Parametrization of a Departure Model

JAN KRCAL
Department of Informatics and Telecommunications
Faculty of Transportation Sciences
Czech Technical University in Prague
Konviktska 20, 110 00 Prague 1
CZECH REPUBLIC
krcal@fd.cvut.cz

Abstract:
Departure model is one of the characteristics, through which we can describe traffic flow on signalized intersections. Management of such intersections or of entire traffic networks requires a highly accurate mathematical model, through which the traffic behavior can be described. There is a direct relationship between the accuracy of such a mathematical model and the accuracy of subsequent simulations of behavior of a traffic intersection. Based on these, we can subsequently control the traffic. The departure model expresses the time necessary for an individual vehicle (standing in a queue) to leave the pre-shifting area after the green light has been signaled. In this article, a new mathematical model is compared with a hitherto model, which is, based on experience and simulations, not fully relevant today. The new model attempts to consider parameters, which influence the resulting departure times. This should render the model more accurate.

Key-Words: departure model, entrance times, intersection, traffic lights

1 Introduction
Department model helps us characterize dynamics of movement of a vehicle/vehicles upon its/their departure from the pre-shifting area. In simpler terms, the departure model is determined by departure times (so-called entrance times), which help us determine the time necessary for a “nth” vehicle waiting in a queue on a red light to pass through the „stop“ line from the onset of the green/yellow-green light.

The term departure model has been introduced in 1947 by Greenshields in his theory of a traffic network [6]. It is undisputable that transportation (vehicles, degree of motorization etc.) has undergone a major development. This is a reason why new departure models were gradually created. They attempted to be more accurate than their predecessors, more or less successfully for the given time (due to up-to-date data).

The resulting departure model (or rather the entrance times) was represented by average times of individual vehicles in a queue (Table 1). If we used one of these models today, we would encounter several problems. For example, the duration of the orange-red phase (used in Medelska’s model [3]) has been reduced from 3 seconds to 2 seconds. Another example of potential insufficiency is that moving off of the vehicles begins today in more than 50% already after the orange-red phase has been signaled. Therefore it is difficult today to use these entrance times without being greatly inaccurate. In this article, a newly created model (described below) is compared with the departure model of Medelska. It does not use the resulting entrance times, but a mathematical apparatus, which led to these entrance times of the departure model. All is, of course, based on real data.

<table>
<thead>
<tr>
<th>Vehicle sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Greenshields</td>
<td>3.8</td>
<td>6.9</td>
<td>9.6</td>
<td>12.0</td>
<td>14.2</td>
</tr>
<tr>
<td>Fischer</td>
<td>3.1</td>
<td>5.4</td>
<td>7.5</td>
<td>9.5</td>
<td>11.4</td>
</tr>
<tr>
<td>Medelská</td>
<td>2.3</td>
<td>5.5</td>
<td>8.3</td>
<td>10.8</td>
<td>13.1</td>
</tr>
<tr>
<td>Medelská (revised)</td>
<td>1.2</td>
<td>3.9</td>
<td>6.6</td>
<td>9.1</td>
<td>11.6</td>
</tr>
<tr>
<td>USMD</td>
<td>3.6</td>
<td>6.5</td>
<td>8.9</td>
<td>11.2</td>
<td>13.4</td>
</tr>
<tr>
<td>Webster</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 1: Departure models of different authors

2 Departure model according to Medelska
Departure model according to Prof. Medelska was created already in 1966 (see Table 1, 3rd row) and revised in 1971 (see Table 1, 4th row). The revision did not involve a change of mathematical apparatus, but a computation of new entrance times of updated data. A significant shift is evident, caused primarily by moving off of vehicles already in the orange-red phase. Let’s look at the mathematical model that was used for the computation of the entrance times. Although Medelska first computed entrance times for a homogeneous traffic flow (for private vehicles and motorcycles only), the
resulting times (Table 1) are for a non-homogeneous traffic flow (all types of vehicles). After Medelska computed average values of entrance times for vehicles in a sequence \(1 \ldots n\), she approximated these average values with the help of the following curves:

- \(ax^2 + bx + c = y\) \hspace{1cm} (Eq. 1)
- \(ax^3 + bx^2 + cx + d = y\) \hspace{1cm} (Eq. 2)
- \(a - \log y^2 = x\) \hspace{1cm} (Eq. 3)
- \(cx^n = y\) \hspace{1cm} (Eq. 4)

Considered to be most accurate, approximation using a second degree polynomial (Eq. 1) was chosen. This polynomial will thus be used to compare this model with the new one.

### 3 New departure model

The departure model presented here introduces a wholly new approach. Unlike the above described model, this model does not express the average times only through a mathematical equation, but attempts to ascertain and subsequently apply parameters, which influence the entrance times of the departure model. Another highly important information used is knowledge of the departure of the preceding vehicle, which concerns all the vehicles with the exception of the first one. It is clear that if for example the third vehicle leaves in 9 seconds, the following vehicles will be influenced by this departure.

There are many parameters impacting the departure model, such as:

- type of the vehicle
- weather
- road surface
- road angle
- geometrical order of the intersection
- impact of the vehicles in the opposite direction
- health of the driver
- age of the driver
- psychological well-being of the driver
- fatigue of the driver
- …

Apparently, some of these parameters are difficult to measure, although it is beyond doubt that they influence the departure of the vehicle. Among these are primarily health-related parameters.

The basic equation of the new model, which takes into consideration the departure of the preceding vehicle, as well as other parameters, can be expressed as follows:

\[
y_t = \beta u_t + \kappa + \epsilon_t \hspace{1cm} (Eq. 5)
\]

where,

\[
y_t = [y_{1,t}, y_{2,t}, \ldots, y_{m,t}] \hspace{1cm} \text{is the vector of modeled departure times of individual vehicles in the row where its length is expressed by } m \text{ and the time index by } t
\]

- \(\beta\) is matrix of parameters
  \[
  \beta = \begin{bmatrix}
  \beta_{11} & \beta_{12} & \cdots & \beta_{1n} \\
  \beta_{21} & \beta_{22} & \cdots & \beta_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  \beta_{m1} & \beta_{m2} & \cdots & \beta_{mn}
  \end{bmatrix}
  \]
  \[
  n \hspace{0.1cm} \text{is the number of parameters}
  \]

- \(u = [u_{1,t}, u_{2,t}, \ldots, u_{n,t}]\) is the vector of values of variables influencing departures
- \(\kappa = [\kappa_1, \kappa_2, \ldots, \kappa_m]\) is the model’s constant
- \(\epsilon = [\epsilon_1, \epsilon_2, \ldots, \epsilon_m]\) is the noise with the median value of zero and covariance matrix \(R\)

After further adjustments (more in [4]), we arrive at the following system of equations:

\[
y_{1,t} = \beta_{11} u_{1,t} + \beta_{12} u_{2,t} + \cdots + \beta_{1n} u_{n,t} + \epsilon_{1,t}
\]

\[
y_{2,t} = \beta_{21} y_{1,t} + \beta_{22} u_{2,t} + \cdots + \beta_{2n} u_{n,t} + \epsilon_{2,t}
\]

\[
y_{3,t} = \beta_{31} y_{1,t} + \beta_{32} y_{2,t} + \beta_{33} u_{3,t} + \cdots + \beta_{3n} u_{n,t} + \epsilon_{3,t}
\]

\[
y_{m,t} = \beta_{m1} y_{1,t} + \beta_{m2} y_{2,t} + \cdots + \beta_{mm} y_{m-1,t} + \epsilon_{m,t}
\]

which we can write down in the matrix:

\[
\begin{bmatrix}
  y_{1,t} \\
  y_{2,t} \\
  y_{3,t} \\
  \vdots \\
  y_{m,t}
\end{bmatrix}
= \begin{bmatrix}
  u_{1,t} \\
  u_{2,t} \\
  u_{3,t} \\
  \vdots \\
  u_{m,t}
\end{bmatrix}
+ \begin{bmatrix}
  \beta_{11} & \beta_{12} & \cdots & \beta_{1n} \\
  \beta_{21} & \beta_{22} & \cdots & \beta_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  \beta_{m1} & \beta_{m2} & \cdots & \beta_{mn}
\end{bmatrix}
\begin{bmatrix}
  y_{1,t} \\
  y_{2,t} \\
  y_{3,t} \\
  \vdots \\
  y_{m,t}
\end{bmatrix}
\]

(6)

### 4 Two different departure models and real data

Both models have been tested on approximately 2,254 vehicles. The data was collected on three signalized...
intersections (there were different entrances within one intersection). Individual vehicles were categorized either as private vehicles (incl. motorcycles) or as commercial vehicles (all others). Although other parameters were also measured, only one parameter was used for the sake of the best illustration – type of vehicle.

The basic assumptions for the simulation were as follows: there are 5 vehicles in the queue, the traffic flow is non-homogeneous and the measuring starts with the onset of the yellow-red phase.

The computed average times of the departure of vehicles 1 – 5 were as follows:

<table>
<thead>
<tr>
<th>Vehicle sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>𝑡</td>
<td>1.9126</td>
<td>4.9185</td>
<td>7.2098</td>
<td>9.3624</td>
<td>11.3315</td>
</tr>
</tbody>
</table>

If we put through these points a polynomial curve of a second degree according to an equation (Fig. 1), we arrive at resultant values of entrance times according to Medelska’s model.

Before we compare these entrance times with the new model, it is worth it to compare them with the data in Table 1. Note that the entrance times in the table are computed from the onset of the green light (in contrast to data used in this model). Therefore, a significant difference is observable here.

Figure 2 shows comparison of the overall error in the Medelska’s model (described above), which uses approximation of the polynomial of a second degree, with the overall error of the new model based on real data. The error was computed using a method of least squares.

As is clear from this figure, Medelska’s model is worse by approximately 47%, or rather the overall sum total of errors of the Medelska’s model is by 47% worse than in the new departure model. This is a very good result, especially when we realize that in the meantime, only one parameter has been considered.

4 Conclusion

The proposed departure model is capable of computing entrance times of departing vehicles relative to the entrance time of the preceding vehicle and relative to various parameters, which have a significant impact on the departure of a vehicle. These can make the departure model substantially more accurate, because different parameters for different types of intersections are considered. In this simulation, only one parameter – type of vehicle (private vs. commercial) was used. From the results arrived at through a simulation (Figure 2), the Medelska’s model is by 47% worse than the proposed model. Medelska interpolated her measured and averaged entrance times for individual vehicles with a second degree polynomial.

These results are very optimistic and they show that the proposed departure model is a step in the right direction. Since the main strength of the proposed model are particularly parameters, the object of further research is their determination, examination and verification.

References:
