Optimization of a Heat Recuperator Using Genetic Algorithm

M. GHANBARI\textsuperscript{a}, A. NOURI\textsuperscript{b}
\textsuperscript{a,b}School of Mechanical Engineering
\textsuperscript{a,b}Sharif University of Technology
AZADI Road, TEHRAN
IRAN
\textsuperscript{a}ma.ghanbari@gmail.com , \textsuperscript{b}banouri@sharif.edu

Abstract: Heat recuperators are often used to recover the waste heat energy. For recovering more heat, heat exchanger effectiveness should be highest. Heat exchanger effectiveness depends on geometric and flow parameters. Once the flow parameters are fixed, the geometric parameters determine the heat exchanger effectiveness. For a fixed volume of heat exchanger, by increasing heat transfer area, heat exchanger effectiveness increases and on the other hand due to viscous friction, heat exchanger effectiveness decreases. In this paper by using the Genetic Algorithm for a fixed volume of heat exchanger, optimum design parameters are obtained and optimum size of heat recuperator is determined.

Key-Words: Heat Exchanger, Effectiveness, Viscous Friction, Optimization, Genetic Algorithm

1 Introduction
Compact heat exchangers are exclusively used in gas to gas heat transfer and mainly in waste heat recovery applications [1,2]. In this type of heat exchangers, heat transfer rate and pumping power requirement can become comparable which considerably large surface area is require. Because, by increasing the heat transfer area, the effect of viscous friction considerably increases and thereupon pumping power which is due to existence of viscous friction, increases. By lugging the effect of viscous friction in heat exchanger effectiveness expression and then maximizing effectiveness, an optimum status between pumping power and heat transfer rate is obtain. In this status maximum rate of heat transfer and minimum pumping power requires for flowing achieves and optimum size of recuperator is determined. It is carried out by using the Genetic Algorithm [3].

2 Problem Definition and Assumptions
In this study a counter flow heat exchanger is considered and optimization is carried out for a fixed volume of heat exchanger. Heat exchanger composes of hot and cold channels (Fig. 1). There are four designs variable; number of channels in hot and cold zone and hydraulic diameter of channels in cold and hot zone. And there are two constraints for these variables. Therefore there are two independent design variables. It is assume the number of channels in hot and cold zone to be independent variables and other variables are calculated as function of these two variables. For recovering more energy from waste heat, heat exchanger effectiveness should be highest. Hence, heat exchanger effectiveness is set as objective function. By increasing the number of channels, due to increasing heat transfer surface area, heat exchanger effectiveness increases and on the other hand for a fixed volume of heat exchanger, due to viscous friction, heat exchanger effectiveness decreases. In the optimization process an optimum status between these two opponent effects are obtained.

Fig.1: Cross Section of Recuperator

3 Heat Exchanger Effectiveness
For a counter flow heat exchanger as shown in Fig. 1, it can be shown that heat exchanger effectiveness can be obtained from the following expression [4]:

\[
\varepsilon = \frac{BL}{1+BL} \left( \frac{GL + \frac{B}{2}(F+G)L^2}{(1+BL)(T_{h,0} - T_{c,0})} \right)
\]  

(1)
In the above expression, \( L \) is the length of heat exchanger, \( T_{h,in} - T_{c,in} \) is the deference between input temperatures in cold and hot zone. \( B, F \) and \( G \) are as bellow

\[
B = \frac{4n_c n_h \text{Nu} k}{m c_p (n_c + n_h)} \quad (2)
\]

\[
F = \frac{Po \dot{m}_c \mu}{2c_p n_c \rho_c^2 D_c^2} \quad (3)
\]

\[
G = \frac{Po \dot{m}_h \mu}{2c_p n_h \rho_h^2 D_h^2} \quad (4)
\]

In equations (2), (3) and (4); \( \text{Nu} \) is the nusselt number, \( k \) is conduction heat transfer coefficient of fluids, \( n_c, n_h \) are the number of channels in cold zone and hot zone respectively. \( \dot{m} \) is mass flow rate in hot and cold zone which be assumed to be same. \( c_p \) is the heat capacity at constant pressure. \( Po = f \cdot Re \) is the Poiseuille number, defined as the product of the Darcy friction factor \( f \) and the Reynolds number in the channels. \( \rho_c, \rho_h \) are fluid densities in cold and hot zone respectively. \( D_c \) is the hydraulic diameter in hot zone and \( D_h \) is the hydraulic diameter in cold zone. \( D_c, D_h, n_c \) and \( n_h \) are design variables and there is two constraint for these variables

\[
n_c D_c^2 + n_h D_h^2 = A \quad (5)
\]

\[
n_c D_c = n_h D_h \quad (6)
\]

\( A \) is the cross section area of heat recuperator. From equations (5) and (6), \( D_c, D_h \) can be calculated as function of \( n_c, n_h \) as bellow

\[
D_h^2 = \frac{A n_c}{n_c n_c + n_h^2} \quad (7)
\]

\[
D_c^2 = \frac{A n_h}{n_c n_h + n_h^2} \quad (8)
\]

Therefore there are two independent variables, \( n_c, n_h \), which should be optimized. The method which is used for optimization is Genetic Algorithm.

4 Genetic Algorithm

A genetic algorithm (GA) is a search technique used in computing to find exact or approximate solutions to optimization and search problems. Genetic algorithms are categorized as global search heuristics. Genetic algorithms are a particular class of evolutionary algorithms (also known as evolutionary computation) that use techniques inspired by evolutionary biology such as inheritance, mutation, selection, and crossover [5,6].

4.1 Methodology

Genetic algorithms are implemented as a computer simulation in which a population of abstract representations (called chromosomes or the genotype of the genome) of candidate solutions (called individuals) to an optimization problem evolves toward better solutions. Traditionally, solutions are represented in binary as strings of 0s and 1s, but other encodings are also possible. The evolution usually starts from a population of randomly generated individuals and happens in generations. In each generation, the fitness of every individual in the population is evaluated, multiple individuals are stochastically selected from the current population (based on their fitness), and modified (recombined and possibly randomly mutated) to form a new population. The new population is then used in the next iteration of the algorithm. Commonly, the algorithm terminates when either a maximum number of generations has been produced, or a satisfactory fitness level has been reached for the population. If the algorithm has terminated due to a maximum number of generations, a satisfactory solution may or may not have been reached.

A typical genetic algorithm requires two things to be defined: a genetic representation of the solution domain and a fitness function to evaluate the solution domain. A standard representation of the solution is as an array of bits. Arrays of other types and structures can be used in essentially the same way. The main property that makes these genetic representations convenient is that their parts are easily aligned due to their fixed size that facilitates simple crossover operation. Variable length representations may also be used, but crossover implementation is more complex in this case. Tree-like representations are explored in Genetic programming and graph-form representations are explored in Evolutionary programming.

The fitness function is defined over the genetic representation and measures the quality of the represented solution. The fitness function is always problem dependent. A representation of a solution might be an array of bits, where each bit represents a different
object, and the value of the bit (0 or 1) represents whether or not the object is in the knapsack. Not every such representation is valid, as the size of objects may exceed the capacity of the knapsack. The fitness of the solution is the sum of values of all objects in the knapsack if the representation is valid or 0 otherwise. In some problems, it is hard or even impossible to define the fitness expression; in these cases, interactive genetic algorithms are used. Once the genetic representation and the fitness function are defined, GA proceeds to initialize a population of solutions randomly, and then improve it through repetitive application of mutation, crossover, inversion and selection operators.

4.2 Initialization
Initially many individual solutions are randomly generated to form an initial population. The population size depends on the nature of the problem, but typically contains several hundreds or thousands of possible solutions. Traditionally, the population is generated randomly, covering the entire range of possible solutions (the search space). Occasionally, the solutions may be "seeded" in areas where optimal solutions are likely to be found.

4.3 Selection
During each successive generation, a proportion of the existing population is selected to breed a new generation. Individual solutions are selected through a fitness-based process, where fitter solutions (as measured by a fitness function) are typically more likely to be selected. Certain selection methods rate the fitness of each solution and preferentially select the best solutions. Other methods rate only a random sample of the population, as this process may be very time-consuming. Most functions are stochastic and designed so that a small proportion of less fit solutions are selected. This helps keep the diversity of the population large, preventing premature convergence on poor solutions. Popular and well-studied selection methods include roulette wheel selection and tournament selection.

4.4 Reproduction
The next step is to generate a second generation population of solutions from those selected through genetic operators: crossover (also called recombination), and/or mutation. For each new solution to be produced, a pair of "parent" solutions is selected for breeding from the pool selected previously. By producing a "child" solution using the above methods of crossover and mutation, a new solution is created which typically shares many of the characteristics of its "parents". New parents are selected for each child, and the process continues until a new population of solutions of appropriate size is generated. These processes ultimately result in the next generation population of chromosomes that is different from the initial generation. Generally the average fitness will have increased by this procedure for the population, since only the best organisms from the first generation are selected for breeding, along with a small proportion of less fit solutions, for reasons already mentioned above.

4.5 Termination
This generational process is repeated until a termination condition has been reached. Common terminating conditions are:
a) A solution is found that satisfies minimum criteria
b) Fixed number of generations reached
c) Allocated budget (computation time/money) reached
d) The highest ranking solution's fitness is reaching or has reached a plateau such that successive iterations no longer produce better results
e) Manual inspection
f) Combinations of the above

5 Optimization
Optimization is performed for a given conditions. This is: \( A = 20 \times 20 \text{cm}^2, \quad L = 1.5 \text{m}, \quad m = 0.1 \text{kg/s}, \quad k = 57.3 \times 10^{-3} \text{W/m.k}, \quad c_p = 1100.0 \text{j/kg.k}, \quad \mu = 369.8 \times 10^{-7} \text{N.s/m}^2, \quad \rho_c = 0.7 \text{kg/m}^3, \quad \rho_h = 0.4354 \text{kg/m}^3 \). For square channels \( Po = 56.92 \) [7]. The Nusselt number for laminar flow in square channels with constant wall temperature amounts to \( Nu = 3.61 \) [7]. Genetic Algorithm is used for optimization with below features

- Population Type: Double Vector
- Population Size: 500
- Creation Function: Uniform
- Initial Range: [1 ; 5000]
- Fitness Scaling: Rank
- Selection: Tournament
- Tournament Size=4
- Reproduction: Elite Count=2
- Cross Over Fraction=0.7
- Mutation: Uniform
- Rate=0.1
Crossover Function: Scattered
Migration: Direction is Forward
Fraction=0.2
Interval=20
Initial Penalty: 10
Penalty Factor: 100
Stopping Criteria: When generations reach to 100

Also an upper bound for number of channels is determined. It is assume that maximum number of channels to be 5000 in both zones. It is should be note that the model is constructed only in thermodynamic point of view and no constraint for mechanical construction limitations or cost limitation is made into account.

Best fitness value in each generation through optimization process is plotted in Fig. 2.

And average Distance between individuals in each generation through optimization process is plotted in Fig. 3.

The results of optimization are as below
\[ n_c = 2593 \]
\[ n_h = 1971 \]

And heat exchanger effectiveness for this number of channels is
\[ \varepsilon = 0.85841 \]

Other parameters can be estimated from above results. For example
\[ D_h = 3.396 \text{ mm} \]
\[ D_c = 2.581 \text{ mm} \]

It is seen that the model do not select the maximum number of channels which maximum heat transfer surface area achieves. Because by increasing the number of channels more than this, the effect of viscous friction dominates and consequently heat exchanger effectiveness reduces.

Diagram of heat exchanger effectiveness through number of channels is plotted in Fig. 4.

By neglecting the effect of viscous friction by setting \( F = 0, G = 0 \) in heat exchanger effectiveness expression, then
\[ \varepsilon = \frac{BL}{I + BL} = \frac{NTU}{I + NTU} \quad (9) \]

If the model is solved for this condition, the following results are achieved
\[ n_c = 5000 \]
\[ n_h = 5000 \]
And heat exchanger effectiveness becomes
$$\varepsilon = 0.9658$$

It is seen that in absence of viscous friction, there is no limitation on amount of heat transfer area and the model selects the maximum number of channels which is possible.

6 Conclusion

In this paper optimization of a gas to gas heat exchanger is performed by using the Genetic Algorithm. In the gas to gas heat exchangers considerably large surface area requires. Therefore viscous friction which is the primary responsible cause for the pressure drop and pumping power requirement is an important consideration in heat exchanger design. By lugging the effect of viscous friction in heat exchanger effectiveness expression and maximizing it, by Genetic Algorithm, optimum design parameters of heat exchanger obtained. In absence of viscous friction, there is no limitation on amount of heat transfer area in heat exchanger in thermodynamic point of view. Viscous friction exerts an upper bound for heat transfer area in heat exchanger.

References: