Estimating Non-Maturity Deposits

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Abstract: - We use certain statistical models for studying the demand deposits. For each model we precise the theoretical estimation and we propose an original algorithm for simulations. Using a time series data of market rate, internal rate and non-maturity deposits, we run these models and we estimate the optimal parameters. Based on results and data, we propose a mixed model and original relations for estimating the parameters. After a decision upon a better algorithm and model, we calculate the real values of demand deposits. Some conclusions about the characteristic feature of non-maturity deposits were formulates at the end of the paper.

Key-Words: Non-maturity deposits, interest rate, adjustment models, correlation, sensitivity, risk.

1 Non-maturity deposits

Non-maturity deposits (NMD) such as sight, current accounts, savings, demand deposits accounts etc are a major source of funds for all traditional Banks. The characteristic feature for these kind of deposits is that they have no stated contractual maturity and the balance of these funds can increase or decrease throughout the day without any warning (although in practice the balance is quite stable) as the depositors have always the possibility to add or withdraw funds at any time (the embedded options that clients may exercise) at no penalty.

The behavior of non-maturity deposits reflects rational decision making on the part of customers on two factors: received value and perceived value. Higher interest rate paid relative to competitor rates and more consequential barriers to exit create longer term indeterminate maturity deposits.

One can often observe that the volume of a NMD position fluctuates as clients react to changes in the customer rate and the relative attractiveness of alternative investment opportunities (see Fig. 1):
- when interest rates rise, the total balance of NMD tends to fall as customers become more careful in sweeping their funds into long-term investments to “lock up” the high level of yields (withdrawal).
- when interest rates are low, NMD becomes more profitable compared to alternative short-term investments.

The perceived value is the answer to the question: why do balances remain on deposit for long periods of time even though the financial advantage is negative? There are cases when clients do not react to the changes in the customer rate (see Fig. 2). Non-maturity deposit funding costs generally demonstrate less volatility than market interest rates. As a result, high non-maturity deposit volumes may actually reduce reprising risk and moderate overall interest rate risk (see [7]). The cash
flow modeling of non-maturing deposits requires dividing deposits into stable and unstable balances.

The stable fraction of the current balance is called core level (see Fig.3) and is modeled as a permanent balance (long-term outflow) while the volatile fraction is viewed as overnight money and serves as a “buffer” for volume fluctuations (see [6]). As is observed in other cases, the results obtained for the value of core deposits varies substantially by institution, depending on the institution’s supply of deposits and ability to retain deposits (see [9]). This is important for financial decisions in order to make a distribution over several years of the total value that can be used for investing proposal (Fig. 4). We obtain that 44% could first be invested. The trenches that will follow for investment depends on first investment (see [8]).

### 2. Estimation of NMD balances

We shall discuss different statistical models in order to estimate NMD balances. Let $D_t$ be the value of a non-maturity deposit at time $t$. To refer to the linear regression and log-normal diffusion models, we introduce the discrete variables $y_t = \log(D_t)$ and $\Delta y_t = \log(D_t / D_{t-1})$.

The Linear regression model is characterized by:

$$y_t = a + b(t - \bar{t}) = \alpha + \beta t + \sum_{t=1}^{n} (t - \bar{t})(t - \bar{t})$$

$$a = \frac{1}{n} \sum_{i=1}^{n} (t_i - \bar{t})$$

$$b = \frac{\sum_{t=1}^{n} (t - \bar{t})^2}{n}$$

(1)

The Log-normal diffusion model, used for future balance (of liquidity term structure), is based on the following ingredients:

$$y_t = \alpha + \beta t + \sigma \varepsilon_t$$

(2)

with parameters $\alpha$, $\beta$ being determined from a linear regression on $y_t$, $(\varepsilon_t)$ Gaussian (white) noise term, $\sigma = \sqrt{\text{Var}(\varepsilon_t)}$ the volatility of $y_t$ time series, $\varepsilon_t \sim N(0, \sigma^2)$. If we consider a $q$-confidence interval, then $\varepsilon_t = \Phi^{-1}(q)\sqrt{\sigma}$. Therefore

$$y_t = \log(V(t)) = \alpha + \beta t + \sigma \Phi^{-1}(q)\sqrt{t}$$

(3)

where $q$ is the $p^{th}$ quantile from $N(0,1)$, given by relation $q = z_{1-\alpha/2}$, such that $\Phi^{-1}(q) = 1 - \alpha/2$, $\alpha = 1 - p$. The confidence interval, for $n$ the number of selection from a population $N(\mu, \sigma)$ is determined by:

$$\left(\bar{y}_t - \frac{\sigma}{\sqrt{n}} z_{1-\alpha/2}, \bar{y}_t + \frac{\sigma}{\sqrt{n}} z_{1-\alpha/2}\right)$$

(4)
For $p = 0.95$ and $p = 0.99$, we have $q = 1.6449$ and respectively $q = z_{1-0.05/2} = 2.3263$. For our case (see Fig. 5), the confidence band (interval) is stated between the two star-lines. In other words, with the probability $p = 0.99$ (almost sure), the variable $y_t = \log(D_t)$ have values between these star-lines, in the future too. Also, the regression line (marked by red color) has the equation

$$y = 0.00095 t + 16.$$ 

This line divide the confidence band in two equal parts.

Fig. 5 Confidence interval for $y_t$ for $p = 0.99$.

We shall use following models: -moving average, MA(1), autoregressive model AR(1)-see [1], models based on external factors ( Jarrow and van Deventer model; Janosi, Jarrow Zullo (see [8]), Ornstein-Uhlenbeck process) – see [2].

2.1 MA(1) model for rate of NMD balances

This model is base on the formula:

$$\Delta y_t = \mu + \epsilon_t + \beta \epsilon_{t-1}. \hspace{1cm}(5)$$

it permits to simulate the rate of future non-maturity deposit values, for few months. These values are obtained from $\Delta y_t$ evaluated in Fig.6, showing the decaying rate level of investment:

$$\Delta y_t = \log(D_t / D_{t-1}) = -0.1.$$ 

2.2 AR(1) model for rate of NMD balances

We estimate the parameters for

$$\Delta y_t = \beta_0 + \beta_1 \Delta y_t + \epsilon_t, \hspace{1cm}(6)$$

using Yule Walker estimator (see Fig. 6).

2.2 Correlation: interest rate/market rate

The rate of non-maturity deposit values is described also by the “Jarrow and van Deventer model”, “Janosi, Jarrow Zullo, and Ornstein-Uhlenbeck process”. These models are connected to the interest rate and/or market rate (see Fig 8.). The correlation coefficient between the two rates is very small $\rho = 0.0921074$, and consequently we can consider that the two variables are independent.
2.3. Jarrow and van Deventer model

The model is given by:
\[ \Delta y_t = \alpha_0 + \alpha_1 t + \alpha_2 r_t + \alpha_3 (r_t - r_{t-1}) \, . \] (6)

Using the least square method, we estimate the values of coefficients, components of the array \( \alpha = (\alpha_0, \alpha_1, \alpha_2, \alpha_3) \), via a solution of symmetric algebraic system

\[ M\tilde{\alpha} = P, \, M = M_{\text{int}} l(n-1), \] (7)
\[ P = \frac{1}{n-1}((n-1)\bar{y}_t, \sum_{i=2}^{n} y_i t_i, \sum_{i=2}^{n} y_i r_t, \sum_{i=2}^{n} y_i (r_t - r_{t-1}))^t \]

For our data estimation, we find (see Fig.9) \( \tilde{\alpha} = (0.7828847, -0.0023413, -17.982467, 29.75882) \)

We also estimate NMD stock rate, using external variable \( i_t \). One found:

\[ \Delta y_t = \rho(\Delta y_t)_{\text{JDir}} + (1 - \rho)(\Delta y_t)_{\text{AR}} \, . \] (8)

For simplicity we shall use \( \rho = 0.4 \). The values of \( (\Delta y_t)_{\text{JDir}} \) are well determined by

\[ \Delta y_t = \alpha_0 + \alpha_1 t + \alpha_2 r_t + \alpha_3 i_t \, . \] (9)

This time, the system (7) becomes:

\[ M_h \tilde{\alpha} = P_h, \, M_h = M_{\text{int}, h} l(n-1), \] (10)
\[ P_h = \frac{1}{n-1}((n-1)\bar{y}_t, \sum_{i=2}^{n} y_i t_i, \sum_{i=2}^{n} y_i r_t, \sum_{i=2}^{n} y_i i_t) \]

\[ M_{\text{int}, h} = \begin{pmatrix}
    n-1 & (n-1)\bar{r}_t & (n-1)\bar{i}_t \\
    m_{12} & \sum_{i=2}^{n} (r_t)^2 & \sum_{i=2}^{n} i_t r_t \\
    m_{13} & \sum_{i=2}^{n} (r_t)^2 & \sum_{i=2}^{n} i_t r_t \\
    m_{14} & \sum_{i=2}^{n} (i_t)^2 & \sum_{i=2}^{n} i_t r_t
\end{pmatrix} \, . \]

The results are represented in Figs.11-12,
where we have used
\[ \alpha = (0.5511, -0.0147, 43.7898, 14.93, -0.0147). \]

For all models described we can recalculate the values for NMD balances. We consider that our model estimate better the NMD stock rate (see Fig. 10). For this model we calculate the values of NMD balances. These values are given in Fig. 12, showing the maximum level of investment \( D_t = 2 \times 10^6 \) Eur.

4 Conclusions

Non-maturity deposits have value because they are usually one of the lowest cost sources of funds available to the bank. Modeling non-maturity deposits in balance of IRR is important due to their role in determining earning at risk (EaR) and due to their role in establishing the economic value of equity (EVE).

Using our model (8) could be made some simulations on: how could be influenced the NMD volume by a fast change in values of the interest rate (IRR) or market rate. Simulating assets runoffs, including the impact of prepayments under different rate scenarios, and applying the profile obtained to non-maturity deposits allows for the quantification of explicit IRR.

The algorithms proposed for our model could be implemented in any kind of software, as for example, we have used Excel and Matlab. Also, for better accuracy we have used Monte-Carlo methods in simulations (see [11]).

References: