On – line Parametric Identification and Discrete Optimal Command for the Aircrafts’ Longitudinal Movement

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Abstract: - This paper presents a new ON-LINE parametric identification and discrete optimal command algorithm for mono or multivariable linear systems. The method may be applied with good results to the automatic command of the flying objects’ movement. The simulation results obtained with this real time algorithm, with parametric identification for the longitudinal movement of an aircraft is also presented. This algorithm may be used, with good results, for identification and optimal command of an air-air rocket’s movement in vertical plain regarding to target’s line [1].

Key-Words: algorithm, optimal, control law, observer.

1 Introduction

Because of fast flying parameters’ modify for the modern aircrafts and rockets, performing real time identification and optimal or adaptive command algorithms have to be made. The authors of this paper have made such an algorithm. First of all, an OFF-LINE parametric identification is made, without command, for obtaining the initial values of these parameters for the ON-LINE identification process.

Using the control system (A) and model’s outputs, a discrete optimal command law is projected, using a quality quadratic criterion, which assures the convergence of the difference between control system and model’s outputs. The model’s parameters, obtained by the ON-LINE identification, are used for calculus of the command law.

For the algorithm validation one uses as example the automatic command of an aircraft longitudinal movement; time characteristics, representing evolution of state variables of A and their estimate, are plotted. These variables’ stabilization and the convergence of the errors \( e_i = x_i - \hat{x}_i \) happen in maximum 2 seconds.

The proposed algorithm produces very good results in the case of lateral movement’s stabilization for transport and fights aircrafts or in the case of parametric identification of an air-air rocket’s move-
ment in vertical plain regarding to target’s line.

2 Continuous and discrete model for the longitudinal movement of the aircraft

The control system (the movement of A) may be described by the input – output equations with general forms

\[ \dot{x} = Ax + Bu, \quad y = cx, \]  

where \( x \) is the state vector \((n \times 1)\), \( u \) – the command vector \((m \times 1)\), \( A \) – the system matrix \((n \times n)\), \( B \) – matrix \((m \times n)\), \( y \) – output vector \((p \times 1)\), \( C \) – measurement system matrix \((p \times n)\), \( p \leq n \). The estimated model is described by equations

\[ \dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u, \]

\[ \hat{y} = \hat{C}\hat{x}, \]  

where \( \hat{x} \) is the estimation of the state vector, \( \hat{y} \) – estimation of the output vector \( y \), \( \hat{A}, \hat{B} \) and \( \hat{C} \) – estimate matrices.

The discrete variants of equations systems (1), (2), and (3), (4) are, respectively

\[ x(k+1) = A_dx(k) + B_du(k), \]

\[ y(k) = C_dx(k); \]

\[ \hat{x}(k+1) = \hat{A}_d\hat{x}(k) + \hat{B}_du(k), \]

\[ \hat{y}(k) = \hat{C}_d\hat{x}(k); \]  

matrices \( A_d, B_d, C_d \) and \( \hat{A}_d, \hat{B}_d, \hat{C}_d \) are discrete variants of matrices \( A, B, C \) and \( \hat{A}, \hat{B}, \hat{C} \).

Another description form for the estimated system \( A \) (A dynamics estimation) [2] is

\[ \hat{y}(k+1) = \hat{x}^T(k+1)\hat{b}(k) + \hat{e}(k+1) \]  

or

\[ \hat{y}(k+1) = \hat{b}^T(k)\hat{z}(k+1) + \hat{e}(k+1), \]  

where \( e(k+1) = y(k+1) - \hat{y}(k+1) \),

\[ \hat{b}^T(k) = [\hat{\alpha}^T(k) \quad \hat{\beta}_1(k) \quad \hat{\beta}_2(k) \quad \hat{\beta}_3(k)], \]

with

\[ \hat{\alpha}(k+1) = \hat{A}(k+1)\hat{\alpha}(k) - \hat{\beta}(k), \]

\[ \hat{\beta}(k) = \begin{bmatrix} \hat{\beta}_1(k) \\ \hat{\beta}_2(k) \\ \vdots \\ \hat{\beta}_m(k) \end{bmatrix}, \]

\[ \hat{\alpha}(np \times p), \hat{\beta}(p \times m), \hat{\beta}(m-1) \times p \times m; \]

\[ z^T(k+1) = \begin{bmatrix} y(k) & u(k) & U^T(k) \end{bmatrix}; \]

with

\[ \hat{y}^T(k) = [\hat{y}(k) \quad \hat{y}(k-1) \quad \vdots \quad \hat{y}(k-n+1)]; \]

\[ \hat{U}^T(k) = [u(k-1) \quad u(k-2) \quad \vdots \quad u(k-m+1)]; \]

\[ \hat{y}(np \times 1), U(lm-1) \times m \times 1. \]

If \( m = p \), then equation (10) becomes

\[ \hat{y}(k+1) = \hat{\alpha}^T(k)\hat{y}(k) + \hat{\beta}(k)u(k) + \hat{\beta}^T(k)\hat{U}(k); \]  

if \( m \neq p \), then \( \hat{\beta}^T(k) \) matrix can not be multiplied with \( \hat{U}(k) \) vector because of their dimensions. That’s why, in equation (15) the last term is expressed for each concrete case (function of \( m \) and \( p \) values).

So that, in the case presented below (longitudinal movement) \( n = 4, m = 1 \) and equation (15) becomes

\[ \hat{y}(k+1) = \hat{\alpha}^T(k)\hat{y}(k) + \hat{\beta}(k)u(k), \]  

where

\[ \hat{y}(k) = [\hat{y}(k) \quad \hat{y}(k-1) \quad \hat{y}(k-2) \quad \hat{y}(k-3)]; \]

\[ \hat{\alpha}(k) = [-\hat{\alpha}_1(k) \quad -\hat{\alpha}_2(k) \quad -\hat{\alpha}_3(k) \quad -\hat{\alpha}_4(k)], \]

\[ \hat{\beta}_1(k) \] is a \((p \times 1)\) vector, \( \hat{y} \) is a \((p \times 1)\) vector and \( u(k) = (1 \times 1). \)
3 Identification of the longitudinal movement’s parameters

Below one presents the algorithm for the identification of the longitudinal movement’s parameters. This algorithm’s name is ALGLDR.

Algorithm ALGLDR

Step 1: First of all the off – line system A’s parameters identification is made, using, for example, the least square method (LSM), resulting the parameters vector \( \hat{b}_0 = \hat{b}(0) \); that refers to the coefficients \( \hat{b}_j, j = 1, r, \hat{a}_i = 1, n \) of the discrete transfer functions of the aircraft’s estimated model \( \hat{A} \) (in fig.1 switch I has position 1, \( e \) – disturbances and \( u = u_e \) – the random input); \( \hat{y}(t) \) is then computed and the vectors \( \hat{Y}_0 = \hat{Y}(0) \) and \( U_0 = U(0) \) are memorized. Also, the covariance matrix \( P_0 \) (obtained at the end of identification \( P_0 = P(0) \)) is also memorized. Then, matrices \( A_j, B_j, \hat{A}_j, \hat{B}_j \) are computed and with these state vectors \( x \) and \( \hat{x} \) are computed using equation (5); these vectors (at the end of identification) are memorized;

Step 2: For simulation of time varying of A’s parameters, the parameters of A are modified (for example with 5%) and with the new coefficients \( A_j \) and \( B_j \) matrices are computed;

Step 3: Switch I has now position 2 (on – line control); using algorithm ALGLX [6], matrices \( Q' \) and \( R \) matrices are obtained in rapport with \( \hat{A}_j \) and \( \hat{B}_j \) matrices and after that the matrix \( Q = \begin{bmatrix} C & \hat{Q} & \hat{Q} \end{bmatrix} \) is calculated;

Step 4: \( G \) matrix from (20) is obtained with \( \hat{h} \) extracted from \( \hat{b}(k) \);

Step 5: Command \( u(k) \) is computed with equation (19) using \( \hat{Y}^T(k) \) and \( \hat{\alpha}^T(k) \) with elements (17), \( \hat{b}_0(k) \) extracted from \( \hat{b}(k) \) and component \( u(k-1) \) of \( U(k) \);

Step 6: The vectors \( x(k+1), \hat{x}(k+1) \) are obtained using (5), respectively (7); \( y(k+1) \) and \( \hat{y}(k+1) \) are calculated as bellow
\[
y(k+1) = C(x(k+1), \hat{x}(k+1)) = \hat{C}(x(k+1)). (21)
\]
Vectors \( \hat{y}(k+1) \) and \( U(k+1) \) are memorized and the error
\[
\hat{e}(k+1) = y(k+1) - \hat{y}(k+1)
\]
is computed;

Step 7: The actualization of covariance matrix is made with formula [2]
\[
P(k+1) = P(k) - \frac{P(k)z^T(k+1)p(k)z(k+1)}{\hat{z}^T(k+1)p(k)z(k+1)} P(k)
\]
and with this,
\[
\hat{b}(k+1) = \hat{b}(k) + P(k+1)z(k+1)\hat{e}(k+1),
\]
where \( \hat{z}(k+1) \) has the form (13);

Step 8: \( k \rightarrow k+1 \) and one returns to step 4; if \( k < k_{\text{imposed}} \), the program stops; state variables \( x(t) \) and \( \hat{x}(t) \) are plotted.

4 Identification and optimal command of the aircraft longitudinal movement

For the identification and optimal command algorithm’s validation, presented above, a simulation program was made in the MATLAB medium (the program is presented in Appendix).

The longitudinal movement of the aircraft is described by equation [7]
\[
\begin{bmatrix}
\dot{\Delta \alpha} \\
\dot{\Delta \beta} \\
\dot{\Delta \delta_y}
\end{bmatrix} =
\begin{bmatrix}
-0.007 & 0.012 & -9.81 \\
-0.128 & -0.54 & 0 \\
0.065 & 0.96 & -0.99
\end{bmatrix}
\begin{bmatrix}
\Delta \alpha \\
\Delta \beta \\
\Delta \delta_y
\end{bmatrix} +
\begin{bmatrix}
0 \\
1 \\
-0.94
\end{bmatrix} V_x +
\begin{bmatrix}
\delta_p
\end{bmatrix}
\]
where \( V_x \) is the longitudinal component of flight’s speed, \( \alpha \) – the attack angle, \( \theta \) – the pitch angle, \( \omega_\alpha \) – the pitch angular velocity and \( \delta_p \) – the elevator deflection.

First time off-line identification of the longitudinal movement has been made. One obtained the vector associated to estimated model \( \hat{A} \)
\[
b_0' = b'(0) = [0.013 \ -0.604 \ 4.047 \ -1.017 \ 0.058 \ -0.096 \ 0.106 \ -0.970]
\]
and matrices
\[
\begin{bmatrix}
4.012 & 1 & 0 & 0 \\
0 & 4.046 & 0 & 0 \end{bmatrix}, \quad \hat{B}_x = 
\begin{bmatrix}
0 & -0.096 \\
0.106 & 0 \\
-0.070 & 0
\end{bmatrix}, \quad \hat{C}_x = 
\begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}, \quad \hat{D}_x = 0,
\]
matrices
\[
\begin{bmatrix}
3.984 & 0 & 0 \\
-5.954 & 0 & 0 \end{bmatrix}, \quad B_y = 10^{-4}
\begin{bmatrix}
0.203 \\
0.610 \\
-0.605 \\
-0.201
\end{bmatrix}, \quad A_y = 
\begin{bmatrix}
3.954 & 0 & 0 \\
-9.984 & 0 & 0
\end{bmatrix}, \quad C_y = \hat{C}_y, \quad \hat{D}_y = \hat{D}_y.
\]
After switching of I, it begins the on – line identification regime; considering that A’s parameters change with 5%, it results the vector
\[
b_0'' = [3.78 \ -5.65 \ 3.75 \ -0.93 \ -1.9 \times 10^{-5} \ 5.8 \times 10^{-5} \ -5.7 \times 10^{-5} \ -1.9 \times 10^{-5}].
\]
Using algorithm ALGLX, one calculates matrices \( Q \) and \( R \) [6]: \( Q = 1.548, R = 1 \) and, with these, one obtains \( G \) and \( u \) for each calculus step, vectors...
\( x, \dot{x}, y, \dot{y}, P, \dot{P} \) and one plots time characteristics
\( x_t = \Delta V, \dot{x}_t = \Delta P, x = \Delta \alpha, \dot{x} = \Delta \dot{\alpha}, x_0 = \Delta \theta, \dot{x}_0 = \Delta \dot{\theta}, \) \( x_1 = \omega, \dot{x}_1 = \dot{\omega} \), and \( u = \delta \) (presented in fig.2; the curves are \( x_1 \) – blue, \( \dot{x}_1 \) – red).

**Fig. 2** Dynamics of state variables and of their estimations for the longitudinal movement

### 5 Appendix

clear all; close all;

% Data presentation (step 0)
A=[-0.0558 -0.9968 0.0802 0.04415; 0.598 -0.115 -0.0318 0; 0.305 0.388 -0.465 0; 0 0.0805 1 0];
B=[0.0073 0;-0.475 0.123; 0.153 1.063;0 0];
C=[0 0 1 0];
Ts=0.01;
x=[0.08;0.02;0.03;0.3]*180/pi;
xc=zeros(4,1);

% Off-line identification

%Step 1: Off-line identification (Switch on position 1)
[num1,den1]=ss2tf(A,B,C,D,1);
[num2,den2]=ss2tf(A,B,C,D,2);
sysc1=tf(sysc1);sysc2=tf(sysc2);
[numd1,den1]=tfdata(sysc1,'v');
[numd2,den2]=tfdata(sysc2,'v');
AA1=[den1(1) den1(2) den1(3) den1(4) den1(5)];
BB1=[num1(1) num1(2) num1(3) num1(4) num1(5)];
AA2=[den2(1) den2(2) den2(3) den2(4) den2(5)];
BB2=[num2(1) num2(2) num2(3) num2(4) num2(5)];

% On-line identification

% Step 2: Time variant of the system parameters
numd1=numd1*95/100;
numd2=numd2*95/100;
for i=2:length(dend1)
den1(i)=(den1(i))*(95/100);
den2(i)=(den2(i))*(95/100);
end

% Calculus of matrices Ad, Bd
AA1=[den1(1) den1(2) den1(3) den1(4) den1(5)];
AA2=[den2(1) den2(2) den2(3) den2(4) den2(5)];
BB1=[numd1(1) numd1(2) numd1(3) numd1(4) numd1(5)];
BB2=[numd2(1) numd2(2) numd2(3) numd2(4) numd2(5)];

tho1=poly2th(AA1,BB1);
[Ad1,Bd1,Cd1,Dd1]=th2ss(tho1);

tho2=poly2th(AA2,BB2);
[Ad2,Bd2,Cd2,Dd2]=th2ss(tho2);

Ad=Ad1;Bd=[Bd1 Bd2];
Cd=Cd1;Dd=[Dd1 Dd2];

% Step 3: Usage of algorithm ALGLX
close all;
Q=[100 0 0 0;0 1 0 0;0 0 100 0;0 0 0 10];
R=[1 -0.5; -0.5 1];
[K,P,E] = LQR(Adc,Bdc,Q,R);

% Matrix T presentation
N3=randn(4,2);
for i=1:4
    for j=1:2
        T(i,j)=Bdc(i,j);
        T(i,j+2)=N3(i,j);
    end
end

% Obtaining of matrices Ab,Bb,Kb,Rb
Ab=(inv(T))*Adc*T;
Bb=(inv(T))*Bdc;
Kb=place(Ad,Bb,E);
for i=1:2
    for j=1:2
        K10(i,j)=Kb(i,j);
        K20(i,j)=Kb(i,j+2);
    end
end

k11=K10(1,1);k12=K10(1,2);
k21=K10(2,1);k22=K10(2,2);
Rb=[1 (k12-k21)/(k11-k22);(k12-k21)/(k11-k22) 1];
cc=eig(Rb);
N1=Rb*K20;N2=transpose(N1);
Pb=[Rb*K10 Rb*K20;N2 eye(2)];
Qb=-Pb*Ab+(transpose(Adc)*Pb-Pb*Bb*Kb);

% Variant 2
PPP=transpose(inv(T))*Pb*inv(T);
KKK=inv(R)*transpose(Bdc)*PPP;
EEE=eig(Adc-Bdc*KKK);m=rank(T);

% Variant 1
Q=transpose(inv(T))*Qb*inv(T);
R=Rb;

% Determination of matrices Q si R
Q=(pinv(C'))*Q*(pinv(C));Q=Q(1,1);

% Optimal command determination
for k=1:100
    % Step 4: Calculus of matrix G
    Par=R+(b1')*Q*b1;
    G=(inv(Par))*(b1')*Q;
    yb=0;u=G*(yb-(alf')*Y-b2*ub); uu(:,k)=u;
    % Step 6: x(k+1), y(k+1), xc(k+1), yc(k+1) calculus
    x=Ad*x+Bd*u;y=Cd*x;
    xc=Adc*xc+Bdc*u;yc=Cdc*xc;
    x1(k+1)=x(1);x2(k+1)=x(2);
    x3(k+1)=x(3);x4(k+1)=x(4);
    xc1(k+1)=xc(1);xc2(k+1)=xc(2);
    xc3(k+1)=xc(3);xc4(k+1)=xc(4);
    z=[Y' u' ub']';
    Y=[yc;Y(1:length(Y)-1)]; e=y-yc;
    % Step 7
    lambda=0.95;P=100*eye(n+m);
    bc=bc+P*z*e; alf=bc(1:4);
end

subplot(321);plot(t,x1,'b',t,xc1,'r');grid;
subplot(322);plot(t,x2,'b',t,xc2,'r');grid;
subplot(323);plot(t,x3,'b',t,xc3,'r');grid;
subplot(324);plot(t,x4,'b',t,xc4,'r');grid;
subplot(325);plot(t1,uu(1,:),'r');grid;
subplot(326);plot(t1,uu(2,:),'r');grid;
6 Conclusion

The paper presents an on-line parametric identification and discrete optimal command algorithm for linear systems. For validation, it is used to the command of an aircraft’s longitudinal movement. A simulation program, based to presented algorithm, was made in Matlab. The obtained graphics are time variation of the state variables associated to the control system and to the estimated model of the aircraft $x_i(t), \dot{x}_i(t)$ and the evolution of command $\delta(t)$.

References: