Analog Filter Design by Genetic Algorithms

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Abstract:**-** There are well known classic techniques for filter design, the most common are the Butterworth and Chebyshev. The difference in their response is due to the parameter values in the filter's transfer function. Selection of a specific set produces a specific behaviour; hence a designer must select either one among a limited number of responses.

By using Genetic Algorithms (GA's), this paper will provide a filter designer with the freedom to specify the responses he desires and allow the GA's to calculate the parameter values that would realize this response. The paper presents results of filter design by GA's and its performance is compared with that of Butterworth and Chebyshev designed filters.

Key-Words: **-** Genetic algorithms, filter design, circuit design, optimization techniques.

1 Introduction

In circuit theory, a filter is an electrical network that alters the amplitude and/or phase characteristics of a signal with respect to frequency. Ideally, a filter will not add new frequencies to the input signal, nor will it change the component frequencies of that signal, but it will change the relative amplitudes of the various frequency components and/or their phase relationships.

There well known techniques for filter design, a designer chooses among a specific set of parameters for lowpass filters but to design a highpass or bandpass filter some transformation techniques are used. Hence a designer must be proficient and he has a limited choice of filter types, by using Genetic Algorithms (GA's) the designer has a complete freedom to choose the filter with the desired behaviour regardless of the filter type. By using GA's a designer does not necessarily need to be proficient in filter design.

GS's are search algorithms based on the ideas of natural selection and inheritance. The concept of GA is designed to mimic processes in natural systems necessary for evolution, specifically those that follow the principles of "survival of the fittest" suggested by Charles Darwin. He observed that the living species have adapted themselves to a constantly shifting and changing environment in order to survive. Those weaker members of a species tend to die away, leaving the stronger and fitter to mate, create offspring and ensure the continuing survival of the species. Their lives are dictated by the laws of natural selection and Darwinian evolution. And it is upon these ideas that genetic algorithms are based [1].

GA, a class of adaptive stochastic optimization algorithms, was first developed by John H. Holland. He created an electronic chromosome in a form of a binary string and he selects chromosomes for reproduction using its relative fitness based on the principles of genetics. This will allow the program to narrow the search space in order to find optimized solutions and using the optimized solutions as the input for the next generation. So the population of solutions improves after each generation [2].

2 Genetic algorithm

The basic idea of Genetic algorithm is that it works in iterations and there is an improvement in every step as a result of benefiting from the inherited treats from the previous step. It starts from the initial solution, in each iteration step, an operation is selected and it is applied to the current solution. If the altered solution is acceptable e.g. it is better than the current one, it becomes the current solution for the next iteration, otherwise, it is refused. The iteration process stops when the requested solution or the maximum number of iterations is reached.

In general, genetic algorithms are better than gradient search methods if the search space has several local minima or maxima. Since the genetic algorithm traverses the search space using the genotype rather than the phenotype, it is less likely to get stuck in a local high or low [3, 4].

GA's start with a set of initial random solutions called population and these solution are evaluated by test and are sorted according to their fitness, those solution or individuals having higher fitness values are given a chance to reproduce by an operation called cross-over and the less-fit individuals are discarded. After cross-over some bits “gene” are flipped either from 1 to 0 or vice versa in an operation that mimics mutation in living organisms. A flowchart of a generic genetic algorithm is shown in figure 1.

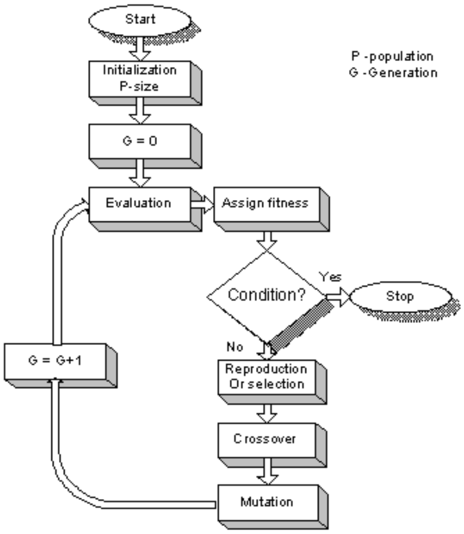


Fig. 1 : Genetic algorithm flow chart

3 Filters and Signals.

Filters are intended to pass some signal frequencies and stop others. Before a design can commence, the designer needs to consider the signals that need to be processed in this way. Once the type of filter has been selected, the appropriate normalized frequency response curve can be used to find the filter order required. The lowest filter order to achieve the desired stop-band attenuation is usually chosen because the filter will be simpler and would have a lower cost [5].

There are many types of filter designs, the most common are Butterworth and Chebyshev which are described briefly below.

The Butterworth lowpass filter provides maximum passband flatness. Therefore, a Butterworth lowpass is often used as anti-aliasing filter in data converter applications where precise signal levels are required across the entire passband. Figure 2 plots the gain response of different orders of Butterworth lowpass filters versus the normalized frequency axis, $\Omega(\Omega = \frac{f}{f_c})$; the higher the filter order, the longer the passband flatness.

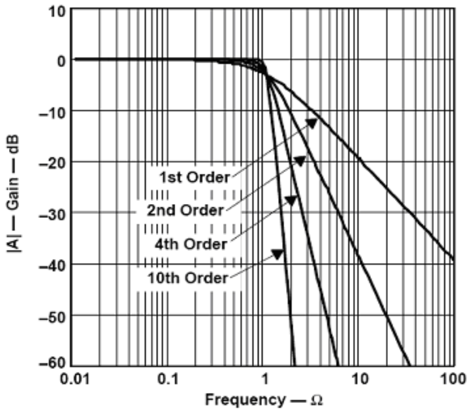


Fig. 2: Amplitude responses of Butterworth lowpass filters

The Chebyschev lowpass filters provide an even higher gain rolloff above f_c. However, as figure 3 shows, the passband gain is not monotone, but contains ripples of constant magnitude instead. For a given filter order, the higher the passband ripples, the higher the filter’s rolloff [6]

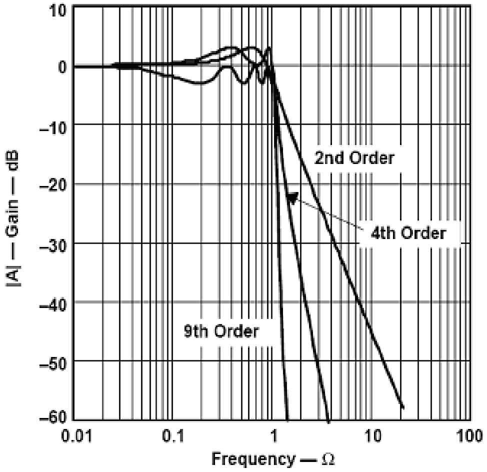


Fig. 3 Gain responses of Chebyschev low-pass filters

With increasing filter order, the influence of the ripple magnitude on the filter rolloff diminishes. Each ripple accounts for one second-order filter stage. Filters with even order numbers generate ripples above the 0-dB line, while filters with odd order numbers create ripples below 0 dB. Chebyschev filters are often used in filter banks, where the frequency content of a signal is of more importance than a constant amplification [6,7].

4 Filter Design by Using Genetic Algorithms

An example of filter design by genetic algorithms (GA) is given below. It is compared with designs by classic methods, namely, Butterworth and Chebyshev.

GA’s are used to design lowpass and highpass filters as examples of the capability of GA’s

For a lowpass filter, the first step is the description of the desired filter response depending on its order. The desired or what we may call theoretical or ideal behaviour of the filter has a constant response of 0 dB from DC to the cutoff frequency, f_c which is chosen to be 100 and then the output drops with a constant slope that depends on filter order. The first order has a slope of 20 dB per decade, the second has a slope of 40 dB per decade and so on. Figure 4 shows the theoretical response produced by implementing the coefficients in the transfer function of the first order filter.

In general, the coefficients of a filter of order *n* are indicated by the transfer function 1.

$$H(S)=k\left(\frac{b_0 s^n+b_1 s^{n-1}+b_2 s^{n-2}+\ldots b_{n-2} s^2+b_{n-1} s+b_n}{a_0 s^n+a_1 s^{n-1}+a_2 s^{n-2}+\ldots a_{n-2} s^2+a_{n-1} s+a_n}\right) \qquad (1)$$

where a_i and b_i are the transfer function coefficients and *n* is the filter order.

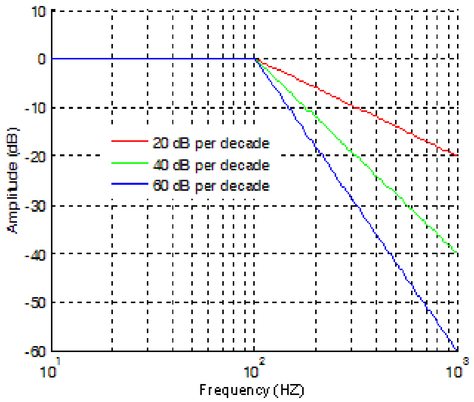


Fig. 4: Theoretical (ideal) response of the filters

In reality, no filter can behave exactly as depicted by the curves in figure 4, however, approximations can be obtained with some errors.

4.1 Evaluation of the cost function (objective function) for filter design by GA’s.

The main task of GA is to vary the transfer function coefficients to minimize errors between the filter responses based on assumed random values in order to make it as close as possible to the theoretical behaviour.

Error function is the difference between two curves compared at several points; one curve is the desired response of the filter and the second is the result produced by filter coefficients produced by the Genetic algorithm. The following equation 2, represents the optimization error function

.

$$Error = \sum_{i=1}^{n} \left| M_{id} - M_{iGA} \right| \qquad (2)$$

Where M_d is the desired response and M_{GA} is the response produced by GA’s. Comparison between them is done at n points. Figure 5 shows errors iin the objective function.

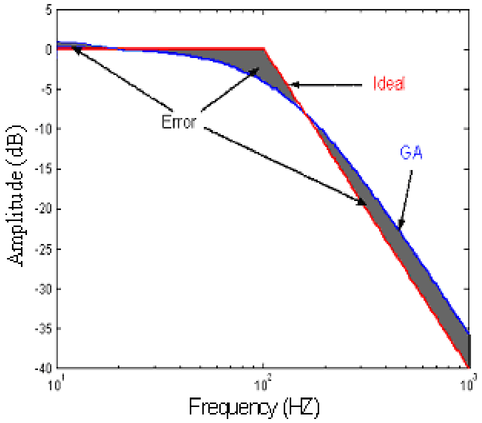


Fig. 5: Errors in the objective function indicated by the dark shaded area.

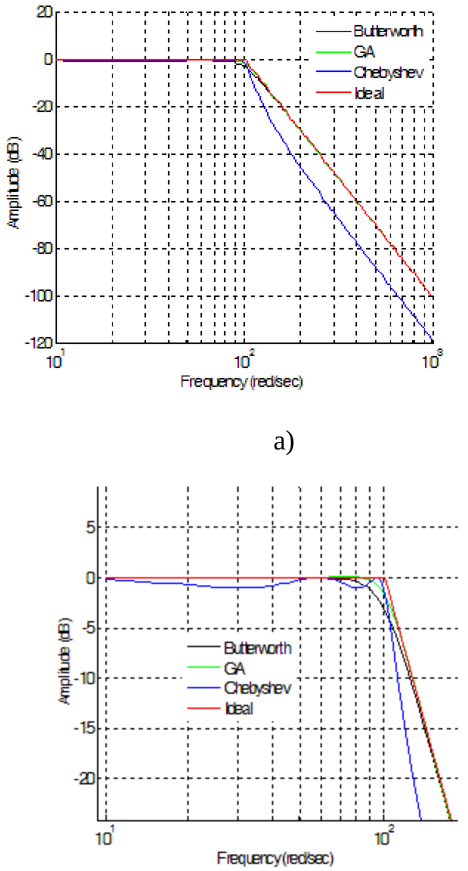
4.2 Lowpass filter design example.

Butterworth and Chebyshev filters are designed according to the transfer function given in equation 1. The ideal low-pass filter has a pass-band that starts at DC and continues with constant response until the specified cutoff frequency. At this cutoff frequency the magnitude will decrease to zero with a specific constant decay that depends on filter order.

For the worked examples in this paper the specifications of GA’s are:

- Cutoff frequency (f_c =100 Hz).
- Number of generations equal to 300.
- Initial population size equal to 100.

Figures 6 and 7 compare the frequency and phase response of GA, Butterworth, Chebyshev and the ideal fifth-order low bass filters.



a)

(b)

Fig. 6: Amplitude response of the fifth order low pass filter, a) a wide view and b) the filter responses in a more detail.

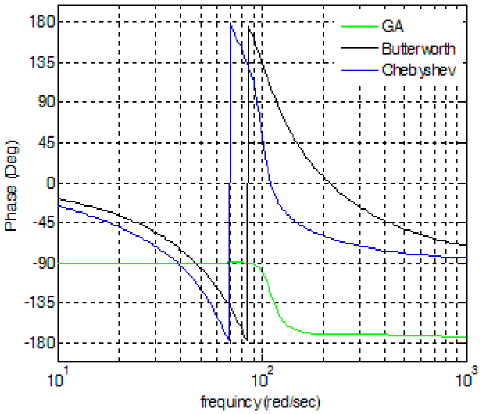


Fig. 7: Phase response of 5^{th} order low pass filter

We observe that the frequency response of the GA and ideal filters is almost identical from 10 Hz to about 1 kHz. Above this, GA shows better performance.

The frequency response of the GA designed filter is flat at the band pass region of the filter which makes the filter being non-dispersive.

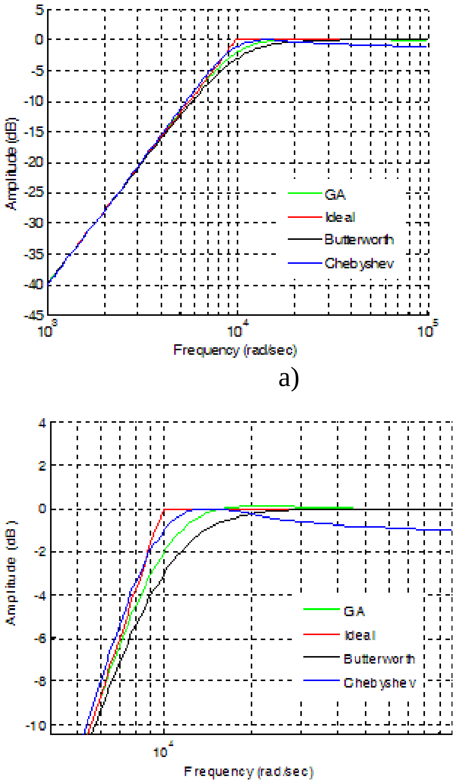
4.3 Highpass filter design example.

Butterworth and Chebyshev filters are designed according to the transfer function given in equation 1. The ideal high-pass filter has a pass-band that starts at cutoff frequency and goes to infinity with a constant response. From cutoff and going down to DC the magnitude will decrease to zero with a specific constant decay that depends on filter order.

For the worked examples the in this paper the specification of GA are:

- Cutoff frequency (f_c =100 Hz).
- Number of generations equal to 300.
- Initial population size equal to 100.

This section presents the results of highpass second order filter design. Figures 8 and 9 show the amplitude and frequency response of these filters, respectively.



a)

b)

Fig. 8 : Amplitude response of the second order highpass filter a) A general view and b) a more detailed view.

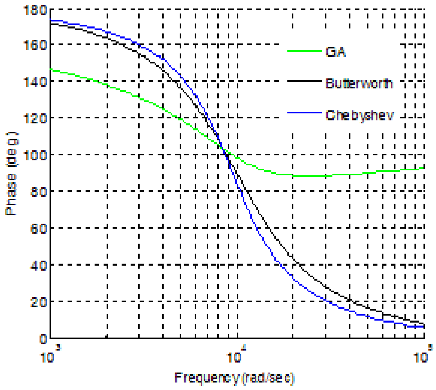


Fig. 9 : Phase response of the second order highpass filters

5 Conclusions

From the work accomplished here we can draw the following conclusions:

It is observed that GA designed filters have a response that is very close to the desired one and can be made closer as the control points are added. Filters designed by GA's have the slope of the frequency response steeper than Butterworth and less ripple at the pass-band than Chebyshev which shows that they are more efficient, i.e. reduced passband ripple and increased stop-band attenuation. Filter design is simplified, a designer can select the desired response and the GA algorithm will do the work for him. A designer does not necessarily have to master the art of filter design.

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