Global Diversification, Hedging Diversification, and Default Risk in Bank Equity: An Option-Pricing Model

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Abstract: Can multiple diversifications provide greater safety for banks? This paper aims to answer this question by applying Vassalou and Xing’s (2004) formula, which is a nonlinear option-based function of the default probability of an individual bank. We find that the extent of global diversification may provide greater safety for banks, but that the extent of hedging diversification may not.

Key-words: Default Risk, International Lending Diversification, Loan Portfolio Swap

1 Introduction

Acharya, Hasan, and Saunders (2006) prove that lending across different national markets (global diversification) does not produce greater safety for banks. On a related diversification issue, banks can use loan portfolio swaps to hedge against adverse moves in the credit quality of their loans (hedging diversification). Can these two diversifications, together, further provide greater safety for banks?

This paper uses Vassalou and Xing’s (2004) formula based on Merton’s (1974) option pricing model to study the determination of optimal bank interest margins and default probability in equity returns. The results of this paper show that an increase in the degree of global diversification may decrease the default risk in the bank’s equity return, whereas an increase in the swapped portion of the bank may not.

2 Global and Hedging Diversifications

All financial decisions are made and values are determined within a one-period horizon, 0 ≤ t ≤ 1. This paper designs the framework to incorporate two distinct diversifications through loan portfolio diversifying globally and loan portfolio swap hedging. In the domestic (home) market, the bank’s demand function is a function of domestic loan interest rate: , . This paper assumes that the bank has some market power in the home lending business, . In the
foreign market, the bank takes the loan rate \( R_X \) determined in the market, since foreign banks find themselves facing much stiffer competition from local banks (Damanpour, 1986). An increase in \( R_X \) increases the bank’s foreign lending activities \( X \) and then decreases its home lending, ceteris paribus. An increase in \( R_M \) decreases \( M \) and then increases \( X \), \( X = X(R_M) \).

In loan portfolio swaps, our model follows Sorensen and Bollier (1994) by seeking to price it; doing so, the bank estimates the required credit-risk adjustment allocated to its counterparty bank’s risk of default as:

\[
CR_B = P_C RV_B - P_B RV_C
\]

where \( P_B \) (\( P_C \)) is the probability that the bank (the counterparty bank) will default on the single default date; and \( RV_B \) (\( RV_C \)) is the value of the option for the bank (the counterparty bank) to replace the swap. If \( CR_B > 0 \) is larger, then the bank will either receive a higher fixed coupon or pay a lower fixed coupon.

At \( t = 0 \), a part of initial funds is invested in domestic and foreign loans maturing at \( t = 1 \). Prior to hedging diversification, the bank’s expected loan repayment at \( t = 1 \) is:

\[
V = (1 + R_M)M + e(w)(1 + R_X)X
\]

where the exchange rate at \( t = 0 \) is assumed to 1, and is random, and that it is given by \( e(w) \), where \( w \) is the state of the world at \( t = 1 \). It is assumed that at \( t = 0 \) the bank swaps a portion \( \alpha \) of the expected repayments from its counterparty. It is further assumed that the exchange rate in the swap contract equals the exchange rate at \( t = 0 \). Thus, \( RV_B \) in equation (1) is defined as:

\[
RV_B = \alpha[(1 + R_M)M + (1 + R_X)X]
\]

3 The Model

At \( t = 0 \) the bank has the following balance-sheet constraint:

\[
M + X + B = D + K = K(\frac{1}{q} + 1)
\]

where \( B \) is the bank’s net position in the default-free asset market with a known market rate \( R \). The bank accepts \( D \) dollars of deposits at \( t = 0 \). The bank provides its depositors with a market rate of return equal to the risk-free rate \( R_D \). Regulations require equity capital, held by the bank, be tied to a fixed proportion \( q \) of the bank’s deposits, \( K \geq qD \). According to Zarruk and Madura (1992), the required ratio of capital-to-deposits \( q \) is assumed to be an increasing function of the amount of the loans held by the bank at \( t = 0 \),

\[
\frac{\partial q}{\partial M} = \frac{\partial q}{\partial X} = q > 0.
\]

At any time during the period horizon, the value of the bank’s risky assets with loan portfolio swap is:

\[
[=(1-\alpha)(1 + R_M)M + e(w)(1 + R_X)X]
\]

\[
\begin{align*}
A^0 + CR_B &= A^0 \quad \text{if embodied risk} = 0 \\
&< A^0 \quad \text{if embodied risk} > 0
\end{align*}
\]

The value of the bank’s earning-asset portfolio then is:
\[ E = A + (1 + R)[K\left(\frac{1}{q} + 1\right) - M - X] + RV_C \]  \quad (6)

The bank’s equity return at \( t = 1 \) may be stated as: \( S = \max\{0, E - Z\} \), where \( Z = (1 + R_D)K / q \). In the model, \( S \) represents the residual equity value of the bank after meeting all of its obligations. The total cost \( Z \) is assumed to be only the deposit payment cost at \( t = 1 \).

The selection of our model’s objective function follows Lin, Chang, and Lin (2009), and Lin, Lin, and Jou (2009). Our argument, which is based on Merton (1974), suggests that the equity of a banking firm can be viewed as a call option on the bank’s risky-asset portfolio. The strike price of the call option is the book value of the bank’s net liabilities. Our approach in expressing the default probability follows Vassalou and Xing (2004). The market value of a banking firm’s risky assets follows a geometric Brownian motion of the form: \( dA = \mu Adt + \sigma AdW \), where \( \mu \) is an instantaneous drift, \( \sigma \) is an instantaneous volatility, and \( W \) is a standard Wiener process. The market value of equity return \( S \) can then be given by:

\[
\text{Max } S = AN(d_1) - Ge^{-\delta}N(d_2) \quad (7)
\]

where

\[
G = (1 + R_D)K / q - (1 + R)[K\left(\frac{1}{q} + 1\right) - M - X] - RV_C
\]

\[
d_1 = \frac{1}{\sigma}[\ln A / G + \delta + \frac{1}{2}\sigma^2]
\]

\[
d_2 = d_1 - \sigma
\]

\[
\delta = R - R_D
\]

\( G \) is the book value of the strike price. \( N(\cdot) \) is the cumulative density function of the standard normal distribution. \( \delta \) is the spread, which is defined as the difference between \( R \) and \( R_D \) in this model.

In addition, this paper models the default risk in the bank’s equity return as:

\[
P_S = N(-d_1) \quad (8)
\]

where

\[
d_1 = \frac{1}{\sigma}[\ln A / G + \mu - \frac{1}{2}\sigma^2]
\]

4 Equilibrium of the Model

Partially differentiating equation (7) with respect to \( R_M \), the first-order condition is given by:

\[
\frac{\partial S}{\partial R_M} = \frac{\partial A}{\partial R_M}N(d_1) - \frac{\partial G}{\partial R_M}e^{-\delta}N(d_2) = 0 \quad (9)
\]

where the second-order condition is required to be satisfied, that is, \( \frac{\partial^2 S}{\partial R_M^2} < 0 \).

Given the equilibrium condition in equation (9), the bank determines the optimal domestic loan rate \( R_M^* \) to maximize the market value of its equity return. To obtain the default probability of the bank’s equity return in equation (8), the optimal domestic loan interest rate \( R_M^* \) is substituted.

5 Comparative Static Results

Differentiating equation (8) evaluated at \( R_M^* \) with respect to \( R_A \) is:

\[1\] Structural changes expressed by the statistical fat tails can be incorporated into equation (8) (Asosheha, Bagherpour, and Yahyapour, 2008). For simplicity, this consideration is ignored in this paper.
\[
\frac{\partial P_S}{\partial R_X} = \frac{\partial P_S}{\partial R_X} + \frac{\partial P_S}{\partial R_M} \frac{\partial R_M}{\partial R_X} \tag{10}
\]

where

\[
\frac{\partial P_S}{\partial R_X} = -\frac{\partial N(d_1)}{\partial R_X} \left( \frac{1}{\sigma_R} \frac{\partial A}{\partial R_X} + \frac{R_M}{A} \frac{\partial G}{\partial R_M} \right) \]

\[
\frac{\partial P_S}{\partial R_M} = -\frac{\partial N(d_3)}{\partial R_M} \frac{1}{\sigma_R} \left( \frac{R_M}{A} \frac{\partial A}{\partial R_M} - \frac{R_M}{G} \frac{\partial G}{\partial R_M} - \frac{R_M}{\partial R_M} \right)
\]

\[
\frac{\partial R_M}{\partial R_X} = -\frac{\partial^2 S}{\partial R_M \partial R_X} \frac{\partial^2 S}{\partial R_M} = \frac{\partial^2 A}{\partial R_M} \frac{N(d_1)}{\partial R_X}
\]

\[
\frac{\partial^2 A}{\partial R_M \partial R_X} = \left[ (1 - \alpha)e(w) + \alpha P_e \right] \frac{\partial X}{\partial R_M} > 0
\]

\[
\frac{\partial N(d_1)}{\partial d_1} - \frac{\partial N(d_2)}{\partial d_2} = \frac{\partial N(d_1)}{\partial R_X} \left( 1 - \frac{AN(d_1)}{Ge^{-\delta N(d_2)}} \right) < 0
\]

\[
\frac{\partial d_1}{\partial R_X} > 0
\]

We define \((R_M/A)(\partial A/\partial R_M) - (R_M/G)(\partial G/\partial R_M)\) as the loan rate elasticity effect. Changes in the domestic loan rate have a more significant impact on the risky-asset portfolio management than on the net-obligation management since banks frequently encounter situations in which loan rate decisions are made in the presence of fixed deposits. The loan rate elasticity effect is negative. The term \(\partial d_1/\partial R_M\) is negative and thus \(\partial P_S/\partial R_M > 0\).

The first term on the right-hand side of equation (10) can be explained as the direct effect. This direct effect is negative since an increase in the foreign market loan rate increases the value of the bank’s risky-asset repayments, and hence, lower default risk in equity return.

The second term of equation (10) can be explained as the indirect effect. The sign of this effect is determined by the product of how changes in \(R_X\) affect \(R_M^*\) as well as how changes in \(R_M^*\) affect \(P_S\).

First, the impact on \(P_S\) from changes in \(R_M^*\) is positive \((\partial P_S/\partial R_M^* > 0)\) since the loan rate elasticity effect is negative. Second, the impact on \(R_M^*\) from changes in \(R_X\) \((\partial R_M^*/\partial R_X)\) is governed by \(\partial^2 S/\partial R_M^* \partial R_X\). The term associated with \(N(d_1)\) in \(\partial^2 S/\partial R_M^* \partial R_X\) is explained as the mean effect on \(\partial A/\partial R_M^*\), while the term associated with \(\partial d_1/\partial R_M^*\) is explained as the variance effect. The mean effect is positive since \(\partial^2 A/\partial R_M^* \partial R_X > 0\). The variance effect is positive as well since there are \(\partial A/\partial R_M^* < 0\) and \(\partial d_1/\partial R_X > 0\). Thus, we have \(\partial^2 S/\partial R_M^* \partial R_X > 0\); accordingly, there is \(\partial R_M^*/\partial R_X > 0\). In light of previous work, we have the positive indirect effect.

**Proposition 1: An increase in the foreign loan market rate decreases the default risk in the bank’s equity return.**

As the bank faces an increasing loan rate in the foreign market, it provides a return to a larger foreign loan base. One way the bank may attempt to augment its total returns is by shifting its lending activities to foreign markets and away from the domestic market. If domestic loan demand is relatively rate-elastic, a smaller domestic lending
business is possible at an increased domestic loan rate. The default risk in the bank’s equity return is generally higher, the higher the degree of global diversification through an increased loan rate in the domestic market (the indirect effect). As stated earlier, the default risk is directly lower, the higher the degree of global diversification (the direct effect).

Proposition 1 provides us with a belief that the total effect will be negative should the direct effect be partially offset by the indirect effect. The rationale is that an increase in the loan rate in the foreign market makes lending in the domestic loan market less attractive relative to that in the foreign market. An increase in foreign lending decreases the default risk in the bank’s equity return while decreasing domestic lending due to an increase in foreign lending, thereby increasing the default risk. As a result, this induces the bank to cut risky lending by increasing global diversification.

Next, differentiating equation (8) evaluated at \( R_M^* \) with respect to \( \alpha \) is:

\[
\frac{dP_S}{d\alpha} = \frac{\partial P_S}{\partial \alpha} + \frac{\partial P_S}{\partial R_M} \frac{\partial R_M}{\partial \alpha} \quad (11)
\]

where

\[
\frac{\partial P_S}{\partial \alpha} = -\frac{\partial N(d_1)}{\partial d_1} \frac{1}{\sigma} \frac{\partial A}{\partial \alpha}
\]

\[
\frac{\partial A}{\partial \alpha} = (P_c - 1)(1 + R_M)M
\]

\[
\frac{\partial R_M}{\partial \alpha} = -\frac{\partial^2 S}{\partial R_M^2} \frac{\partial A}{\partial R_M} \frac{\partial R_M}{\partial \alpha} \frac{\partial^2 S}{\partial R_M^2}
\]

\[
+ \frac{\partial A}{\partial \alpha} \left( \frac{\partial N(d_1)}{\partial d_1} - \frac{N(d_1)}{\partial N(d_2)} \frac{\partial d_1}{\partial \alpha} \right)
\]

The first term on the right-hand side of equation (11) can be interpreted as the direct effect, while the second term can be interpreted as the indirect effect. If the home currency is depreciated, the term \( (P_c - e(w)) \frac{\partial A}{\partial \alpha} \) is negative in sign. The direct effect is positive \( (\frac{\partial P_S}{\partial \alpha} > 0) \).

The sign of the indirect effect is determined by how changes in \( \alpha \) affect the bank’s optimal loan rate, as well as by the relationship between \( P_S \) and \( R_M^* \). \( \frac{\partial P_S}{\partial R_M} > 0 \) is positive. \( \frac{\partial^2 S}{\partial R_M^2} \frac{\partial A}{\partial \alpha} \) is positive if the home currency is depreciated. Thus, \( \frac{\partial R_M}{\partial \alpha} > 0 \) . The following proposition states the result of equation (11).

**Proposition 2:** When the currency in the home country is depreciated, an increase in the swapped portion of the bank increases its default risk in the equity return.

An increase in the amount of the loan portfolio swap transaction increases the bank’s domestic loan rate. The bank now provides a return to a larger hedging base. To make this adjustment, the bank shifts its investments into foreign lending and away from domestic lending. This adjustment indicates that an increase in international lending diversification increases the bank’s default risk in equity return, since an increase
in $\alpha$ decreases the bank’s risky-asset repayment or makes loans more risky to grant. Thus, the multiple diversifications of international lending associated with the increasing loan portfolio swap transaction may produce less safety for the bank.

6 Conclusions

This paper finds that the default risk is negatively related to the foreign market loan rate, and positively to the swapped portion of the bank’s risky-asset portfolio. An increase in the extent of global diversification, captured by an increase in the foreign market loan rate, decreases the default risk, whereas an increase in the extent of hedging diversification, demonstrated by an increase in the hedging portion, increases the default risk.

References


