

# Static Voltage Stability Analysis of a Power System

ELEONOR STOENESCU

University of Craiova, Faculty of Electrotechnics  
Blv. Decebal, nr. 107, Craiova, Dolj  
ROMANIA  
[estoenescu@elth.ucv.ro](mailto:estoenescu@elth.ucv.ro)

JENICA ILEANA CORCAU

Division Avionics  
University of Craiova, Faculty of Electrotechnics  
Blv. Decebal, nr. 107, Craiova, Dolj  
ROMANIA  
[jcorcau@elth.ucv.ro](mailto:jcorcau@elth.ucv.ro), [jcorcau@yahoo.com](mailto:jcorcau@yahoo.com)

TEODOR LUCIAN GRIGORIE

Division Avionics  
University of Craiova, Faculty of Electrotechnics  
Blv. Decebal, nr. 107, Craiova, Dolj  
ROMANIA  
[lgrigorie@elth.ucv.ro](mailto:lgrigorie@elth.ucv.ro)

**Abstract:** Power system stability is an important factor in power system studies. Many papers discuss of different categories of stability and various analysis methods for utility power system will be presented. In this paper a comparison of static voltage stability index found in the literature is presented. Static voltage stability analysis of power systems is performed based on the line voltage stability indexes such as SVSI, FVSI and VSI. The indexes were formulated and used to identify the critical line outages and sensitive lines in the system.

**Key-Words:** - power system, static voltage stability, critical line outage, FVSI, VSI and SVSI.

## 1 Introduction

Power system stability can be divided into steady-state, dynamic or small and transient and long term stability. Power system stability is the ability of an electric power system, for a given initial operating condition, to regain a state of operating equilibrium after being subjected to a physical disturbance, with a system variable bounded so that system integrity is preserved [1].

In general, any power system can be represented by a set of differential algebraic equations (DAE)

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{y}) \quad (1)$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}, \mathbf{y}) \quad (2)$$

Where  $\mathbf{x}$  is a vector of the state variables and describes the dynamics of power systems, such as the dynamics of exciter control systems.

The vector of state variables could also include specific system configurations and operating conditions, such as load, generation, voltage setting points etc.

$\mathbf{y}$  is a vector of algebraic variables and satisfies algebraic constraints, such as a power flow equations,

which is implicit assumed to have an instantaneously converging transient.

The conventional method for modeling and simulating power systems can be classified into two main categories: 1) differential algebraic equation solver based methods, such as implemented in SimPower System Toolbox by Matlab/Simulink, and 2) nodal admittance matrix based circuit simulation methods [2].

Voltage stability and contingency analyses are two important procedures to be conducted especially when voltage security assessment is discussed.

Generally voltage stability can be categorized into two categories namely static and dynamic.

Static voltage stability analysis is commonly performed on a system and the results are indicative in determining the voltage stability condition of a system.

In this paper, static voltage stability and a contingency analysis is performed based on the line voltage stability indexes termed as SVSI, FVSI and VSI.

## 2 Voltage stability analysis

Manny papers discusses the voltage stability assessment of a power system using power flow analysis methods [3].

Static analysis methods could be used to analyze voltage stability problems approximately.

A number of special algorithms analysis using static approached [1], however these approaches are laborious and do not provide sensitivity information useful in a dynamic process. Voltage stability is indeed a dynamic phenomenon [3].

Several voltage stability indexes derived from static power flow analysis were proposed for power systems. The values of the indexes were calculated for each distribution line based on load flow results.

### 2.1 Static voltage stability index FVSI

A fast voltage stability index FVSI was derived in [3] by Musirin.

The index FVSI is derived based on a 2 – bus power system model. It is developed from the quadratic expression of the receiving bus voltage.

In figure 1 a model of the 2-bus power system is presented.

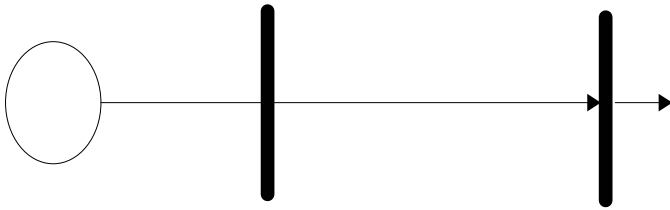


Fig. 1 A two-bus power system

The quadratic equation for the receiving bus is given by [3]

$$V_2^2 - \left( \frac{R_{21}}{X_{21}} \sin \varphi + \cos \varphi \right) V_1 \cdot V_2 + \left( X_{21} + \frac{R_{21}^2}{X_{21}} \right) \cdot Q_2 = 0 \quad (3)$$

Where

$V_1$  and  $V_2$  are voltage sending and receiving buses;

$P_1$  and  $Q_1$  are active and reactive power on the sending bus;

$P_2$  and  $Q_2$  are active and reactive power on the receiving bus;

$S_1$  and  $S_2$  are apparent power on the sending and receiving bus;

$\varphi$  is angle difference between sending and receiving buses;

$R_{21}$  is the line resistance and  $X_{21}$  is the reactance line.

The voltage roots can be derived as

$$V_2 = \frac{\left( \frac{R_{21}}{X_{21}} \sin \varphi + \cos \varphi \right) V_1}{2} \pm \frac{\sqrt{\left[ \left( \frac{R_{21}}{X_{21}} \sin \varphi + \cos \varphi \right) V_1 \right]^2 - 4 \left( X_{21} + \frac{R_{21}^2}{X_{21}} \right) Q_{21}}}{2} \quad (4)$$

In order to maintain real roots of  $V_2$ , the discriminant of equation (3) must greater than or equal zero.

Rearranging the equation gives

$$\frac{4Q_{21}Q_2X_{21}}{V_1^2(R_{21}\sin\varphi + X_{21}\cos\varphi)^2} \geq 0 \quad (5)$$

Since the difference in the angle between the sending bus and the receiving bus  $\varphi$  is normally very small, so,

$$\varphi \cong 0, R_{21}\sin\varphi \approx 0, X_{21} \cong X_{21}.$$

Taking “j” as the receiving bus and “i” as the sending bus, hence, the fast voltage stability index FVSI can be expressed as [4]

$$FVSI = \frac{4Z^2Q_j}{V_i^2X} \quad (6)$$

Where

$Z$  - line impedance

$X$  - line reactance

$Q_j$  - reactive power at the receiving end

$V_i$  - sending end voltage.

### 2.2 Static voltage stability index SVSI

Another static voltage index SVSI. This index is applied to each cable in a SPS system. The SPS system is an especially for switchboards power systems [1].

The cable with the maximum value of the index is the weakest cable in the system. The part of SPS below generator switchboards can be taken as a radial distribution system.

The bus with the maximum value of the index is thus the most sensitive to voltage instability.

For the radial part of SPS, the index could indicate the level of stability at each bus.

The power flow on the connecting line  $S_{21}$ , the voltage on sending bus  $V_1$  and receiving bus  $V_2$  and the line impedance  $Z_{21}$  are defined as:

$$S_{21} = P_{21} + jQ_{21} \quad (7)$$

$$V_2 = |V_2| \cos \varphi_2 + j|V_2| \sin \varphi_2 \quad (8)$$

$$V_1 = |V_1| \cos \varphi_1 + j|V_1| \sin \varphi_1 \quad (9)$$

$$Z_{21} = R_{21} + jX_{21} \quad (10)$$

The real power and reactive power are  $P_{21}$  and  $Q_{21}$ . The line resistance and reactance are  $R_{21}$  and  $X_{21}$ . The voltage magnitude and angle are  $|V|$  and  $\varphi$ .

The voltage static stability index SVSI is defined from the power flow on the line between sending bus 1 and receiving bus 2 in the two-bus system [1]

$$SVSI_{L21} = \frac{2\sqrt{(X_{21}^2 + R_{21}^2)(P_{21}^2 + Q_{21}^2)}}{|V_1|^2 - 2X_{21}Q_{21} - 2R_{21}P_{21}} \quad (11)$$

$SVSI_{L21}$  indicates the steady state voltage stability on the line and thus can indicate the stability level of two bus power system.

If  $SVSI_{L21}$  is less than one the system is stable; if the index  $SVSI_{L21}$  is equal to one the system is at a stability boundary, and if the index  $SVSI_{L21}$  is larger than one, there is the system is unstable or a steady – state voltage collapse occurs.

So, the closer to one the value of the index  $SVSI_{L21}$  is, the closer the system is operating to voltage instability.

### 2.3 Static voltage stability index VSI

VSI is another index which to allows prediction of the voltage instability, this index is described in [4], [7].

The problem of voltage stability may be simply explained as instability of power system to provide the reactive power or the egregious consumption of the reactive power by the system itself.

It is understand as a reactive power problem and is also a dynamic phenomenon [3].

The index VSI gradually approaches the value of one as the load power is increased.

The value of VSI is less than one once the load power is larger than the maximum allowable power and the voltage on bus 2 is unstable.

A voltage stability index VSI was derived in [3]

$$VSI = \frac{S_2}{V_2^2 Y_{22}} \quad (12)$$

The VSI index derived which can be used for monitoring the voltage stability problem of the system.

When  $S_2 = 0$ , the index VSI will be zero and indicates that there will be no voltage problem.

When  $S_2 = 1$  the voltage at load bus will collapse.

## 3 Comparison of the static voltage stability index

The performance of the indexes found in the literature are compared.

Three static voltage stability indexes FVSI, VSI and SVSI were proposed in [3], [7], [1].

They are written as: (5), (11) and 12.

For all the indexes the value of one indicates the stability boundary. Voltage is statically stable when the index value is larger than zero and less than or equal to one. The closer to one the index value is, the closer the system is to voltage instability [7].

The simple two-bus system shown in figure 1 is used as an example to compare the three indexes. All variables are expressed as per unit values. The impedances of the

line are: Short line (SL):  $0,085+0,01j$  and Long line (LL):  $0,17+0,0205j$  [1].

The bus voltage at the sending end bus is assumed to be the one in the power flow. The real power of the load at receiving bus 2 increases from zero to 2,4 p.u.

The values of the voltage stability indexes were calculated at each load level.

The power factor of the load at the receiving end bus 2 was varied to represent high and low power factors (HPF  $\cos \varphi = 0,9$  and LPF  $\cos \varphi = 0,6$ ).

In figure 2 and figure 3 shown the performance comparison of the three voltage indexes at different situation with different combination of line length and load factors are shown.

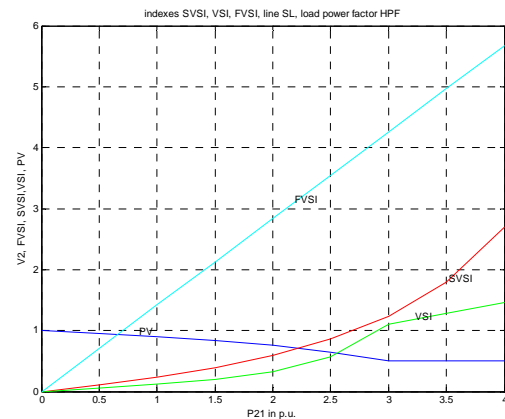


Fig. 2 SVSI, FVSI and VSI with short line and high power factor

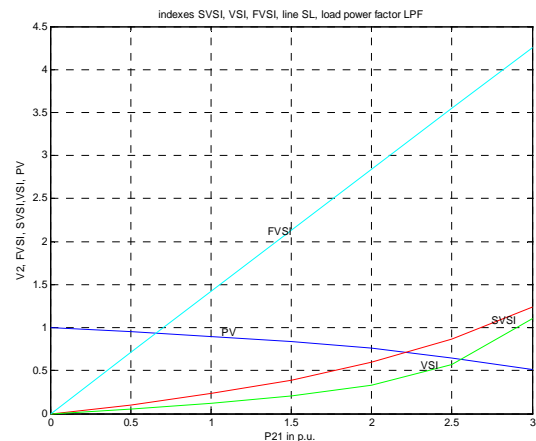


Fig. 3 SVSI, FVSI and VSI with short line and low power factor

In figure 4 and figure 5 compare the three indexes when the line is long and the power factor of the load is high and low, respectively.

In each figure, the PV curve at each different situation drawn.

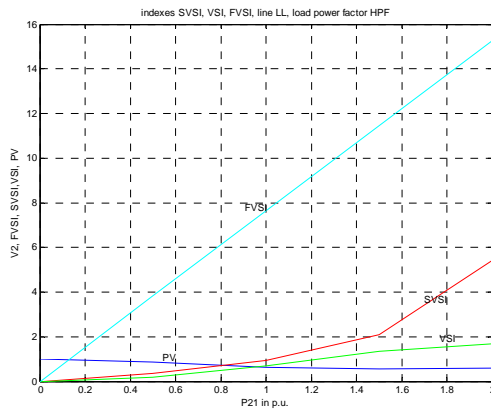


Fig. 4 SVSI, FVSI and VSI indexes with long line and high power factor

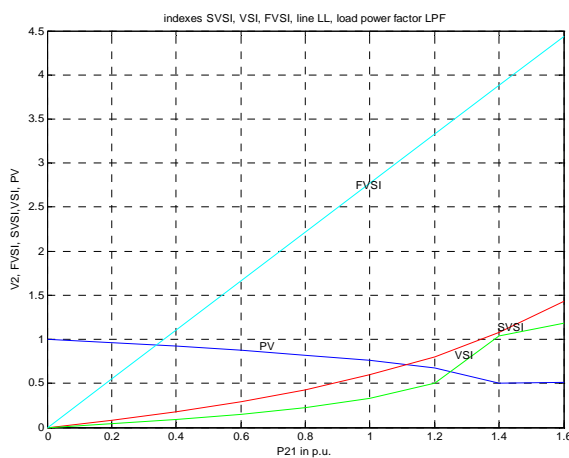


Fig. 5 SVSI, FVSI and VSI indexes with long line and low power factor

All indexes increase as the real power of the load increases. When voltage instability occurs indexes SVSI, and VSI reach one.

The results of comparative studies between the indexes can be observed in figure 2 to 5.

This study presents a new technique for static voltage stability analysis of a power system.

A line stability indexes was formulated and used to identify the critical line outages and sensitive line in the system.

## 4 Conclusion

In this paper, the comparative study shows and analysis methods will be presented. Power system stability is an important factor in an power system. Static analysis methods could be used to analyze voltage stability problems approximately. Several voltage stability indexes derived from static power flow analysis were proposed for an utility power system. A fast voltage

stability index FVSI was presented in [3] by Musirin; the fast index neglected the angle difference between the voltages at both ends of a line. The values of the indexes were calculated for each distribution line based on load flow results.

### References:

- [1] Li Qi. *AC system stability analysis and assessment for shipboard power systems*. Dissertation, December 2004, Electrical Engineering;
- [2] A. Mohamed and G.B. Jasmon. *Voltage Contingency Selection Technique for Security Assessment*, IEE Proc. Generation, Transmission, and Distribution, vol. 136, no. 1, pp. 24-28, Jan. 1989;
- [3] Musirin I., Rahman T. K. A. *Implementation of FVSI for Contingency Ranking in Power System*, AUPEC 2002;
- [4] Chiang H. D., Dobson I., Thomas R. J. *On voltage collapse in electric power systems*. IEEE Trans. Power Syst., 1990, 5(2), 601-611;
- [5] Mohamed a., Shaaban H., Kahla., A. *A fast efficient accurate technique for circuit contingency evaluation*. Int. J. electrical power syst. 1998, 181-189;
- [7] Subramani C., Subhransu Sekhar Dash, Jagadeeshkumar. *Voltage stability based collapse prediction and weak cluster identification*. International Journal of Electrical and power Engineering 3(2), 124-128, 2008, ISSN 1990-7958.