Mathematical Theory of Information Technology

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Abstract: - A theoretical framework for a technological study of information processes, mathematical theory of information technology, is developed based on system theory principles. In the mathematical theory of information technology such problems as reliability, equivalence, stability, robustness, and realizability of information technologies are studied. Concepts of a technological operator, composition of technological operators, and technological conditions are considered for computational processes and systems. Their mathematical models are developed and studied from the perspective of information technology quality. Classes of technological operators are separated based on practical problems. Relations between technological operators are studied. The aim is the development of efficient methods and algorithms for computer hardware engineers, software engineers and system engineers.

Key-Words: - Information technology, mathematical model, technological process, technological operator, robustness, information processing

1 Introduction

Important information and communication technology (ICT) related projects fail too often. According to Standish Group’s report [7], less than 20 % of software development projects of large organizations succeed, being delivered in time and in budget. To improve this situation, it is necessary to analyze information processing in general and software development processes, in particular, and to elaborate more advanced and efficient software technology based on scientific principles.

Analysis shows that there are three levels of computation, communication and networking representation. The first level indicates what is done in a computational/communication process or in a process in a network. From this perspective, processes are represented as sequences of events or actions. The mathematical theory that studies processes on this level is process algebra. The second level tells us not only what is done in a computational/communication process or in a process in a network but also how it is done. From this perspective, processes are represented by algorithms, programs, and scenarios. The major mathematical theory that studies processes on this level is the theory of algorithms. The third level of process description explains us not only what is done in a process and how it is done but also with what means everything is performed in the process. From this perspective, processes are represented by technologies.

Now education of engineers in information technology area, such as software engineers, computer hardware engineers, and system engineers, goes only on the first two levels [8]. For instance, traditional computer science programs did not equip their graduates with the practical network or system administration skills that organizations needed to expand and maintain their IT infrastructures, or the web development skills required to take advantage of the many opportunities opened up by the Internet. Besides, graduates often do not acquire a sufficient understanding of organizational processes to be able to support IT applications from a user or organizational perspective.

The last 20 years or so has seen a dramatic growth in the demand for professionals trained in information and communication technologies [8]. This growth has been driven by tremendous technological advances, such as the emergence of computer networking, including wireless networking, graphical user interfaces, and the Internet, the World-Wide Web, and their applications, as well as by a much greater realization on the part of many organizations and individuals of the importance of information and communication technologies.

Emerging millennia ago, technology plays more and more important role in society. Modern society cannot exist without technologis that support its normal functioning in general and information technologies, in particular. Many production technologies, such as semiconductor technologies,
have been developed. However, Only recently, the development of technological knowledge and advancement of mathematics made it possible to elaborate a mathematical theory of technology [2 - 5]. Its starting point is an informal definition of technology as a specific system of knowledge that describes how different things are produced or various services are provided. Basing on this definition, two classes of technologies (general and specific technologies) are introduced to reflect the situation existing in industry and engineering. This provides for the construction of a general mathematical model of a specific technology as well as for the development of a relevant mathematical apparatus and exact methods for an investigation and design of various technologies (in industry, management, information processing and so on).

The basic concept of the theory, a technological operator, reflects operations performed by people and machines, as well as natural processes included in technological processes, e.g., biotechnologies utilize biological processes, while chemical technologies are based on chemical processes.

The mathematical theory of technology utilizes new mathematical disciplines, such as theory of named sets, fuzzy set theory, and theory of structured multidimensional models, as well as traditional fields such as algebra, probability theory, and the theory of algorithms. In the mathematical theory of technology, such problems as stability, reliability, equivalence, constructability, and realizability of technologies are studied. The aim is the development of efficient methods and algorithms for building various technologies in industry, construction, business, medicine, and so on.

To know properties of hardware or software of information processing systems, such as computers, we evaluate corresponding properties of technologies based on these systems and performance of these systems in evaluated technologies. Thus, evaluation of systems is reduced to evaluation of technologies because in information processing, it is important how the system works and not how this system looks.

In the paper, we use mathematical notation, such as symbol \( \forall \) that means "for any" or "for all" and symbol \( \exists \) that means "there is" or "there exists".

2 Methodological Framework
To build a mathematical model of a system, it is necessary to understand this system. Understanding is usually expressed through definitions. In spite of being one of central phenomena of the modern civilization, technology is difficult to define. That is why many encyclopedias and dictionaries do not have such a definition. Those definitions that are given describe rather than define technology. Let us consider some of them.

Originally, the word 'technology' comes from the Greek word 'techne', which means art or craft. This suggests that:

- technology is a set of crafts or techniques for making things,
- or (in the modern environment),

technology is a collection of methods that people use for building, manufacturing and producing, or

technology is a systematic application of scientific knowledge to some practical purpose or activity [9]. At the same time, technology is contemplated as the technical means people use to improve their surroundings. Another definition also interprets technology as electronic or digital products and systems considered as a group (for instance, a store specializing in office technology [1].

All existing definitions show that the term technology has many meanings and is subject to diverse understanding. On the one hand, technology is interpreted as physical objects (tools, mechanisms, devices, and machines) and processes (instrumental actions that use these physical objects). On the other hand, technology is interpreted as knowledge how to build these tools, how to organize these processes and how to apply them to achieve a desired outcome (some practical purpose). We call this knowledge technological.

Thus, we discern three sides (aspects) of technology:

- Knowledge in the brains, instruction manuals, books, etc.
- Technology as an abstract structure
- Technological practice

In turn, technological practice also has three facets:

- Dynamic objects
- Functions
- Processes

Static objects tools, mechanisms, devices, and machines
Knowledge in technology is pivotal, e.g., technological transfer concentrates on problems of knowledge exchange because giving machines without knowledge how to use them is not a technology transfer. If we want to build a theory of technology, we have to elaborate mathematical models of technological knowledge. Technological knowledge naturally falls into two categories: concrete and general technologies.

A concrete technology $T$ is a system of completely specified methods and procedures for representation of a definite kind of processes used for obtaining a specific result.

A concrete technology describes: what is necessary to do (goals), how to do (procedure/program), and in what conditions it is done (by what means, with what materials, and who has to do this).

A process, method or procedure $M$ is called technologically specified if conditions and means for its realization are included in its description.

A general technology $GT$ is a cluster of knowledge about concrete technologies.

A technological process is a process that is realized as it is prescribed by some technology.

Information technology, IT, is a type of technology that deals with the use of computers and telecommunications to retrieve, store and transmit information.

Technologies used in society belong to one of the following three classes:

1. Production/construction technologies are aimed at creation and/or transformation of some things (which can be material or ideal, such as algorithms, knowledge, rules, laws or ideas).
2. Service technologies are aimed at providing some service.
3. Utilization technologies are aimed at utilization of some product.

Service technological operators, in turn, perform actions of three sorts:

1. Support of a given activity, e.g., maintenance, including repair, of a system.
2. Organizing of a given activity.
3. Providing commodities, including intellectual, emotional, and spiritual ones.

### 3 Mathematical Models of Concrete Technologies

The basic concept of the mathematical theory of technology is a technological operator.

**Definition 3.1.** An elementary technological operator $T_o$ (or an elementary technology) is a triad $(Cb, Op, Cf)$

where:

- $Cb$ denotes a description of the initial technological conditions or input specification of $T_o$, that is, such conditions that must be satisfied for the beginning of $T_o$ execution;
- $Cf$ denotes a description of the finalizing technological conditions or output specification of $T_o$, that is, such conditions that must be satisfied when $T_o$ gives its result;
- $Op$ denotes the operation realized by $T_o$.

Technological conditions have a complex structure and their description includes three components:

- descriptions DAA of all active agents (performers and operational devices) AA;
- descriptions DPA of all passive agents (objects of activity) PA;
- descriptions DCC of the context conditions CC.

**Example 3.1.** Let us consider a technology $T$ for designing some software product. It has the following components. DAA: three programmers (who can write programs in C++ and Java, know methods of system analysis, and have, at least, two years of experience), three computers, and one workstation.

DPA: algorithms and programs with given specifications.

DCC: one programmer works at home and two programmers come to the office and work there from 9 a.m. to 6 p.m.

Active agents consist of two types:

1. Performers or executors (producers, managers, and controllers).
2. Operational devices (completely controlled, partially controlled, and autonomous).

Passive agents consist of three types: raw materials (input data), work materials, products (output data).

Context conditions have different types: temporal conditions (such as physical time, system time, etc.), spatial conditions (such as physical space, information space, state space, etc.), and combined conditions (such as velocity, productivity, etc.).

State space may include such parameters as temperature, light conditions, pressure, etc.

**Definition 3.2.** A complete technological operator $T_c$ is a triad of the following form:

$$T_c = (T_{pr}, T_m, T_{pt})$$

Here

$T_{pr}$ is an operator that describes execution of preparatory operations;

$T_m$ is an operator that describes execution of the main operation;
Tpt is an operator that describes execution of concluding operations.

**Example 3.2.** A technology of computer simulation includes, as a rule, such preparatory operations as: coming to a computer, sometimes switching the computer on, log in, preparation of parameters for simulation, etc. Concluding operations may be: saving the results of simulation and/or analysis of these results, log out, switching the computer off, etc.

**Definition 3.3.** A composite technological operator $Tp$ is a technological operator that consists of other technological operators.

In what follows, we primarily consider information technologies. The main input, output and working material of information technologies is information in the form of data and knowledge.

Types of information technologies are presented in the following table:

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Information in the form of data</td>
<td>Information in the form of data</td>
<td>computation</td>
</tr>
<tr>
<td>2. Information and materials</td>
<td>Physical objects</td>
<td>Automated manufacturing</td>
</tr>
<tr>
<td>3. Information and materials</td>
<td>Technical devices, e.g., computers</td>
<td></td>
</tr>
<tr>
<td>4. Information and materials</td>
<td>Service</td>
<td>Communication</td>
</tr>
<tr>
<td>5. Information in the form of data</td>
<td>Information products in the form of programs, texts, music, images, etc.</td>
<td>Programming, writing, painting, performing, etc.</td>
</tr>
</tbody>
</table>

**Table 1.** Types of information technologies

**Definition 3.4.** A composition of technological operators is called technological if it is determined by a technological operator.

Sequential composition is considered in the next section.

**Definition 3.5.** A technology is a technological composition of complete technological operators. Thus, a general form of the mathematical model of technology has the following structure:

$$ T = \{ T_c, T_1, \ldots, T_n \} $$

Here $T_c$ is the composition operator of the technology $T$ for operators $T_1, \ldots, T_n$.

It means that technology is a composition of technological operators and complex technological operators are composed of simpler technological operators. Decomposition of technological operators allows one to study properties of the technology by exploring properties of elements of its decomposition and utilizing composition of properties [6].

Computer programs can illustrate this construction. As it is demonstrated in [2], any technology is, in particular, an algorithm and a program. On the other hand, any program may be considered as an incomplete technology. Incompleteness means that program in itself does have for example directions what type of computer it needs for its execution, the volume of necessary memory etc. Thus, the structure of a program reflects the general structure of a technology. Really, a program can be perceived as a collection of modules (operators $T_1, \ldots, T_n$) with a small, algorithmic core (operator $T_c$) that expresses how the modules are used to obtain a desired effect. In a sense, the core is expressed in an application specific language, with control constructs implemented by the modules.

### 4 Relations between Production Technologies

To understand what technologies are doing and how they work, as well as to build new efficient technologies and to improve existing ones, it is necessary to know properties of technologies and relations between technologies. Here we study such relations as covering, inclusion and decomposition.

**Definition 4.1.** A technological operator $A$ is a part of a technological operator $B$ if there is a decomposition $B = \{ B_i, B_1, \ldots, B_n \}$ such that $A = B_i$ for some $i \in \{1, 2, 3, \ldots, n\}$.

In this case, we also say that the technological operator $A$ is included in the technological operator $B$.

**Definition 4.2.** A technology $T_1$ is a part of a technology $T_2$ if there is a representation of $T_1$ and $T_2$ by technological operators $A$ and $B$ where $A$ is a part of $B$.

In this case, we also say that the technology $T_1$ is included in the technology $T_2$.

Inclusion of technologies allows one to model the process of hierarchical construction of technologies when simpler technologies are used to build a complex technology.

**Theorem 4.1.** If a technological operator $A$ is a part of a technological operator $B$, a technological
operator $B$ is a part of a technological operator $C$, and composition operators $B$, and $C$ are composable into one composition operators $D$, then the technological operator $A$ is a part of the technological operator $C$.

Our main interest in this section is in production technologies represented by production technological operators. Such operators are aimed at creation and/or transformation of some things (which can be material or ideal, such as algorithms, knowledge, rules, laws or ideas). It means that a production operator $D$ gets some input and transform it into definite output. Thus, production operator $D$ determines a mapping $f_D : X_d \rightarrow Y_d$ where $X_d$ is the domain of all possible inputs for $D$ and $Y_d$ is the domain of all possible outputs for $D$. Note that the mapping $f_D$ may be multivalued.

For production technological operators (production technologies), it is possible to define their sequential composition. Let us consider two technological operators $A$ and $B$.

**Definition 4.3.** A technological operator $D$ is a sequential composition of technological operators $A$ and $B$ if application of the technological operator $D$ consists of two stages: at first, the technological operator $A$ is applied and taking output of $A$ as its input, the technological operator $B$ is applied.

The sequential composition $D$ is denoted by $A \circ B$. We call technological operators $A$ and $B$ sequential parts of the technological operator $D$.

**Example 4.1.** It is necessary to make a decision whether to start oil digging in the area that is close to a national park. A technology $T$ of this decision-making can be decomposed in three parts $A$, $B$, and $C$, i.e., $T$ is represented as sequential composition $A \circ B \circ C$. Here $A$ is a technology of collecting ecological data about the national park and people who live in the neighborhood, $B$ is a technology of computer simulation of the processes caused by different decisions, and $C$ is a technology of making the decision based on the results of computer simulation.

Note that sequential composition is not always defined.

Taking sequential composition, we see that it is a kind of a general technological composition of technological operators, that is, $D = A \circ B = (D_\circ, A, B)$ where the composition operators $D_\circ$ represents a technology of taking the output of the technological operator output of $A$ and giving it as input for the technological operator $B$. In such a way, it is possible to compose any number of production technological operators because composition of two binary sequential composition operators is a ternary sequential composition operator. Thus, Theorem 4.1 implies the following result.

**Corollary 4.1.** If a technological operator $A$ is a sequential part of a technological operator $B$ and a technological operator $B$ is a sequential part of a technological operator $C$, then the technological operator $A$ is a sequential part of the technological operator $C$.

Another important relation between production technologies called covering reflects the situation when one technology can produce everything that the other technology can produce. This relation is formalized in the following way.

Let us consider two production technological operators $A$ and $B$.

**Definition 4.4.** A technological operator $B$ covers a technological operator $A$ if $X_A \subseteq X_B$, $Y_A \subseteq Y_B$ and for any $x$ from $X_A$, we have $f_A(x) \subseteq f_B(x)$. We denote this relation by $A \ll B$.

**Definition 4.5.** A technological operator $B$ strictly covers a technological operator $A$ if $X_A \subseteq X_B$, $Y_A \subseteq Y_B$ and for any $x$ from $X_A$, we have $f_A(x) = f_B(x)$. We denote this relation by $A \ll_a B$.

Let us consider production technological operators $A$, $B$, and $C$.

**Proposition 4.1.** a) If $A \ll B$ and $B \ll C$, then $A \ll C$.

b) If $A \ll_a B$ and $B \ll_a C$, then $A \ll_a C$.

Let us consider production technological operators $A$, $B$, and $C$ and assume that sequential compositions $A \circ B$ and $B \circ C$ are defined.

**Proposition 4.2.** If $A \ll_a B$, then $A \circ C \ll_a B \circ C$.

Production technologies can be deterministic and nondeterministic.

When a technology $T_g$ is represented by a technological operator $A$, then $D_A$ denotes the domain from which input data for $A$ are taken and $C_A$ denotes the domain that contains all tentative output of $A$.

**Definition 4.6.** A technological operator $A$ is called output deterministic in a domain $D_0 \subseteq D_A$ if for any $x \in D_0$, the operator $A$ always gives one and the same output.

When $D_0 \subseteq D_A$, the technological operator $A$ is called output deterministic. For instance, technology of addition in calculators and computers is output deterministic for one-digit numbers, e.g., adding 2 and 2 on any calculator or computer, we always get 4.

**Proposition 4.3.** The sequential composition of output deterministic technological operators is an deterministic technological operator.

**Definition 4.7.** A technological operator $A$ is called output nondeterministic in a domain $D_0 \subseteq D_A$.
if for the same \( x \in D_0 \), the operator \( A \) may give different output.

In contrast to deterministic technological operators, the sequential composition of output nondeterministic technological operators is not always output nondeterministic.

**Definition 4.8.** A technological operator \( A \) is called *operationally nondeterministic* if the operator \( A \) may use different operations to produce the same output.

**Proposition 4.4.** The sequential composition of an operationally nondeterministic technological operator with any technological operator is an operationally nondeterministic technological operator.

**Corollary 4.1.** The sequential composition of operationally nondeterministic technological operators is an operationally nondeterministic technological operator.

**Definition 4.9.** A technological operator \( A \) is called *conditionally nondeterministic* if the operator \( A \) can produce the same output in different technological conditions.

**Proposition 4.5.** The sequential composition of conditionally nondeterministic technological operators is a conditionally nondeterministic technological operator.

4 Conclusion

We have constructed a mathematical model of information technology based on the concept of a technological operator, dynamics of which represents technological processes. Then we have used this model to formalize important relations between information technologies such as inclusion, covering, and decomposition. Results obtained in the paper allow one to analyze compound technological operators using relations between their components.

Besides, there are many other important relations between technologies, such as comparative efficiency, interoperability, reliability, mutual security, dependability, etc. The mathematical theory of technology provides means for formalization, study and evaluation of these relations.

**References:**


