Abstract: - Combination of logical and continuous controls is a typical feature of many power engineering plants. Logical and continuous control systems are usually designed separately and this design practice may lead to unpredictable interactions between logical and continuous control and deteriorate the overall control performance. On the other hand, it is sometimes claimed that the unified framework of hybrid systems and hybrid control enables us to design controllers that can handle both continuous and logical control tasks and to achieve a coordinated control that avoids undesirable continuous/logical interactions. This paper describes an attempt to test the validity of these claims in an experimental way. Laboratory scale plant that was designed for experiments with hybrid systems is used as an experimental testbed. Control performance achieved with separate design of logical and continuous control is compared with the performance achieved with an unified design of logical and continuous control based on hybrid model predictive control. The results of this comparison show that unified design can indeed achieve better control performance than standard separate design. On the other hand, the complexity of the hybrid control law is fairly high and its implementation using standard industrial control hardware would not be easy. In this way, the presented comparison indicates that hybrid model predictive control is a promising alternative to more traditional design approaches, however at its current state of development is not ripe for practical applications.

Keywords: - Hybrid systems, model predictive control, interaction of logical and continuous control

1. Introduction

Many power engineering plants as well as other complex engineering systems feature a combination of logical and continuous controls. Logical control is often responsible for safety related and limiting functions. Its tasks include preventing the values of the process variables from leaving the safe operation limits, avoiding emptying and overflow of tanks, starting and shutdown of process equipment etc. Logical controllers are also used to control discrete-valued control inputs such as on/off valves and switches, gears etc. On the other hand, the regulatory and supervisory control is usually performed by continuous controllers.

Common design practice normally relies on separate design of logic and continuous control. As many non-trivial interaction may arise between continuous and logic parts of the control system, this practice may result in a very suboptimal behaviour of the whole system. This fact is one of the motivations behind the recent developments in the field of hybrid systems and hybrid control.

The theory of hybrid systems allows to describe both continuous valued dynamics and logical switching as well as other discrete phenomena within the framework of one unified modelling formalism. Since hybrid model describes both continuous and logical (or more generally discrete-valued) parts of the whole systems, including continuous/logical interactions, the hybrid controller designed on the basis of this model can be expected to control the whole plant in a co-ordinated manner and avoid the deteriorating effects of interactions between separately designed logical and continuous control systems.

However, it is a well known fact that expectations though reasonably well founded in the theory and practical reality may often be two very different things. For this reason, this paper attempts to make an experimental comparison of a separate design of logic and continuous control on the one hand and co-ordinated design based on hybrid model on the other hand. This comparison is done using a case study of a relatively complex laboratory scale plant that was designed for experiments with the control of hybrid systems.

Although the number of control techniques developed for continuous systems seems to be almost
infinite, only a very few of them can be extended to the control of hybrid systems. Among them the model predictive control is probably the most developed and promising one and model predictive control will also be the approach used in this study.

2. Experimental Testbed

The experiments described in this paper were performed using a laboratory scale plant that was designed and built in order to represent typical hybrid phenomena often occurring in the field of process control. A detailed description of this plant has been given in a recent paper presented at IFAC world Congress [4]. Since the full text of [4] is easily available from the IFAC website, the description of this plant in this paper can be short and limited to the most important features.

Plant structure is sketched in Fig. 1 and its real appearance is shown in Fig. 2. The basic components of the plant are three water tanks. Tanks 2 and 3 have special shapes that introduce changes in dynamics. The tanks are thermally insulated to make the heat losses negligible (as thermal insulation hides tank shapes, all tanks look the same in the plant photo). Water from the reservoir mounted under the plant is drawn by Pump 1 and Pump 3 (delivery flow rate 0-4.5 l/min) to the respective tanks. The delivery rates can be continuously changed by changing the armature voltage of the DC motors driving the pumps. The flow rates are measured using turbine flow-meters. To compensate for pump non-linearity, it is beneficial to use slave flow rate controllers, which are denoted with FC in Fig. 1. The higher level controllers can then use the flow rates and not the armature voltages as their manipulated variables.

Fig. 1. Functional and block scheme (model) of the laboratory scale plant. The meaning of the symbols is as follows: FT, LT, TT are flow, level and temperature transmitter respectively, FC – flow controller, S – solenoid valve, M – motor, \( r_1 = 5.64 \text{ cm}, r_2 = 5.8 \text{ cm}, r_3 = 6 \text{ cm}, r_4 = 2.9 \text{ cm} \), tank height \( l_{\text{max}} = 80 \text{ cm}, l_1 = l_2 = 40 \text{ cm} \)

Fig. 2. Photograph of the laboratory scale plant
The flow from Pump 3 is fed directly to Tank 3. The flow from Pump 1 goes through a storage water heater (2 kW) and it is further controlled by a solenoid valve S1. The power consumption of the heater can be changed continuously. Another continuously controlled heater is mounted on the bottom of Tank 2 (800 W). The temperatures are measured with Pt1000 sensors at the points shown in Fig. 1. In addition to the pumps, the delivery flow rates of which can be changed continuously, the plant includes another way of manipulating the flow: solenoid valves. These discrete valued actuators control the flow from Tank 1 to Tank 2 (valves S3, S4, $k_c=5$ l/min each). This flow is changed in three steps: no valve open, one open, both valves open. The valves close and open instantaneously. Tank 1 can be by-passed by closing S1 and opening S2. The air-water heat exchanger with cooling fan at the output from Tank 2 keeps the water temperature in the reservoir roughly constant during the experiments. The water levels are measured using pressure sensors. These can be configured either as continuous sensors or as level switches (indicating three different water levels).

3. Control objective and mathematical model of the testbed

The plant described in the previous section can be utilised in many ways and many control scenarios can be defined. An example of a control setting that is fairly different from the scenario used in this paper can be found in [5] where this plant was used by the authors for experiments with switching controllers. The scenario considered in this paper uses two tanks (Tank 1 and 2). This scenario (inspired in part by [5]) can be formulated as follows.

Tank 1 serves as a buffer that receives water from an upstream process. Water flow rate and temperature are disturbances. The main control objective is to deliver the water to a downstream process at a desired temperature, while the flow demand of the downstream process is variable and hence it also acts as a disturbance. Power output of heater $H_2$ is a continuous manipulated variable. Valves S3 and S4 are used as a discrete valued manipulated variable to control the flow from Tank 1 to Tank 2 in three steps. Valve S1 can be used to close water flow to Tank 1, if tank overflow is to be avoided. There is no valve at the output of Tank 2, however an effect equivalent to closing a valve at the output of Tank 2 in order to prevent emptying Tank 2 can be achieved by switching off Pump 2.

The main control objective features continuous valued controlled variable (temperature $\vartheta_2$) and continuous valued manipulated variable (power output of heater $H_2$). However it also necessarily includes several auxiliary objectives. Tank levels must be kept within specified limits, and overflow as well as emptying of the tanks must be avoided. It is also necessary to avoid the necessity to close valve S1 in order to prevent Tank 1 overflow. In a real control situation, closing S1 would mean that water from the upstream process cannot flow to the buffer but must be re-routed to the environment. Similarly it is necessary to avoid the necessity to switch off Pump 2 in order to prevent Tank 2 from underflow. The standard way to satisfy these auxiliary objectives would be to use separately designed control logic to close or open valves at predefined water levels.

Mathematical model of the laboratory scale plant described in the previous section can be derived using mass and energy balances. The reader is referred to [4] for a detailed derivation of the model. For the purpose of this paper it will be sufficient to give just the relevant part of the plant model (excluding Tank 3). Assuming incompressibility of the liquid in the tanks, constant liquid heat capacity $c$, negligible heat losses and ideal mixing, the plant model can be written as

\begin{align}
\dot{\vartheta}_1(t) &= \frac{1}{A_1} \left[ q_{1i}(t) \sigma_1(t) - 0.1k_c \sigma_1(t) \sqrt{g \vartheta_1(t)} \right] - \frac{1}{A_1} q_{1o}(t) \sigma_1(t) \vartheta_1(t) \frac{\partial \vartheta_1(t)}{\partial A_1} \\
\dot{h}_1(t) &= \frac{1}{A_1} \left( \frac{1}{A_2} \right) \left( 0.1k_c \sigma_1(t) \sqrt{g \vartheta_1(t)} - q_{20}(t) \sigma_1(t) \right) h_2(t) \leq l_1 \\

\dot{\vartheta}_2(t) &= \frac{1}{\rho} \frac{H(t)}{A} \left( \frac{1}{A_2} \right) \left[ 0.1k_c \sigma_1(t) \sqrt{g \vartheta_1(t)} \left( \vartheta_1(t) - \vartheta_2(t) \right) + \frac{H(t)}{\rho} \right] \frac{\partial \vartheta_2(t)}{\partial h_2(t)} \\
\dot{h}_2(t) &= \frac{1}{\rho} \frac{H(t)}{A} \left( \frac{1}{A_2} \right) \left[ 0.1k_c \sigma_1(t) \sqrt{g \vartheta_1(t)} \left( \vartheta_1(t) - \vartheta_2(t) \right) + \frac{H(t)}{\rho} \right] \frac{\partial h_2(t)}{\partial h_2(t)} \\

h_2(t) &\leq l_1 \\

h_2(t) &> l_1
\end{align}

where $A = \pi r^2$, discrete valued input $\sigma_0$ assumes values 0,1 (S1 closed, S1 open), $\sigma_1$ assumes values 0,1,2 (no valve open, S3 open, S3 and S4 open), $\sigma_1$ assumes values 0,1 (Pump 2 switched off , Pump 2 running with flow rate $q_{20}$ depending on the flow demand of the downstream process), $H$ is power output of heater $H_2$, $k_c$ is flow coefficient of valves S3 and S4 ($k_c=5$ l/min).
4. Control design and experiments

Standard procedure to design a control system satisfying the specified objectives is divided into two separate tasks: logical and continuous control. Logic part of the control system is described by a set of simple rules. Normal and desirable state of the logical control inputs can be characterized by $\sigma_l=1$; $\sigma_0=0$. Thus, water from the upstream process flows to the buffer (Tank 1) and further to the supply (Tank 2) and the supply is able to meet the demanded downstream process while water levels of both tanks remain within acceptable ranges $h_{1min} \leq h_1 \leq h_{1max}$, $h_{2min} \leq h_2 \leq h_{2max}$. If water levels deviate from these ranges, the following rules apply:

A. If $h_1 < h_{1min}$, set $\sigma_1$ to zero to avoid Tank 1 emptying

B. If $h_1 > h_{1max}$, set $\sigma_0$ to zero to avoid Tank 1 overflow, if also $h_2 < h_{2min}$ set $\sigma_1$ to 2 to accelerate the recovery of both water levels to normal ranges

C. If $h_2 < h_{2min}$ set $\sigma_1$ to zero to avoid Tank 2 emptying

D. If $h_2 > h_{2max}$, set $\sigma_1$ to zero to avoid Tank 2 overflow

Switching of logical manipulated variables among discrete values is done with a small hysteresis to avoid too frequent switching.

Continuous control system is then designed as a simple SISO control loop, where heater power output is used as a manipulated variable and temperature $\theta_i$ is a controlled variable. The design can be done by a variety of approaches including model predictive control. It can be seen from (4) that logical control inputs act as disturbances and they may have adverse effects on control performance.

The situation changes when logical and continuous control are designed in a unified way. System described by (1)-(4) is then treated as a hybrid system marked by a mixture of continuous and logical inputs and dynamics switching depending on the current water level in Tank 2. There are currently not many design approaches available that could be used for this class of systems. One of the most promising approaches is model predictive control (MPC). In this method, the computation of manipulated variables is based on a numerical solution of an optimization problem that is solved online in discrete sampling instants. At each sampling instant the values of manipulated variables are computed according to the chosen performance objective and applied to the controlled plant. Thus, the extension of MPC to the control of hybrid systems can be relatively straightforward in principle. As the model of the control plant now contains discrete valued variables (in general not only inputs but also state and output variables can be discrete-valued) the optimization problem to be solved is no more standard constrained quadratic programming problem but becomes a mixed integer programming problem.

In principle, it is even possible to extend MPC to general non-linear hybrid systems, however the resulting numerical optimization problem becomes complicated enough to be practically intractable. For this reason, non-linear hybrid models must be approximated with piecewise affine (PWA) systems. The general form of a discrete-time PWA system can be expressed by the following equations

$$x(k+1) = M_i x(k) + N_i u(k) + f_i$$
$$y(k) = C_i x(k) + D_i u(k) + g_i$$

where each dynamics $i=1,2,..N_D$ is active in a polyhedral partition $D$ that is defined by guard lines. These guard lines are described by

$$G_i^c x(k) + G_i^a u(k) \leq G_i^b$$

That means, the dynamics $i$ represented by matrices and vectors $[M_i, N_i, f_i, C_i, D_i, g_i]$ is active in the part of state-input space which satisfies constraints (5.8).

State, input and output variables can be both continuous valued and discrete valued. It is possible to consider also continuous time PWA systems as it is done e.g. in [6], however continuous time formulation is not a suitable basis for MPC design.

Model predictive control of PWA systems is an evolving field, however it has already reached a certain level of maturity. The main references are [2] and more recently [3]. The application of model predictive control for PWA systems is now much easier by the existence of specialized Matlab Toolboxes such as Hybrid Toolbox [1] and Multi-Parametric (MPT) Toolbox [7]. In this paper, MPT Toolbox will be used.

Model described by Eqs. (1)-(4) is non-linear and as a first step of control design it must be approximated with a PWA system. It can be observed that even if the laboratory scale plant considered in this paper is by no means extremely complex it can be accurately modeled using basic and simple physical laws, its approximation with a PWA model will not be particularly easy and it will be marked by a relatively high number of partial models $N_D$.

An obvious source of partial models are logical control inputs $\sigma_0, \sigma_1, \sigma_2$. Different values of these inputs result in considerable changes of dynamic behavior of the system and moreover they are in product with other input and/or state variables in (1)-(4). Linearization with respect to these logical inputs would be difficult and it is better to associate one partial model with each combination of values of these inputs. This results in 12 partial models.
Equations (3) and (4) include dynamics switching at water level $l_i$. That means the number of 12 must be doubled and we obtain 24 partial models as an absolute minimum for modeling this plant.

However, these 24 partial models are not in PWA form because they include nonlinear functions: square root, reciprocal value and product of the state variables. These non-linear functions must be approximated by suitable piecewise affine approximations. The number of approximating functions depend on desired accuracy.

Square root can be approximated by a piecewise affine approximation that is designed to give the same values at $h=0$ and $h=l_{max}$ and to be least squares optimal between these two points.

$$\sqrt{h} \approx \begin{cases} kh & 0 \leq h \leq h_a \\ kh + c_2 & h_a \leq h \leq l_{max} \end{cases}$$ (7)

Considering the maximum value $l_{max}=0.8$ m, the numerical values of these coefficients are as follows

$$k_1 = 3.5278; k_2 = 0.7109$$

$$c_2 = 0.3257; h_a = 0.1156m$$ (8)

Other approximations can also be considered. It is possible to use closer approximation with more than two partial models. It is possible to take advantage of the fact that normally the water level will be between $h_{min}$ and $h_{max}$ and make the approximation equal to the original function at $h_{min}$ and $h_{max}$ thus improving approximation accuracy in the working range.

However, if approximation (7), (8) is used, equation (1) can be written in the form of a PWA system in the following way

$$\dot{h}_i(t) = 0 \quad 0 \leq h_i \leq l_{max}$$

$$\dot{h}_i(t) = -0.1k_1 \sqrt{h} \dot{h}_i(t) \quad 0 \leq h_i \leq h_a$$ (9)

$$\dot{h}_i(t) = -0.2k_2 \sqrt{h} \dot{h}_i(t) \quad 0 \leq h_i \leq h_a$$ (10)

$$\dot{h}_i(t) = -0.1 \sqrt{h} \dot{h}_i(t) - \sqrt{c_2} \quad 0 \leq h_i \leq l_{max}$$ (11)

The situation where $h_i=1$ and is described by the same set with the term $q_0(t)/A_i$ added to each of equations (9)-(13).

Equation (3) can be converted to the PWA form in a similar way, just the number of partial models is higher because this equation also includes dynamics switching depending on whether $h_{2} \leq l_{2}$ or not. Equations (2) and (4) pose a more difficult problem due to the presence of $h(t)$ in the denominator. The approximation of this function with a piecewise linear function is more complicated than was the case with square root. A relatively simple approximation can be achieved by using the fact that one of the control objectives is to keep the water levels within specified ranges $h_{min} \leq h \leq h_{max}$, $h_{2min} \leq h \leq h_{2max}$ and to replace the term $1/h(t)$ with its mean value in the interval ($h_{min}$, $h_{max}$) in the case of Eq. (2) and in the intervals ($h_{2min}$, $l_i$) and ($l_i$, $h_{2max}$) in the case of Eq. (4).

Using these approximations a set of partial piecewise affine models is obtained. These models must be finally converted to discrete time. This conversion is slightly complicated by the fact that Matlab has no standard routine for discretization of systems of the form

$$x(t) = A_i x(t) + B_i u(t) + o_i$$ (14)

However it can easily be derived that assuming standard discretization with zero order hold at the inputs, the formulae for computation of $N_i$ and $f_i$ in (5) have the same form

$$N_i = e^{A_T \tau} \int_0^T e^{-A_T \tau} B_i d\tau; f_i = e^{A_T \tau} \int_0^T e^{-A_T \tau} o_i d\tau$$ (15)

Thus the computation of discretized model is possible by using c2d command first with arguments $(A_i, B_i, C_i, D_i)$ to obtain $N_i$ and then with arguments $(A_i, o_i, C_i, 0)$ to obtain $f_i$.

To compare control performance of the systems using separate design and hybrid model predictive control, the following control experiment was performed. Starting from the state $q_0 = q_{20} = 1 l/min, h_1 = 0.5 m, h_2 = 0.3 m, \theta_1 = 50^\circ C, \theta_2 = 40^\circ C$, the setpoint was increased from $40^\circ C$ to $60^\circ C$.

Separate design used logic rules defined in the beginning of this section. The continuous control algorithm was MPC with linear performance function and control horizon 2. The unified design used MPC with the same performance function and control horizon, however this MPC algorithm could make use of the logical control inputs. The responses are shown in the following figures.
It can be seen from these figures that unified design is able to achieve better results. The control time is much shorter. Fig. 3 shows that this improvement is due to the ability of the unified design to make use of logical manipulated variables. Unlike separate design, these variables can be used not only to keep the water levels within specified limits but also to accelerate control responses. In the beginning $\sigma_1$ is used to accelerate the setpoint response by increasing the inflow of warmer water to Tank 2. When controlled variable reaches setpoint, $\sigma_1$ is used just to keep the water levels within specified range.

6. Conclusion
The purpose of this paper was to explore the potential of hybrid MPC for unified design of logical and continuous control. The paper had mainly experimental character and consequently its results are connected with the particular plant considered and they cannot be regarded as completely general. In spite of that, they lead to some interesting observations. MPC based on hybrid model has full information how the controlled variable is affected by logical manipulated variables and it can make use of this information to improve control responses. It could clearly be observed that the setpoint response was improved by adding the effect of opening valve S4 to the effect of increasing heater power output.

On the other hand, a necessary precondition for the application of hybrid MPC is approximation of the plant model with a PWA system. The complexity of the PWA model expressed by the number of partial systems is fairly high even in the case of this relatively simple laboratory scale plant. Some part of this complexity is due to the special shape of Tank 2. However, it is evident that even if there were no change of the diameter of Tank 2, the complexity of the PWA model and consequently also the complexity of the MPC control law would remain very high. In this way, the presented comparison indicates that hybrid model predictive control is a promising alternative to more traditional design approaches for the future. However further developments will be necessary to make it suitable for routine practical applications.

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