\left(\frac{k2\pi}{3}\right)\right)+\frac{Q_2}{k}\left(\sin\frac{k4\pi}{3}\right)\right) \tag{45}$$

and

$$b_k = \frac{1}{\pi}\left(\begin{array}{l}\displaystyle\int_{\frac{2\pi}{3}}^{\pi}\left(\frac{1}{2}+\frac{1}{2}\cos\left(\left(\frac{3\bar{x}\pi}{2\pi}\right)-\left(\frac{4\pi^2}{2\pi}\right)\right)\right)\sin(k\bar{x})\,d\bar{x} \\ \displaystyle+\int_{\pi}^{\frac{4\pi}{3}}\left(\frac{1}{2}+\frac{1}{2}\cos\left(\left(\frac{3\bar{x}\pi}{2\pi}\right)-\left(\frac{2\pi^2}{2\pi}\right)\right)\right)\sin(k\bar{x})\,d\bar{x}\end{array}\right) \tag{46}$$

By plugging-in equations (39) and (42) into equation (46), we obtain:

$$b_k = \frac{1}{\pi}\left(\frac{Q_1}{k}\left[\cos(k2\pi)-\cos\left(\frac{k2\pi}{3}\right)\right]-\frac{Q_2}{k}\left[\cos\left(\frac{k4\pi}{3}\right)-1\right]\right)$$

The details of the implementation of this algorithm are shown in Appendix A for the specific cases of cosine, and Gaussian membership functions.

3 Systems Design And Implementation

Based on the foregoing, we hereby document the development of an embedded "*Fuzzy* controller" to measure temperature and pressure and produce output that can represent input to other sub-systems or systems. The implemented device circuit incorporates a mid-range 40-Pin Enhanced Flash PIC16F877A Microcontrollers, MPX4115A piezoresistive pressure sensors, LM35D precision integrated-circuit temperature sensors and a Lumex 2x16 Alphanumeric Liquid Crystal Display (LCD) (HD44780 compliant). The PIC16F877A Microcontrollers were encoded with derived Fourier computations using the Hi-Tech ANSI C Language. To test the proposed techniques and as a demonstration of the capability of our approach, we specifically applied the techniques to data obtained from a natural gas distribution company in Lagos Nigeria. Table 1 (Appendix A) presents parameter values obtained for Mole % of Methane in the composition of natural gas. The corresponding results of sample output (mole % of methane) are presented in Tables 2 and 3 (Appendix A). The full code for the Simulation (in MATLAB) and the code for hardware implementation (Hi-Tech ANSI C) are available.

Itemised below are some of the details of systems design and the implementation of the embedded *Fuzzy* controllers. The electrical circuit is shown in Fig. 9, while the corresponding photo-image of the device is presented in Fig. 10.

3.1 Circuit Description

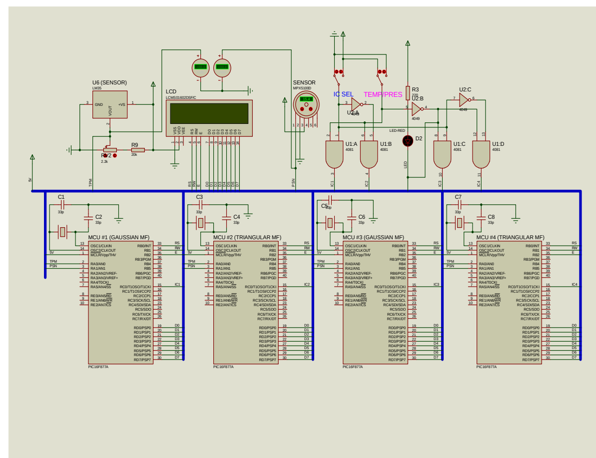

Fig. 9: Electrical Circuit of the Device

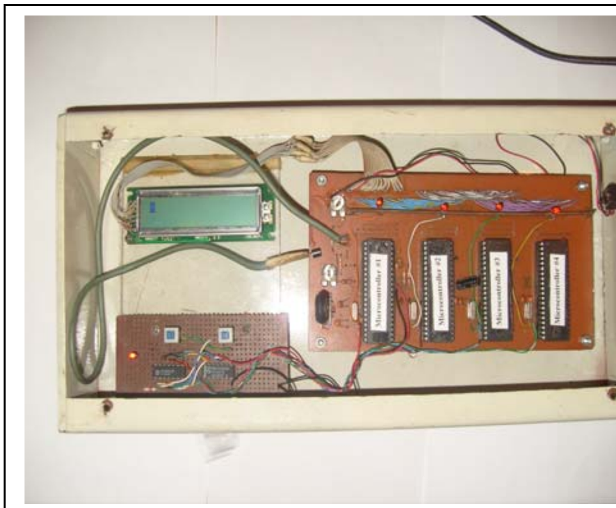

Fig. 10: Image of the Device

Whereas the circuit in Fig. 9 consists of the following major hardware components:

- Microchip 40-Pin Enhanced Flash PIC16F877A Microcontrollers
- LM35D precision integrated-circuit temperature sensor
- MPX4115A piezoresistive pressure sensor, and
- LCM-S01602DSF/C Liquid Crystal Display (HD44780-compliant LCD),

There are only four units of the PIC16F877A Microcontrollers deployed in the circuit. Each Microcontroller is configured with XT 4MHz Crystal. Moreover, the circuit incorporates the LCM-S01602DSF/C Liquid Crystal Display (LCD) output unit capable of displaying 2 x 16 characters. The four (4) Microcontrollers are moreover grouped into two functional sections viz:

3.1.1 Section 1: Temperature

This section consists of two (2) Microcontrollers. Microcontroller #1 executes the program code for temperature input from the LM35D Sensor, it also conditions and convert signals to digital form, and computes grade of membership of Guassian/cosine *Fuzzy sets.* Similarly, Microcontroller #2 executes the corresponding program code for temperature input from the LM35D Sensor, digitises the signals, and computes membership grades of triangular *Fuzzy sets*.

3.1.2 Section 2: Pressure

This section consists of two (2) additional Microcontrollers to handle the pressure readings from the MPX4115A Sensor.

3.1.3 Switching Between Sections 1 and 2

Switching between Section 1 and Section 2 is achieved with the Switch labelled IC SEL; while switching between the two Microcontrollers in each section is achieved with the switch labelled TEMP/PRES.

4 Summary and Conclusion

Fuzzy logic has become very relevant in machine, process or systems control, and particularly as a means of making machines more capable and responsive by resolving intermediate categories in between states hitherto classified on bivalent logic. In the past the use of Fuzzy set theory has been popularised for handling overlap domains in control engineering but this has been in the context of triangular membership functions. In actual practice such domains are hardly triangular and in fact for most engineering applications are usually Gaussian and sometimes cosine. In this paper, we have derived Fourier series representation for computation of membership functions for such distributions. Furthermore, we have established its efficacy by applying it to a Natural Gas Distribution Network based on a Fuzzy controller device built on this principle. In particular, we presented the development of an embedded “*Fuzzy* controller” to measure temperature and pressure and produce an output that can represent input to additional sub-systems or systems. This device has clearly demonstrated that the proposed technique can indeed be incorporated in engineering systems for the dynamic computation of grade of membership in the overlap and non-overlap regions of *Fuzzy set*s and thus provides a basis for the design and implementation of embedded *Fuzzy* controller (in hardware) for mission critical applications.

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