Multivariable Fuzzy Logic Control of Aerodynamic Plant

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Abstract: - Many plants are difficult to precisely represent by a simple model because of complex nonlinear relationships, imprecise or insufficient measurements, many interactions and disturbances and noise effects. This makes their control difficult mainly because of the heavy, obscure and unreliable design procedure. For such cases the fuzzy logic controller has proved efficient since it employs linguistic expert knowledge and empirical rules to produce simple for completion and implementation nonlinear controllers, which are robust, economic and over perform the linear controllers. The aim of this investigation and the main contributions are related to the design of a two-variable FLC for a laboratory-scale aerodynamic plant (helicopter) and to the testing of the control system in real time using MATLAB™.

Key-Words: - Aerodynamic Nonlinear Plant, MATLAB™, Real Time Control, Two-Variable Fuzzy Logic Controller Design

1 Introduction
Many plants are difficult to precisely represent by a simple model because of complex nonlinear relationships, imprecise or insufficient measurements, many interactions and disturbances and noise effects. The employment of linear controllers for their control is often restricted in a narrow operating range. The more appropriate nonlinear control algorithms are however specific and complex both to design and to implement.

The fuzzy logic control technique is a good compromise between nonlinearity and simplicity. It has been proved [1-3] that it can improve the control system performance making the system more robust to various hazardous signal and plant model uncertainties.

Fuzzy control has gained popularity in many industrial applications – mechanical and transportation systems, home appliances, building and process automation, environmental and power systems [4-10]. However, there lacks experience in its implementation for the control of multivariable nonlinear plants.

2 Problem Formulation
The plant to be controlled is shown in Fig.1. It is a laboratory-scale Two Rotor Aerodynamic System (TRAS) that in certain aspects resembles the behaviour of a helicopter [11, 12]. From the control point of view it exemplifies a high order nonlinear system with significant cross-couplings. The TRAS consists of a beam pivoted on its base in such a way that it can rotate freely both in the horizontal and vertical planes. At both ends of the beam there are rotors (the main and tail ones) driven by 12 V DC motors. A counterbalance arm with a weight at its end is fixed to the beam at the pivot. The state of the beam is described by four process variables: horizontal and vertical angles - azimuth and pitch, measured by incremental encoders fitted at the pivot, and two corresponding angular velocities, measured by rotor velocity sensors. Two additional state variables are the angular velocities of the rotors - azimuth and pitch propeller velocities, measured by speed sensors (tachometers) coupled with the driving DC motors. In a real helicopter the aerodynamic force is controlled by changing the angle of attack. In the laboratory set-up the angle of attack is fixed. The aerodynamic force and hence the beam position is controlled by varying the speed of rotors changing the DC motor input voltage $U_h$, $U_v$. Significant cross couplings are observed between actions of the rotors. Each rotor influences both position angles.

The TRAS system has been designed to operate in real time with external, PC-based digital controller. The
control computer communicates with the position, speed sensors and motors by a dedicated I/O board and power interface. The I/O board with DAC and ADC RT-DAC4/USB is controlled by the real-time software, which operates in the MATLAB/ Simulink RTW/RTWT environment [13]. The DC motors are controlled via pulse-width modulated (PWM) voltage. Therefore the control actions are normalized in the range [-1, +1], which corresponds to [-24, +24], V.

Approximate azimuth, pitch and combined as the depicted in Fig.2, models of the plant have been derived [11] under various assumptions. They all are nonlinear, of high order, two-variable cross-coupled, so complex and difficult to use for design of the controllers.

The following designations are used: 
- $a_\alpha/a_{\omega\alpha}$ - azimuth/pitch angle of the beam [rad]
- $\Omega_\alpha/\Omega_{\omega\alpha}$ - azimuth/pitch velocity [rad/s] - $\Omega_\alpha = da_\alpha/dt, \Omega_{\omega\alpha} = da_{\omega\alpha}/dt$
- $U_a/U_{\omega\alpha}$ - azimuth/pitch DC motor input voltage
- $o_{\alpha}/o_{\omega\alpha}$ - azimuth/pitch motor angular velocity [rad/s] - nonlinear functions $o_{\alpha}=H_\alpha(U_{\alpha\alpha}), o_{\omega\alpha}=H_{\omega\alpha}(U_{\omega\alpha})$
- $F_\alpha/F_{\omega\alpha}$ - azimuth/pitch aerodynamic force
- $l_{\alpha}/l_{\omega\alpha}$ - arm of the azimuth / pitch aerodynamic force [m] - nonlinear functions $l_{\alpha}=l_{\alpha}(\alpha), l_{\omega\alpha}=l_{\omega\alpha}(\alpha)$
- $J_\alpha/J_{\omega\alpha}$ - total azimuth/pitch inertial moment [kg.m^2] - nonlinear functions $J_\alpha=J_\alpha(\alpha), J_{\omega\alpha}=J_{\omega\alpha}(\alpha)$
- $M_{\omega\alpha}/M_\alpha$ - total azimuth / pitch torque [N.m]
- $K_\alpha/K_{\omega\alpha}$ - azimuth/pitch angular moment [N.m.s]
- $f_{\alpha}/f_{\omega\alpha}$ - azimuth/pitch friction coefficient [N.m]
- $R_{\alpha}/R_{\omega\alpha}$ - inverse pitch moment [N.m] - $R_{\alpha}(\alpha,\Omega_\alpha)=f_{\alpha}+f_{\alpha}$
- $J_{\omega\alpha}/J_{\alpha}$ - azimuth/pitch angular moment [N.m.s]
- $G_\alpha/G_{\omega\alpha}$ - azimuth/pitch aerodynamic moment - $G_\alpha(o_{\alpha},\Omega_{\omega\alpha}), G_{\omega\alpha}(o_{\omega\alpha},\Omega_\alpha)$

In the derivation of the model in Fig.2 are assumed first order differential equations for the propellers (rotors), viscous friction (proportional to angular velocity), valid fluid mechanics equations for description of propeller-air flow relationships. It is considered that

$$M_\alpha = J_\alpha \frac{d^2 \alpha}{dt^2}, \quad M_{\omega\alpha} = M_{\omega\alpha 1} + M_{\omega\alpha 2} + M_{\omega\alpha 3} + M_{\omega\alpha 4} + M_{\omega\alpha 5} + M_{\omega\alpha 6}$$

with components:
- $M_{\omega\alpha 1}=-k_1 \cos(\alpha) - k_2 \sin(\alpha)$ - moment of the gravitation force with $k_1$ and $k_2$, determined from the mass and the geometric dimensions of the beam and the mounted upon it devices.
- $M_{\omega\alpha 2} = l_{\omega\alpha} F_\alpha(o_{\omega\alpha})$ - moment of the driving force with $l_{\omega\alpha}$ the arm of the pitch aerodynamic force
- $M_{\omega\alpha 3} = -k_3 \Omega_{\omega\alpha}^2 \sin(\alpha) \cos(\alpha)$ - moment of the centrifugal forces with $k_3$ determined from the mass and the geometric dimensions of the beam and the mounted upon it devices
- $M_{\omega\alpha 4} = -k_4 \Omega_\alpha \cos(\alpha)$ - moment of pitch friction forces with constant coefficient $k_4$
- $M_{\omega\alpha 5} = -k_5 \Omega_\alpha \Omega_{\omega\alpha}$ - moment of pitch friction forces with constant coefficient $k_5$
- $M_{\omega\alpha 6} = l_{\alpha} F_\alpha(o_{\alpha})$ - moment of azimuth control $U_\alpha$ with constant coefficient $k_6$
- $M_{\omega\alpha 7} = k_7 \cos(\alpha)$ - moment of pitch disturbance.

In a similar way the pitch beam position is described:

$$M_\alpha = J_\alpha \frac{d^2 \alpha}{dt^2}, \quad J_\alpha = k_8 \cos^2(\alpha) + k_9$$

$M_{\omega\alpha} = M_{\omega\alpha 1} + M_{\omega\alpha 2} + M_{\omega\alpha 3} + M_{\omega\alpha 4}; \quad M_{\omega\alpha 1} = l_{\omega\alpha} F_\alpha(o_{\alpha}) \cos(\alpha)$ with $l_{\omega\alpha}$ arm of $F_\alpha(o_{\alpha})$; $M_{\omega\alpha 2} = -k_6 \Omega_\alpha \Omega_{\omega\alpha}$; cross-moment $M_{\omega\alpha 3} = k_7 \cos(\alpha) U_\alpha$; disturbance moment $M_{\omega\alpha 4}$. Also $F_\alpha(o_{\alpha})$ and $F_\alpha(o_{\omega\alpha})$ are approximated by fifth order polynomials.

The final nonlinear model includes also relationships for the propellers:

$$I_{\alpha} \frac{d\omega_{\alpha}}{dt} = U_\alpha - H_{\alpha}^{\prime}(\omega_{\alpha}), \quad I_{\omega\alpha} \frac{d\omega_{\omega\alpha}}{dt} = U_{\omega\alpha} - H_{\omega\alpha}^{\prime}(\omega_{\omega\alpha})$$

where $\omega_{\alpha}(U_\alpha)$ and $\omega_{\omega\alpha}(U_{\omega\alpha})$ are approximated by polynomials of order seven and five respectively.

The TRAS model is of sixth order, has two inputs ($U_\alpha, U_{\omega\alpha}$) and four outputs ($o_{\alpha}, \omega_{\alpha}, o_{\omega\alpha}, \Omega_{\omega\alpha}$), from which only the first two are the controlled variables. The model is complex and approximate. In order to be used for the purpose of control, linearization is applied using Taylor’s series expansion of the nonlinear relations [INTECO] around a nominal static operating point – usually the horizontal beam equilibrium is reached for $U_{\omega\alpha,\text{nom}} = 0.3$ by positioning of the weight.

The helicopter system is a benchmark for testing various advanced control techniques because of its high complexity expressed in many degrees of freedom, which are difficult to reduce and are the basis for uncertainties.

The control objectives for the design of the fuzzy controllers are the following:
- fast and accurate reference tracking
- insensitivity to measurement noise and changes of the plant dynamic properties
- compensation of load disturbances and plant coupling.
The controllers developed in fulfillment of the control objectives are mainly based on robust, classical PID, multivariable decoupling and/or linearising techniques [1-8, 11, 12]. A common drawback is the need of a precise plant mathematical model. The linear controllers perform well only in the neighbourhood of a given operation point. The advanced controllers are usually of high order and have complex and obscure design. Therefore the fuzzy logic controller (FLC) seems appealing since it employs linguistic expert knowledge and empirical rules to produce simple for completion and implementation nonlinear controllers, which are robust, economic and over perform the linear [14-20].

The aim of this investigation is to design a two-variable FLC for a laboratory-scale aerodynamic plant (helicopter) and to test the control system in real time using empirical knowledge about the plant and MATLAB™ facilities.

3 Design of Two-variable Fuzzy Logic Controller

The block diagram of the suggested controller to be designed is shown in Fig.3 [18-20]. The two-variable FLC comprises two separate controllers for the two coupled plant variables \((\alpha_h, \alpha_v)\). Each separate controller consists of a fuzzy controller, an output denormalising factor \(K_{ah} (K_{av})\) and a PID linear controller in series and has two inputs – the normalized errors \((e^{nh}_h, e^{nv}_v)\), and one output – the normalized voltage \(u^{nh}_h (u^{nv}_v)\). The measured values for \((\alpha_h, \alpha_v)\) are compared with their references \((r_h, r_v)\) to form the two error signals \((e_{nh}, e_{nv})\), which are normalized by the factors \(K_{ah}\) and \(K_{av}\) respectively. Each separate controller is designed independently, considering expert knowledge about the plant behaviour along individual and cross-coupling channels.

The design of each separate controller starts from tuning the parameters of the PID controller using low order approximate plant model and a design criterion for desired allocation of the roots of the characteristic equation of the closed loop system.

The PID controllers have transfer function of the type \(C_{PID}(s)=k_p+k_i/s+k_ds\) and the calculated gains are given in Table 1. The normalization factors \(K_{ah}=1\) and \(K_{av}=2\) are computed to ensure that the error signals have all values in the range [-1,1] for the expected reference changes and disturbances. The denormalisation factors are tuned to \(K_{ah} = 0.5\) and \(K_{av}=0.1\) to comply with the limited output signals of the analog PID controllers and their tuning parameters.

Next the fuzzy controllers are designed employing empiric rules, Mamdani model and five discretization levels for the main input error and for the output of the separate controller and two - for the cross-coupling error input. The linguistic values for each of the two linguistic variables are: P – positive; N – negative; NG – negative great; NS – negative small; Z – zero; PS – positive small; PG – positive great. The accepted membership functions (MFs) and the designed fuzzy rules in Fuzzy Associative Matrix (FAM) for Fuzzy Controller 1 and Fuzzy Controller 2 are shown in Fig.4 and Fig.5 respectively.

The fuzzy controllers are developed by the help of the Fuzzy Logic Toolbox of MATLAB™ [21]. The max-min composition is selected as inference method and the Centre-of-Gravity for defuzzification in order to ensure smooth fuzzy controller output. The universes of discourse of the linguistic variables are normalized in the range [-1,+1].

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<thead>
<tr>
<th>System</th>
<th>Desired poles locations</th>
<th>PID tuning parameters</th>
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<tbody>
<tr>
<td>Azimuth PID1</td>
<td>(\alpha=-0.55)</td>
<td>0.4285 0.0786 0.7428</td>
</tr>
<tr>
<td>Pitch PID1</td>
<td>(\alpha_1=-0.606)</td>
<td>0.1936 0.6997 0.3717</td>
</tr>
<tr>
<td>Pitch PID2</td>
<td>(\alpha_2=-0.228 \pm 1.392i)</td>
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**Table 1. PID tuning parameters**

Fig.3. Two-variable fuzzy logic controller
4 Real-Time Fuzzy Control

The designed fuzzy controllers represent static nonlinear corrections of the gains of the linear analog PID controllers aiming at better compensation for the plant multivariable and nonlinear behaviour and thus improving the control system performance.

The closed loop system for the control of the beam position angles $\alpha_h$ and $\alpha_v$ of the laboratory-scale aerodynamic plant by the designed two-variable FLC is tested via simulations and in real time operation. The FLC is completed in a Simulink model. In Fig.6 the simulated system step responses to various reference changes are shown. In Fig.7 the time responses in real time operation are recorded. The sampling time is 0.01s.

For comparison the time responses of a closed loop system with only the PID controllers are shown in Fig.8. The simulated and the real time fuzzy control systems have very close time responses to the same reference changes, which shows that the complex model of the plant and its Simulink completion describe precisely the real processes and can be successfully used for further simulation investigations. However this complex mathematical model cannot be used for design and tuning of the PID and the fuzzy controllers.

The performances of the fuzzy and the PID control systems are not very different. The fuzzy system has slower time responses and is more sensitive to real time measurement noise, which urged the use of especially tuned exponential filters. In both systems the time responses of the pitch angle are better – overdamped, but with high frequency oscillations at the first reference change.

The performance of the fuzzy control system can be improved by increasing the number of MFs in the cross coupling error in order to more precisely reflect the impact of the cross coupling in the control action, which is significantly expressed.

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<tr>
<th>$u_h^n$</th>
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<td>NG</td>
<td>NS</td>
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<th>$u_v^n$</th>
<th>$e_v^n$</th>
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<td>P</td>
<td>NS</td>
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<td>N</td>
<td>PS</td>
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Fig.4. Fuzzy Controller 1: MFs for main error, cross-coupled error and output and fuzzy rules

Fig.5. Fuzzy Controller 2: MFs for main error, cross-coupled error and output and fuzzy rules
5 Analysis of Results and Conclusion

The main results of this investigation are the following.

- A two-variable FLC is designed for the control of a nonlinear two-variable plant – a laboratory scale helicopter. It is based on PID controllers and fuzzy nonlinear static correction. The PID controllers are tuned using a simplified plant model. The fuzzy controllers are developed employing empiric rules and expert knowledge about the operation and the cross coupling of the plant.

- The fuzzy control is tested by simulations and in real time operation and the time responses to various reference changes are recorded and compared with the real time PID control system responses.

- All the investigation is carried in MATLAB, which proved to be a perfect environment for designing of FLCs, testing of the control systems via simulation and in real time operation and prototyping of intelligent control algorithms.

- This investigation shows that a highly nonlinear multivariable plant can be satisfactorily controlled by simple controllers PID and fuzzy, designed in a transparent way using a black box low order linear plant model and empiric knowledge from observations on the plant operation.

Future work is envisaged in improving the fuzzy controllers using more MFs, neural networks for their determination and tuning and trying different FLC structures in order to increase more the quality of the fuzzy control in comparison with the PID control.

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References: