Abstract: In a centralized loan portfolio construction with decentralized loan portfolio management, changes in the bank’s degree of capital market imperfection have direct effects on the bank’s interest margin through the centralized as well as the decentralized loan rate determinations. We find that the decentralized loan rate managed by the outside loan manager is positively related to the bank’s degree of capital market imperfection. The centralized loan rate managed by the bank is positively related to its degree of capital market imperfection under strategic complements, but negatively under strategic substitutes.

Key-words: Centralized vs. decentralized loan rate; Information asymmetry; Black-Scholes formula.

1 Introduction

It is widely recognized that many financial institutions employ outside portfolio managers to manage part or all of their investable funds. The most likely institutions include pension funds, private endowments, and private trusts. Although the employment of an outside portfolio manager is less likely to be currently observed in banking firms than that in the previous financial institutions, there may be instances when money-center banks have experienced diversified financial problems due to their information asymmetry of agricultural, real estate, and oil-related lending businesses. We do an alternative examination and argue that concerns with bank diversified lending quality may prompt money-center banks to adopt the outside administrator’s portfolio management.

In their paper, Elton and Gruber (2004) argue that the centralized portfolio is unlikely to be optimum since the individually managed portfolios themselves are constructed without taking into account the portfolios of the other managers. Hence, information asymmetry takes place. Their model explains why and how the centralized and decentralized decision makers make their own portfolio (risk) decisions, but not their own operation (rate and equity return) decisions, at least not major ones. In this paper, we use a two-stage model of option-based valuation to examine a bank’s choice of its centralized loan rate in the first stage.
stage, and the determination of the outside manager’s decentralized loan rate in the second stage. In the decentralized loan-rate setting stage, we find that an increase in the degree of capital market imperfection increases the outside manager’s optimal loan rate (and thus the bank’s margin). In the centralized loan-rate setting stage, we show that an increase in the degree of capital market imperfection decreases (increases) the bank’s optimal loan rate under strategic substitutes (strategic complements).

2 The Model

All financial decisions are made and values are determined within a one-period horizon, \( 0 \leq t \leq 1 \). At \( t = 0 \), the bank accepts \( D \) dollars in deposits. At \( t = 1 \), the bank provides depositors with a market rate of return equal to the risk-free rate, \( R_D \). Capital regulation requires equity capital \( K \) held by the bank tied to a fixed proportion \( q \) of the bank’s deposits, \( K \geq qD \). The required ratio of capital-to-deposits \( q \) is an increasing function of the amount of the loans held by the bank at \( t = 0 \), \( \partial q / \partial qL > 0 \).

We assume that the bank (the centralized decision maker) employs only one outside loan manager to manage part of its investable risky assets. Both the bank and its selected outside manager make term loans \( L \) and \( L_E \), respectively, at \( t = 0 \), which mature and are paid off at \( t = 1 \). Both the two loan markets faced by the bank and the outside manager are imperfectly competitive.

The outside manager faces a loan demand function, which is a function of its loan interest rate, \( R_E \), and the degree of capital market imperfection, \( L \). Loan demand faced by the bank is a downward-sloping function of the loan rate, \( \partial L / \partial R_L < 0 \). The demand for loans is a positive function of the interest rate on the outside manager’s lending rate, \( \partial L / \partial R_E > 0 \). The demand for loans is a negative function of the bank’s degree of capital market imperfection, \( \partial L / \partial c < 0 \). This assumption implies that the bank will decrease its lending with the presence of a higher degree of capital market imperfection.

When the capital requirement constraint is binding, the bank has the following liquidity constraint:

\[ L + L_E + B = D + K = K(\frac{1}{q} + 1) \]  

(1)

where \( B \) is the amount of liquid assets. The bank can lend and borrow in the money market at a given market rate, \( R \).

At any time during the period horizon, the repayment of the risky loans under the bank’s determination is:

\[
\begin{cases} 
V = (1 + R_L)L = V^0 & \text{if no loan losses} \ (2) \\
< V^0 & \text{if loan losses}
\end{cases}
\]

Given the constraint in equation (1), the value of the bank’s earning-asset portfolio is:
The value of the bank’s equity return at \( t=1 \) is the residual value of the bank after meeting all of the obligations:

\[
S = \max \{0, A - (1 + R_D)K / q\} \quad (4)
\]

The bank’s total obligations or total costs, \((1 + R_D)K / q\) in our model, are only the deposit payment costs.

We follow a number of previous authors, for example, Mullins and Pyle (1994) and Lin, Chang, and Lin (2009), and assume the objective of the bank to be the maximization of market value of equity return. To do this, besides the first fold of the centralized decision maker’s task of deciding how much to invest in each portfolio, the second fold is to give the outside manager instructions that will result in its making an optimal loan rate \((R_E)\) determination from the point of view of the overall plan.

Our model applies Merton’s method (1974) of the option-based valuation that the bank’s equity capital can be viewed as a call option in its risky assets. In our model, the strike price of the call option is the book value (or default-free value) of the bank’s liabilities net of the outside manager’s lending repayments, as well as the default-free money market funds. We note that the value of the outside manager’s lending repayments is default-free because the cost of information asymmetry (through premium, \(cL^2_E / 2\)) has been explicitly introduced to the model. To illustrate the feature above, we express equation (4) as:

\[
Max \ S = VN(d_1) - Z e^{-\mu} N(d_2) \quad (5)
\]

where

\[
Z = \frac{(1 + R_D)K}{q} - \{(1 + R_E)(L_E - cL^2_E / 2) \}
+ (1 + R)[K(\frac{1}{q} + 1) - L - L_E]
\]

\[
d_1 = \frac{1}{\sigma} (\ln V / Z + \mu + \frac{1}{2} \sigma^2)
\]

\[
d_2 = d_1 - \sigma
\]

\[
\mu = R - R_D
\]

The cumulative standard normal distributions of \(N(d_1)\) and \(N(d_2)\) are the risk-adjusted factors of the first and second terms in equation (5) respectively. \(\mu\) is the net deposit spread rate, which is defined as the difference between \(R\) and \(R_D\).

3 Decentralized Loan-Rate Setting Stage

The two-stage setting unwinds in the two distinct stages of the determinations of \(R_L\) and \(R_E\). In the first stage of centralized portfolio construction, the bank’s loan rate is determined and remains fixed for the remainder for the model. In the second stage of decentralized portfolio management, the bank’s loan rate is revealed and the outside manager’s loan rate is set. We solve this two-stage model using backward induction. Partially differentiating equation (5) with respect to \(R_E\), the first-order condition is given by:

\[
\frac{\partial S}{\partial R_E} = \frac{\partial V}{\partial R_E} N(d_1) - \frac{\partial Z}{\partial R_E} e^{-\mu} N(d_2) = 0 \quad (6)
\]

where

\[
\frac{\partial V}{\partial R_E} = (1 + R_L) \frac{\partial L}{\partial R_E} > 0
\]

\[
\frac{\partial Z}{\partial R_E} = \frac{(R - R_D)K}{q^2} \left( \frac{\partial L}{\partial R_E} + \frac{\partial L}{\partial R_E} \right) - \frac{1}{2} \left[ \frac{L_E (1 - cL^2_E)}{2} + (1 + R_E)(1 - cL^2_E) - (1 + R)(\frac{\partial L}{\partial R_E} + \frac{\partial L}{\partial R_E}) \right]
\]

where a sufficient condition for an optimum is \(\partial^2 S / \partial R^2_E < 0\). In equilibrium condition (6), the first term on the right-hand side is positive in sign since \(L\) and \(L_E\) is gross substitutes. The second term is positive based on the equilibrium condition.

Implicit differentiation of equation (6) with respect to \(c\) yields:

\[
\frac{\partial R_E}{\partial c} = -\frac{\partial^2 S}{\partial R_E \partial c} / \frac{\partial^2 S}{\partial R^2_E} \quad (7)
\]

where
\[
\frac{\partial^2 S}{\partial R_L \partial c} = -\frac{\partial^2 Z}{\partial R_L \partial c} e^{-\mu} N(d_2) + \frac{\partial V}{\partial R_L} \left( \frac{\partial N(d_1)}{\partial d_1} - N(d_1) \frac{\partial N(d_2)}{\partial d_2} \right) \frac{\partial d_1}{\partial c}
\]

\[
\frac{\partial^2 Z}{\partial R_L \partial c} = -\frac{2(R - R_L)K(q')^2}{q^3} \frac{\partial L}{\partial c} \left( \frac{\partial L}{\partial R_L} + \frac{\partial L_E}{\partial R_L} \right) + \left[ \frac{L^2}{2} + (1 + R_L) \frac{\partial L_E}{\partial R_L} \right] < 0
\]

\[
\frac{\partial d_1}{\partial c} = \frac{1}{\alpha} \left( \frac{c \frac{\partial V}{\partial c}}{V} - \frac{c \frac{\partial Z}{\partial c}}{Z} \right)
\]

The term, \( \frac{\partial d_1}{\partial c} \), can be interpreted as the difference between the information asymmetry elasticity of loan repayments, \( (c/V)(\frac{\partial V}{\partial c}) \), and that of net obligations, \( (c/Z)(\frac{\partial Z}{\partial c}) \). The information asymmetry elasticity of loan repayments is unambiguously negative because an increase in the degree of capital market imperfection makes the bank’s risky loans more costly to grant. The information asymmetry elasticity of net obligations is indeterminate in sign. However, our model provides us with a hunch that this difference should be negative since information asymmetry is a loan-repayment issue rather than a net-obligation one in our setting of the paper. Thus, we have \( \frac{\partial d_1}{\partial c} < 0 \). In light of previous work, we establish the following proposition.

**Proposition 1:** An increase in the degree of capital market imperfection increases the outside manager’s loan rate.

Intuitively, there is the potential for adverse selection in the capital market, perhaps because the outside manager has gained some information as to the quality of the bank’s earning-asset portfolio, and can use this information to exploit the centralized decision maker. When the outside manager has a higher degree of information asymmetry in the capital market, the outside manager provides a return to a less lending base by increasing loan rate \( R_E \). Applying Kashyap, Rajan, and Stein’s argument (2002, p.42), we further argue that the higher the degree of information asymmetry, the more costly employing the outside manager is to the centralized decision maker.

**4 Centralized Loan-Rate Setting Stage**

To choose the optimal centralized loan rate, the centralized decision maker must take the decentralized loan rate determination into account. Under a unique market equilibrium assumption, \( R_E(R_L) \) completely characterizes the outside manager’s loan-rate equilibrium as a function of the bank’s loan rate. We can substitute \( S(R_L, R_E, c) \) to obtain \( S(R_L, R_E(R_L), c) \). Accordingly, the bank solves the following objective, \( \max_{R_L} S(R_L, R_E(R_L), c) \), where \( S \) is defined in equation (5). Partial differentiating equation \( S \) with respect to \( R_L \), the first-order condition is given by:

\[
\frac{\partial S}{\partial R_L} = \frac{\partial V}{\partial R_L} N(d_1) - \frac{\partial Z}{\partial R_L} e^{-\mu} N(d_2) = 0 \tag{8}
\]

where

\[
\frac{\partial V}{\partial R_L} = L + (1 + R_L) \alpha_L
\]

\[
\frac{\partial Z}{\partial R_L} = \frac{(R - R_L)Kq' \alpha_L + \alpha_E}{q^2} \left[ \frac{L}{2} \left( 1 - \frac{cL}{L_E} \right) \frac{\partial R_E}{\partial R_L} \right] - \left( 1 + R_L \right) \alpha_L
\]

\[
\alpha_L = \frac{\frac{\partial L}{\partial R_L}}{\frac{\partial R_L}{\partial R_L}} + \frac{\frac{\partial L}{\partial R_E}}{\frac{\partial R_E}{\partial R_L}}
\]

\[
\alpha_E = \frac{\frac{\partial L_E}{\partial R_L}}{\frac{\partial R_L}{\partial R_L}} + \frac{\frac{\partial L_E}{\partial R_E}}{\frac{\partial R_E}{\partial R_L}}
\]

A sufficient condition for an optimum is \( \frac{\partial^2 S}{\partial R_L^2} < 0 \). The terms \( \alpha_L \) and \( \alpha_E \) represent how changes in centralized loan rate-setting affect the bank’s loan amount and outside manager’s loan amount, respectively. We apply Bulow, Geanakoplos, and Klemperer (1985) and demonstrate that a crucial strategy in determining the nature of the interactive strategy is whether the bank regards both centralized and decentralized loans as strategic substitutes or strategic complements. We discuss this in more detail below.
The term \( \partial R_E/\partial R_L \) is equal to
\[-(\partial^3 S/\partial R_E \partial R_L)/(\partial^2 S/\partial R_E^2). \] If the numerator is negative (and thus \( \partial R_E/\partial R_L < 0 \)), the bank regards both centralized and decentralized loans as strategic substitutes, accordingly, \( \alpha_L < 0 \) and \( \alpha_E > 0 \). Based on rather general assumptions, it is reasonable to believe that the impact on the bank’s lending from a change in the centralized loan rate-setting (the own-rate effect) is more significant than the impact on the outside manager’s lending from a change in the centralized loan rate-setting (the cross-rate effect), at least in the short run. Thus, the sign of \( (\alpha_L + \alpha_E) \) is negative.

But if the numerators \( \partial^2 S/\partial R_E \partial R_L \) are positive (and thus \( \partial R_E/\partial R_L > 0 \)), the bank regards both centralized and decentralized loans as strategic complements. \( \alpha_L \) is still negative since the direct effect \( (\partial L/\partial R_L) \) is in general more significant than the indirect effect \( (\partial L/\partial R_E)(\partial R_E/\partial R_L) \). \( \alpha_E \) follows a similar argument as in the case of \( \alpha_L \). Accordingly, \( (\alpha_L + \alpha_E) \) is also negative.

Condition (8) implies that the bank sets its optimal loan rate at the point where the marginal loan repayment of loan rate equals the marginal net-obligation payment.

Implicit differentiation of equation (8) with respect to \( c \) yields:
\[
\frac{\partial R_L}{\partial c} = -\frac{\partial^2 S}{\partial R_L \partial c} \frac{\partial^2 S}{\partial R_E^2}
\] (9)

where
\[
\frac{\partial^2 S}{\partial R_L \partial c} = \frac{\partial^3 V}{\partial R_L \partial c} N(d_1) - \frac{\partial^2 Z}{\partial R_L \partial c} e^{-\mu} N(d_2)
\]

\[
+ \frac{\partial V}{\partial R_L} \left( \frac{\partial N(d_1)}{\partial d_1} \frac{\partial N(d_2)}{\partial d_2} \right) \frac{\partial d_1}{\partial c}
\]

In the term \( \partial^2 S/\partial R_L \partial c \), the difference between the first term and the second term can be interpreted as the mean profit effect, while the third term can be interpreted as the variance or risk effect. The sign of this mean profit effect is indeterminate. The sign of this variance effect is negative since the three terms in this effect are all negative in sign.

When the mean profit effect is negative, the variance effect reinforces the mean profit effect. The total effect of an increase in \( c \) on the optimal bank interest rate is unambiguously negative. The negative total effect implies that the bank regards loan rate-setting and information asymmetry as strategic substitutes in Bulow, Geanakoplos, and Klemperer’s (1985) sense. But if the mean profit is positive, it is reasonable to believe that the negative variance is insufficient to offset the mean profit effect since \( c \) is modeled in the mean profit effect directly, but indirectly in the variance effect. Under the circumstances, the total effect of an increase in \( c \) on the optimal bank interest rate is positive. This positive total effect implies the bank regards loan rate-setting and information asymmetry as strategic complements. The result of equation (9) is stated in the following proposition.

**Proposition 2:** An increase in the degree of capital market imperfection decreases (increases) the bank’s optimal loan rate under strategic substitutes (strategic complements).

The outside manager can use some information to exploit the centralized decision maker. When the bank has a higher degree of capital market imperfection, it provides a return to a larger lending base due to be exploited by its outsider manager. One way the bank may attempt to increase its total returns is by increasing total loan repayments at a reduced loan rate under strategic substitutes.

**5 Conclusions**

This paper explores the optimal bank interest margin determination based on an option-based firm-theoretic model under the centralized loan portfolio construction with decentralized loan management. The model demonstrates how capital market imperfection and risk conditions jointly determine the optimal bank interest margin decision. We find that an increase in the bank’s (the centralized decision maker’s) degree of capital market imperfection increases the decentralized (the outsider manager’s) loan rate and thus the bank’s interest margin, increases...
its loan rate under strategic complements, and decreases its loan rate under strategic substitutes. One way the bank may attempt to augment its total equity returns is by employing a outside manage for diversified lending activities since there is information asymmetry in different loan markets. Insofar as such changes in optimal loan rates in our two-stage framework affect the bank’s ability to sustain information asymmetry, these effects are relevant considerations in any centralized restructuring of the decentralized portfolio management.

References