Study on the Stability Problem of Hydroelectric Station Penstock under External Pressure

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Abstract: - The stabilization problem of steel penstock under external pressure is one of main control conditions of penstock design of hydroelectric station. At present, because of factors of assumption conditions and boundary simplification, the various calculating methods of this problem have their limitations, sometimes even the results have great differences. In this paper, the influences that diameter-to-thickness ratio of the steel tube, size and space of reinforce rings to losing stability and breach for reinforcing ring steel penstock is analyzed by artificial neural network and simulated annealing. Test results show that the method is feasible and its validity is verified. This paper provides an efficient solving path for stability problem of reinforcing ring steel penstock under external pressure.

Key-Words: - Penstock; Stability calculation under external pressure; Neural network; Simulated annealing algorithm

1 Introduction
The big diameter transportation water tube in hydroelectric station building is a thin cylinder-shell structure with a series of reinforcing ring. This structure is easy to lose stability under external pressure. If the penstock loses stability, great economic loss will occur. With losing stability and breach for reinforcing ring, penstock is being plastic deformation, and dry crinkle crack of concrete leads to non-linear contact, so analysis of this problem is very difficult. At present various methods of this problem are affected by some subjective factors as hypothesis condition and predigesting boundary, sometimes results have great differences.

The paper applies neural network model (Martin and Menhaj, 1994) and simulated annealing algorithm (Metropolis et al., 1953; Kirkpartric et al., 1983) to simulate external pressure stability of reinforcing rings penstock. It utilizes powerful non-linear mapped power of artificial neural network to deal with all kinds of complex factors of losing stability and breach for reinforcing ring steel penstock. An improved BP algorithm is used to overcome drawback of neural network such as the convergence rate is slow, easy to fall into local minimum and so on.

2 Design Method of the Stiffened Penstock Shell Stability

2.1. Mises method
Mises hypothesizes the place of reinforcing ring being fixed. The losing stability deformation of penstock shell is along with circumference of the penstock and shows many waves. Its formula as follows (Wang et al., 2003):

\[
p_{cr} = \frac{Et}{(n^2 - 1)\left(1 + n^2\frac{r}{\pi^2r^2}\right)} + \frac{Et}{12(1 - \mu^2)r^3} \cdot \left(\frac{2n^2 - 1 - \mu}{\pi r^2}\frac{n^2r^2}{1 + \frac{n^2r^2}{\pi r^2}}\right)^2 \quad (1)
\]

\[
n = 2.74 \left(\frac{r}{\pi}\right)^{\frac{1}{2}} \left(\frac{r}{\pi}\right)^{\frac{1}{2}} \quad (2)
\]

Bearing capacity of external pressure \( p \) is calculated by cylinder formula:
\[ \sigma_s \cdot t = p \cdot r \] (3)

Where \( p_{cr} \) - critical external pressure, \( n \) - breach wave numbers, \( t \) - thickness of penstock shell, \( l \) - distance of stiff rings, \( r \) - radius of penstock, \( E \) - elastic module \( \mu \) - Poisson ratio.

### 2.2. Numerical method

Due to error can not be avoided in manufacture and fixed process of penstock, thickness of penstock shell is asymmetry and there is initial crack between penstock and concrete. These factors lead to breach occurring in some defect place firstly. In this case, Mises method isn’t worked. Although some breach examples of embedding penstock on engineering building and model experiment had proved the validity of hypothesis in Amstutz (1970) method for losing stability wave shape: \( \eta (\varphi) = a \cos(\varphi) + b \cos(\varphi) + c \), but Amstutz method can be used only in mill finish steel tube. Liu and Ma (1990) proposed a semi-analytic finite element computing methods. This method plots the structure to cylinder shell element and circle plate element. Element is connected by nodal circle among element.

The stiffness equation of structure are as follows:

\[
[K_E] + q[K_G^*] = \{P\} \quad (4)
\]

\[
[K_E] = \sum [K] \quad (5)
\]

\[
[K_E] = \sum (q^e / q) [K_G^*]^e \quad (6)
\]

Where \([K_E] \), \([K_G^*] \) - elastic stiffness matrix and geometrical stiffness matrix \( e \) - The code of element; \( q \) - external pressure of structure; \( \{\Delta\} \), \( \{P\} \) - displacement vectors and nodal force vector of the structure.

If the determinant of total stiffness matrix \( ([K_E] + q[K_G^*]) \) is zero, the Characteristic equation of stability analysis are as follows:

\[
| [K_E] + q[K_G^*] | = 0 \quad (7)
\]

\[
| [A] + \lambda[I] | = 0 \quad (8)
\]

\[
[A] = -[K_E]^{-1}[K_G^*] \quad (9)
\]

\[
\lambda = 1 / q \quad (10)
\]

The reciprocal of maximum characteristic root \( 1 / \lambda_{max} \) is the losing stability loading.

For longer penstock shell and bigger distance of reinforcing ring, the calculating result of this numerical algorithm is consistent with Mises method.

But for short shell, its result is 10%—15% bigger than Mises method.

### 2.3. Lai-Fang Method

Lai-Fang method is proposed by Lai and Fang (1990). They gave a derivation of expression by Amstutz’s hypothetic condition. Its essential theory applies not only hypothesis of elastic theory but also more hypothesis as follows: ① The breach wave shape of penstock shell under external pressure shows 3 half of wave shape ② The reinforcing ring cannot move along to axis of penstock; ③ The longitudinal deformation is 0: \( U = 0 \); ④ The reinforcing ring is absolute stiffness. The author utilized boundary condition of losing stability of penstock to solved \( P_{cr} \):

\[
P = E'P \frac{1 - \mu}{r^2 (\eta^2 - 1) \left( \frac{1 - \mu}{2} \lambda^2 + \eta^2 \right)} + E'(t)^3 \frac{16}{12} \left( \frac{r}{r} \right)
\]

(11)

Where \( \lambda = \frac{\pi r}{l} \); \( \eta = \frac{n \pi \alpha}{2} \), \( \alpha \) - The half deflection angle of breach wave shape for the center of cylinder shell.

By solving the equation \( dP/d\eta = 0 \), gain \( \eta \) value as follows Table 1, then calculate \( P_{cr} \) by expression (11).

In Lai and Fang (1990), the authors prove the reliability of the formula through experimentation. The results show: there is a good relationship between calculation value and experiment value when the distance of reinforcing ring is less (\( l > r \)), but when \( l \) is biggish (\( l > 2r \)), the computing result is 16.9% less than experiment value.

### 3. Simulation of the Stability Problem of Penstock under External Pressure

#### 3.1. The computing sample data

The computing model of the stability analysis of the penstock shell under external pressure is shown in Fig.1. The rolled steel adopts 16Mn, (elastic module
\[ E = 2.1 \times 10^5 \text{ Mpa}, \quad \text{Poisson ratio: } \mu = 0.3, \quad \sigma_s = 340 \text{ Mpa}, \]

Table 1 The \( \eta \) value of the radial pressure approaching critical pressure.

<table>
<thead>
<tr>
<th>( \frac{r}{l} )</th>
<th>( \frac{100t}{r} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.6</td>
</tr>
<tr>
<td>0.2</td>
<td>6.75</td>
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<tr>
<td>0.4</td>
<td>12.2</td>
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<td>0.6</td>
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</tr>
<tr>
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<td>22.5</td>
</tr>
<tr>
<td>3.0</td>
<td>24.1</td>
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</table>

The ratio of the penstock radius \( r \) to penstock thickness shell \( t \) (relatively radius \( r/t \)) is from 20 to 400, step length: 20. The ratio of distance of reinforcing ring \( l \) to penstock radius \( r \) (relatively distance of reinforcing ring \( l/r \)) : [0.1, 0.2, 0.3, 0.5, 0.8, 1.4, 2.0, 3.0, 4.0], total have 180 models.

By formula (1), (2) to calculate critical external pressure \( p_{cr} \) and wave numbers \( n \) of losing stability

### 3.2. Simulation

In the paper, we select four layers BP neural networks, input layer has 3 neuron (relatively radius \( r/t \), relatively distance of reinforcing ring \( l/r \), wave number of losing stability \( n \)), the neuron number of two hidden-layer are respectively 11 and 4, the output layer is critical pressure (Dong et al., 2006). The structure of network is shown in Fig.2. The neuron transfer function is the Sigmoid differentiable function, the outputs neuron use the Purlin linearity transfer function. The neural network sample data is obtained by formula (1), (2). We use improved BP algorithm and simulated annealing method to train the network (Vogl, 1988; Dong et al., 2006; Dong et al., 2006).

Fig.1. The figure of reinforcing ring penstock.

By formula (1), (2) to calculate critical external pressure \( p_{cr} \) and wave numbers \( n \) of losing stability

Fig.2. The structure of neural network.

The training process of the neural network is shown in Fig.3.
The above-mentioned models are tested by using convergent network, the computing results is shown by Table 2.

The computational result fitting figure of simulation and Mises theory is as follows Fig.4.

![Fig.3. The evolvement process of BP network.](image1)

![Fig.4. Fitting figure of the computational result of simulation and Mises theory.](image2)

<table>
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<tr>
<th>$r/t$</th>
<th>Critical pressure $P_{cr}$ (Mpa)</th>
<th>$l/r$</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.5</th>
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<td></td>
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<td>(901.90)</td>
<td>(554.52)</td>
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<td>(0.36)</td>
<td>(0.23)</td>
<td>(0.11)</td>
<td>(0.07)</td>
</tr>
</tbody>
</table>

4. Calculated Result Analysis

1) Fig.4 shows: with an increasing of $r/t$, the losing stability capability of penstock under external pressure is decrease acutely. The critical external pressure $P_{cr}$ decrease with an increasing of $r/t$.

Within $r/t = 20 \sim 260$, the critical external pressure is acutely decrease, beyond the range, the change is less.
For example: within $l/r=0.1\sim3.0$, $r/t$ from 20 to 260, $P_{cr}$ will decrease $0.13\%\sim0.16\%$ of initial value ($r/t=20$ , $P_{cr}$); When the diameter of penstock is increased to a certain value, the stability power of the penstock under external pressure will change very less. For example: $=3.0$ , $P_{cr}$ value ( , ). When the diameter of penstock is increased to a certain value, the stability power of the penstock under external pressure will decrease. While reinforcing ring, the function of reinforcing ring shows: with an increasing of the distance of reinforcing ring, the axial wave number increase. But in axial direction, with an increasing of the reinforcing ring's space, the axial stiffness decrease and $n$ is less. For the penstock of big diameter and thick space, the losing stability wave shape shows the form of more waves, but for the sparse space and small diameter is the form of less wave.

6) The design of penstock: Fig.4 shows: the curve cluster of ‘$\log(p_{cr}) \sim n \frac{\pi}{2 \lambda} \frac{\pi}{r}$’ are divided two district of upper and lower by the plastic losing stability curve. If the penstock diameter and relatively reinforcing ring’s space are biggish, the penstock appears elastic losing stability under smaller external pressure; $\log(p_{cr})$ value locates the district below the elastic losing stability curve. But the penstock diameter and relatively distance of reinforcing ring are less, the stability power of penstock under external pressure is powerful and can bear great pressure; the $\log(p_{cr})$ value locates the district upon the elastic losing stability curve. When we design the penstock, if $r/t$ value has been confirmed by the demand of using and building, we may select appropriate $l/r$ value according to Fig.4, so that the critical external pressure meet anti-external pressure request and be close to the losing stability curve.

5. Conclusion
The paper analyzed characteristic and drawbacks of different calculating method of stability problem of penstock shell under external pressure and proposes simulation method. The feasibility is proved by tested sample. The algorithm provides a new approach for the design of embedding reinforcing penstock in the hydropower station building engineering. It possess reference value in solving high dimension non-linear optimization problem on water conservancy and electric power engineering.

References:


