

# Finite Element Approach to Electric Field Distribution Resulting from Phase-sequence Orientation of a Double-Circuit High Voltage Transmission Line

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*Abstract:* - This paper proposes a mathematical model of electric fields caused by high voltage conductors of electric power transmission systems by using a set of second-order partial differential equations. This study has considered the effect of conductor phase-sequence orientation on electric fields emitted around a double-circuit, extra high-voltage, power transmission line. Six typical-used phase-sequence orientations in Thailand for 500-kV double-circuit transmission lines are investigated. Computer-based simulation utilizing the two dimensional finite element method in the time harmonic mode, instructed in MATLAB programming environment with graphical representation for electric field strength has been evaluated. The simulation results show that the phase-sequence orientation is one among key factors to influence the electric field distribution around the transmission line.

*Key-Words:* - Phase-sequence, Electric Fields, Finite Element Method (FEM), Transmission Line, Computer Simulation

## 1 Introduction

For decades, due to the increasing of electrical power demands in Thailand, Electricity Generating Authority of Thailand (EGAT) decides to enlarge transmission capacity by installing 500-kV extra high-voltage power transmission lines in both AC and DC. In the AC system, double-circuit transmission lines consist of six conductors running in parallel. Orientation of the six conductors results in electric field distribution that may cause some serious harm to electronic equipment or living things. From literature, basic electromagnetic theory [1] or image theory [2] are widely used for electric field calculation in high voltage power transmission lines. Even the study by EPRI [3], the basic electromagnetic theory was employed to analyze electric field strength resulting from orientation of conductor phase-sequences. So far, there is no report on electric field calculation in this scope by using Finite Element Method (FEM).

The FEM is one of the most popular numerical methods used for computer simulation. The key advantage of the FEM over other numerical methods in engineering applications is the ability to handle nonlinear, time-dependent and circular geometry problems. Therefore, this method is suitable for solving the problem involving electric field effects around the transmission line caused by circular cross-section of high voltage conductors.

In this paper, 500-kV, double-circuit, extra high-voltage power transmission lines are studied with six conductor phase-sequence orientation. Computer-based simulation utilizing the two dimensional finite element method in the time harmonic mode, instructed in MATLAB programming environment with graphical representation for electric field strength has been evaluated.

## 2 Modeling of Electric Fields involving Electric Power Transmission Lines

A mathematical model of electric fields ( $\mathbf{E}$ ) radiating around a transmission line is usually expressed in the wave equation (Helmholtz's equation) as Eq.(1) [4-5] derived from Faraday's law.

$$\nabla^2 \mathbf{E} - \sigma \mu \frac{\partial \mathbf{E}}{\partial t} - \varepsilon \mu \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \quad (1)$$

..., where  $\varepsilon$  is the dielectric permittivity of media,  $\mu$  and  $\sigma$  are the magnetic permeability and the conductivity of conductors, respectively.

This paper has considered the system governing by using the time harmonic mode and representing the electric field in complex form,  $\mathbf{E} = E e^{j\omega t}$  [6], therefore,

$$\frac{\partial \mathbf{E}}{\partial t} = j\omega \mathbf{E} \quad \text{and} \quad \frac{\partial^2 \mathbf{E}}{\partial t^2} = -\omega^2 \mathbf{E}$$

..., where  $\omega$  is the angular frequency.

From Eq.(1), by substituting the complex electric field, Eq.(1) can be transformed to an alternative form as follows.

$$\nabla^2 E - j\omega\sigma\mu E + \omega^2 \varepsilon\mu E = 0$$

When considering the problem of two dimensions in Cartesian coordinate  $(x,y)$ , hence

$$\frac{\partial}{\partial x} \left( \frac{1}{\mu} \frac{\partial E}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{1}{\mu} \frac{\partial E}{\partial y} \right) - (j\omega\sigma - \omega^2 \varepsilon) E = 0 \quad (2)$$

Analytically, there is no simple exact solution of the above equation. Therefore, in this paper the FEM is chosen to be a potential tool for finding approximate electric field solutions for the PDE described in Eq.(2) [7].

### 3 System Description with the FEM

#### 3.1 Discretization

This paper determines a four-bundled, double-circuit, 500-kV power transmission line. Fig. 1 shows the power transmission line with the low-reactance orientation type. Height of conductors shown in the figure is the maximum sag position. The lowest conductors are C and A' at the height of 13.0 m above the ground level [8]. Each phase conductor is 795 MCM (0.02772 m - diameter). The overhead ground wire has 3/8 inch - diameter. Fig. 2 displays the domain of study discretizing by using linear triangular elements.

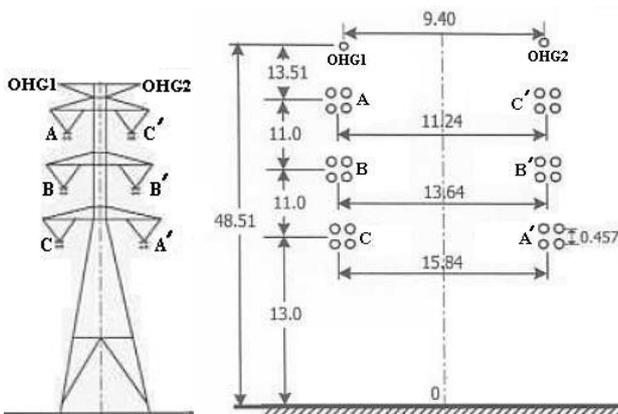


Fig.1 500-kV double-circuit, four-bundled power transmission line with low-reactance orientation

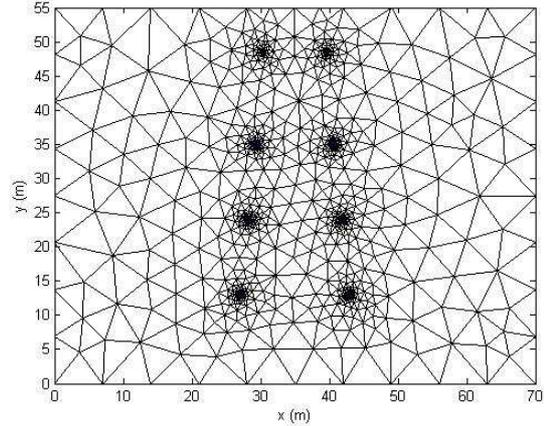


Fig.2 Discretization of the system given in Fig. 1

#### 3.2 Finite Element Formulation

An equation governing each element is derived from the Maxwell's equations directly by using Galerkin approach, which is the particular weighted residual method for which the weighting functions are the same as the shape functions [9-10]. According to the method, the electric field is expressed as follows.

$$E(x, y) = E_i N_i + E_j N_j + E_k N_k \quad (3)$$

..., where  $N_n$ ,  $n = i, j, k$  is the element shape function and the  $E_n$ ,  $n = i, j, k$  is the approximation of the electric field at each node  $(i, j, k)$  of the elements, which is

$$N_n = \frac{a_n + b_n x + c_n y}{2\Delta_e}$$

..., where  $\Delta_e$  is the area of the triangular element and,

$$\begin{aligned} a_i &= x_j y_k - x_k y_j, & b_i &= y_j - y_k, & c_i &= x_k - x_j \\ a_j &= x_k y_i - x_i y_k, & b_j &= y_k - y_i, & c_j &= x_i - x_k \\ a_k &= x_i y_j - x_j y_i, & b_k &= y_i - y_j, & c_k &= x_j - x_i. \end{aligned}$$

The method of the weighted residue with Galerkin approach is then applied to the differential equation, Eq.(2), where the integrations are performed over the element domain  $\Omega$ .

$$\int_{\Omega} N_n \left( \frac{\partial}{\partial x} \left( \frac{1}{\mu} \frac{\partial E}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{1}{\mu} \frac{\partial E}{\partial y} \right) \right) d\Omega - \int_{\Omega} N_n (j\omega\sigma - \omega^2 \varepsilon) E d\Omega = 0$$

, or in the compact matrix form

$$[M + K]\{E\} = 0 \tag{4}$$

$$M = (j\omega\sigma - \omega^2\varepsilon) \int_{\Omega} N_n N_m d\Omega$$

$$= \frac{(j\omega\sigma - \omega^2\varepsilon)\Delta_e}{12} \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$K = \nu \int_{\Omega} \left( \frac{\partial N_n}{\partial x} \frac{\partial N_m}{\partial x} + \frac{\partial N_n}{\partial y} \frac{\partial N_m}{\partial y} \right) d\Omega$$

$$= \frac{\nu}{4\Delta_e} \begin{bmatrix} b_i b_i + c_i c_i & b_i b_j + c_i c_j & b_i b_k + c_i c_k \\ & b_j b_j + c_j c_j & b_j b_k + c_j c_k \\ & & b_k b_k + c_k c_k \end{bmatrix}$$

Sym

..., where  $\nu$  is the material reluctivity ( $\nu = 1/\mu$ ).

For one element containing 3 nodes, the expression of the FEM approximation is a 3x3 matrix. With the account of all elements in the system of  $n$  nodes, the system equation is sizable as the  $n \times n$  matrix.

### 3.3 Boundary Conditions and Simulation Parameters

In this paper, 500-kV, double-circuit, extra high-voltage power transmission lines are studied with six conductor phase-sequence orientation [3] as shown in Table 1. The boundary conditions applied here are that to set zero electric fields at the ground level and the OHGW. In addition, the boundary condition of conductor surface in 500-kV power lines are assigned as given in [3, 8, 11]. This simulation uses the system frequency of 50 Hz. The power lines are bared conductors of Aluminum Conductor Steel Reinforced (ACSR), having the conductivity ( $\sigma$ ) =  $0.8 \times 10^7$  S/m, the relative permeability ( $\mu_r$ ) = 300, the relative permittivity ( $\varepsilon_r$ ) = 3.5. It notes that the free space permeability ( $\mu_0$ ) is  $4\pi \times 10^{-7}$  H/m, and the free space permittivity ( $\varepsilon_0$ ) is  $8.854 \times 10^{-12}$  F/m [12].

Table 1 Six types of phase sequences

type1	type2	type3	type4	type5	type6
A A'	A B'	A A'	A B'	A C'	A G
B B'	B A'	B C'	B C'	B B'	B G
C C'	C C'	C B'	C A'	C A'	C G

## 4 Results and Discussion

This paper employs MATLAB programming to simulate electric field distribution for six phase-sequence orientations. Electric field simulated for each type can be depicted in Fig. 3 - 8.

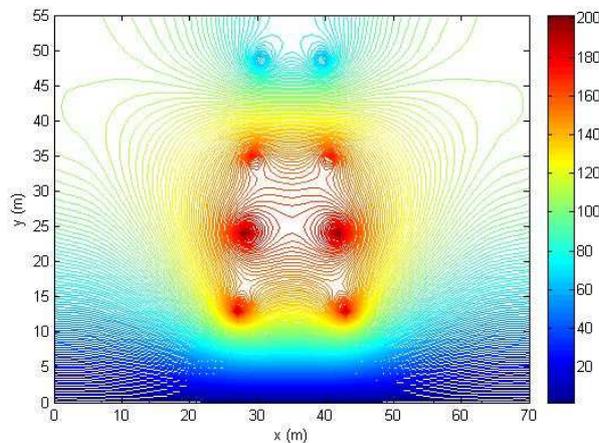


Fig.3 Electric field contour (kV/m) for type 1

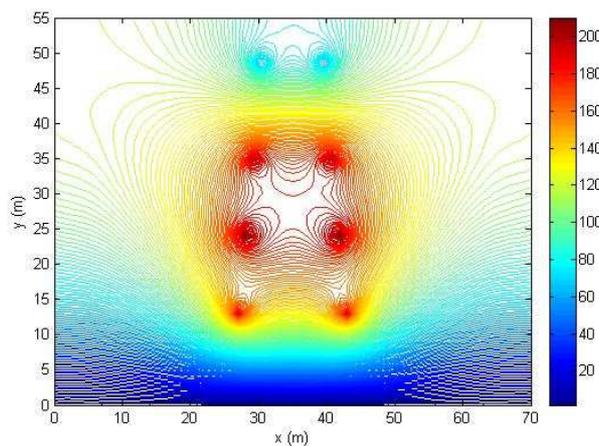


Fig.4 Electric field contour (kV/m) for type 2

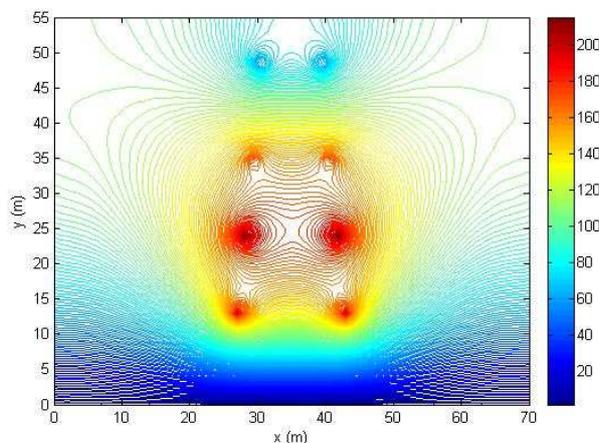


Fig.5 Electric field contour (kV/m) for type 3

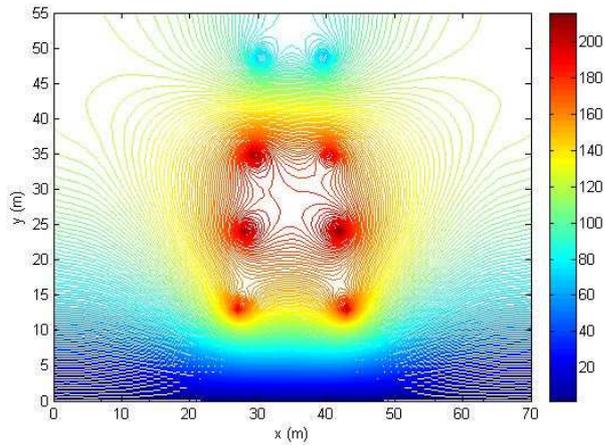


Fig.6 Electric field contour (kV/m) for type 4

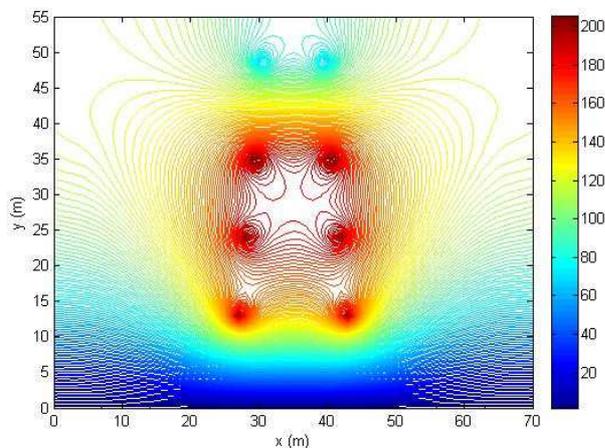


Fig.7 Electric field contour (kV/m) for type 5

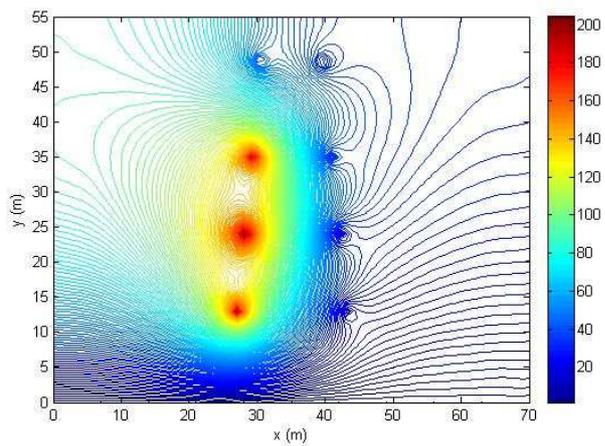


Fig.8 Electric field contour (kV/m) for type 6

From the simulated results, the orientation type has the key effect on electric field distribution around the power transmission line. By observing the electric field strength at a specified height level above the ground with 70-m horizontal span, type 1, 2, 3 and 5 are symmetric in electric field distribution along the vertical axis. Type 4 and 6 are asymmetric

due to unbalanced phase sequencing, especially type 6 with all the ground wires located on the right hand side.

When consider at some selected positions for more detail, symmetry in electric field distribution can be clearly explained. Fig. 9 and 10 show the electric field plot at the height of 0.1 m below the lowest conductor position. Similarly, Fig. 11 and 12 also describe the electric field plot at the height of 0.1 m above the highest conductor position. They confirm that type 1, 2, 3 and 5 are symmetric while type 4 and 6 are asymmetric as concluded by [3].

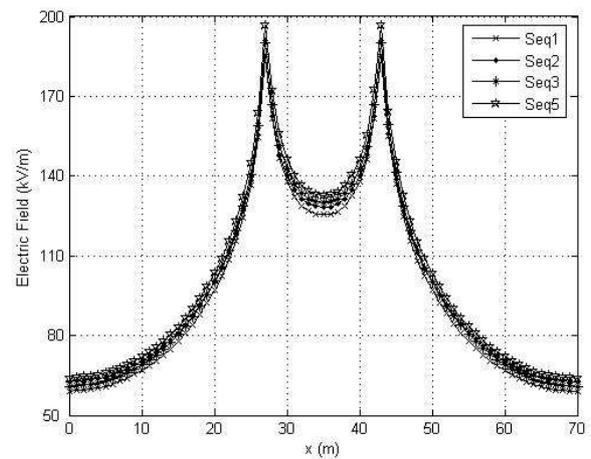


Fig.9 Electric fields of the symmetric orientation at the height of 0.1 m below the lowest conductor position

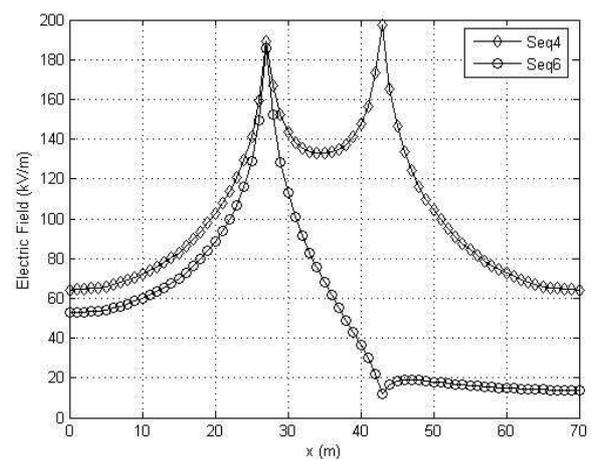


Fig.10 Electric fields of the asymmetric orientation at the height of 0.1 m below the lowest conductor position

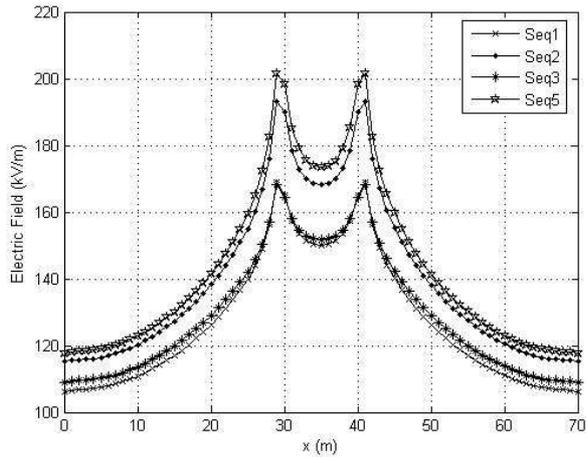


Fig.11 Electric fields of the symmetric orientation at the height of 0.1 m above the highest conductor position

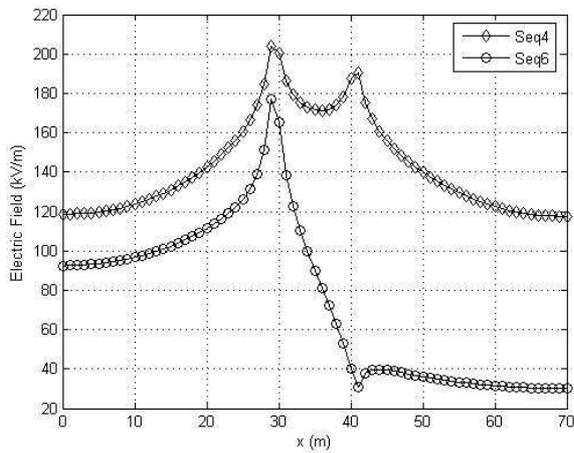


Fig.12 Electric fields of the asymmetric orientation at the height of 0.1 m above the highest conductor position

### 5 Conclusion

This paper has studied electric field distribution resulting from six typical conductor phase-sequence orientations. 500-kV, double-circuit, four-bundled power transmission lines of Electricity Generating Authority of Thailand (EGAT) are investigated. FEM developed by using MATLAB programming is employed. As a result, phase-sequence orientation is one of key factors to influence electric field distribution in electric power transmission lines. With the orientation of type 1, 2, 3 and 5, the electric field distribution is symmetric, while the left two orientations (type 4 and 6) give asymmetric field distribution.

### References:

- [1] Olsen, R.G., Deno, D., Baishiki, R.S., Abbot, J.R., Conti, R., Frazier, M., Jaffa, K., Niles, G.B., Stewart, J.R., Wong, R., and Zavadil, R.M., Magnetic Fields from Electric Power Lines Theory and Comparison to Measurements, *IEEE Transactions on Power Delivery*, Vol.3, No.4, 1988, pp. 2127-2136.
- [2] Li, L., and Yougang, G., Analysis of Magnetic Field Environment near High Voltage Transmission Lines, *Proceedings of the International Conferences on Communication Technology*, 1998, pp. S26-05-1 - S26-05-5.
- [3] Electric Power Research Institute, *Transmission-Line Reference Book 345 kV and Above*, Fred Weidner & Son Printers, Inc., USA, 1975.
- [4] Chari, M.V.K., and Salon, S.J., *Numerical Methods in Electromagnetism*, Academic Press, USA, 2000.
- [5] Weiner, M., *Electromagnetic Analysis Using Transmission Line Variables*, World Scientific Publishing, Singapore, 2001.
- [6] Christopoulos, C., *The Transmission-Line Modeling Method: TLM*, IEEE Press, USA, 1995.
- [7] Pao-la-or, P., Kulworawanichpong, T., Sujitjorn, S., and Peaiyoung, S., Distributions of Flux and Electromagnetic Force in Induction Motors: A Finite Element Approach, *WSEAS Transactions on Systems*, Vol.5, No.3, 2006, pp. 617-624.
- [8] Pin-anong, P., *The Electromagnetic Field Effects Analysis which Interfere to Environment near the Overhead Transmission Lines and Case Study of Effects Reduction*, [M.Eng. thesis], School of Electrical Engineering, Department of Electrical Engineering, King Mongkut's Institute of Technology Ladkrabang, Bangkok, Thailand, 2002.
- [9] Preston, T.W., Reece, A.B.J., and Sangha, P.S., Induction Motor Analysis by Time-Stepping Techniques, *IEEE Transactions on Magnetics*, Vol.24, No.1, 1988, pp. 471-474.
- [10] Kim, B.T., Kwon, B.I., and Park, S.C., Reduction of Electromagnetic Force Harmonics in Asynchronous Traction Motor by Adapting the Rotor Slot Number, *IEEE Transactions on Magnetics*, Vol.35, No.5, 1999, pp. 3742-3744.
- [11] Iyyuni, G.B., and Sebo, S.A., Study of Transmission Line Magnetic Fields, *Proceedings of the Twenty-Second Annual North American, IEEE Power Symposium*, 1990, pp. 222-231.
- [12] Hayt, Jr.W.H., and Buck, J.A., *Engineering Electromagnetics (7th edition)*, McGraw-Hill, Singapore, 2006.