

Control of Delayed Integrating Processes Using Two Feedback Controllers – R_{MS} Approach

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Abstract: - The objective of this paper is to demonstrate the utilization of algebraic controller design in an unconventional ring while control integrating processes with time delay. In contrast to many other methods, the proposed method is not based on the time delay approximation. A control structure combining a simple feedback loop and a two-degrees-of-freedom control structures is considered. This structure can be conceived as a simple feedback loop with inner stabilizing loop. The control design is performed in the ring of retarded quasipolynomial (RQ) meromorphic functions (R_{MS}) - an algebraic method based on the solution of the Bézout equation with Youla-Kučera parameterization is presented. Final controllers may be of so-called anisochronic type and ensure feedback loop stability, tracking of the step reference and load disturbance attenuation. Among many possible tuning methods, the dominant pole assignment method is adopted.

Key-Words: - Time delay systems, algebraic control, Bézout identity, Youla-Kučera parameterization

1 Introduction

Integrating models appear while modeling mass or energy accumulation, rotation of machineries, etc. These processes contain undesirable pole which need to be shifted by the feedback loop. As well, the great deal of technological and other processes own an input-output time delay. The presence of a delay entails problems with controllers design due to the fact that the delay significantly influences the feedback properties of a control system. The combination of integrating behavior of the system and delays makes a controller design more difficult.

There have been investigated various principles for control of integrating processes with a time delay. Some of these methods utilize standard PI or PID controllers, e.g. [1]-[3]. Some ideas are based on generalized Smith predictor, e.g. [4]-[6] or predictive approaches, mainly using state-space description [7].

One of significant approaches in modern control theory is a family of algebraic methods. Unlike some traditional state-space models, algebraic tools are based on fractional description of systems. Any transfer function can be expressed as a ratio of two elements in the appropriate ring. Traditional transfer function is represented by a polynomial fraction. This description is employed for algebraic control strategy for integrating delayed systems in [8] where the control structure with two feedback controllers (Fig.1) is considered. Another frequently used ring, besides polynomials, is designed as R_{PS} (ring of Hurwitz stable and proper rational functions) [9], [10]. Algebraic control philosophy in this ring then

exploits Bézout identity (Diophantine equation) along with Youla-Kučera parameterization to obtain stable and proper controllers. Algebraic controller design methods mentioned above requires rational approximation of exponentials expressing delays, usually via first order Padé approximation.

This contribution presents an algebraic method avoiding any time delay approximation. A ring of stable and proper retarded quasipolynomial meromorphic functions (R_{MS}) for this purpose is utilized. A term of this ring is a ratio of two quasipolynomials where the denominator quasipolynomial is stable and the whole ratio is proper with respect to highest s-powers. The only effort to design controllers in this ring for integration delayed systems is in [11] where a simple 1DOF control structure was utilized.

In this paper, an algebraic approach based on Bézout identity and the Youla-Kučera parameterization using control system with two controllers is considered. The presented feedback system can be comprehend and solved in double meaning. First, it is possible to take the system as a whole. It means that overall input-output transfer functions are utilized for controller design. This approach was presented in [12] where it is compared with linear quadratic (LQ) polynomial approach [8]. Second, the control system can be viewed as a simple control feedback with inner (stabilizing) feedback loop. In this case, inner loop is solved first and the main loop subsequently. The final controllers ensure (in both cases) feedback loop stability, step reference tracking and load disturbance attenuation, and they are tuned by pole

assignment method described in [13]. The paper discusses and compares results of both cases. Illustrative example also demonstrates and verifies the usefulness and applicability of the proposed method.

2 System Description in R_{MS} Ring

2.1 R_{MS} Ring

Algebraic control methods are based on input-output system formulation in the form of transfer function. Conventional transfer functions as a ratio of two polynomials are not directly applicable for models containing delays due to exponentials resulting from the Laplace transform of delays. In order to express the numerator and denominator in polynomials, the first order Padé approximation is then usually utilized. However, there is also possible to use another way. Rational approximation can be avoided so that the transfer function can be performed in the ring of stable and proper RQ-meromorphic functions, R_{MS} .

Any function over this ring is a ratio of two retarded quasipolynomials $y(s)/x(s)$ in general. A denominator quasipolynomial $x(s)$ of degree n means

$$x(s) = s^n + \sum_{i=0}^{n-1} \sum_{j=1}^h x_{ij} s^i \exp(-\vartheta_{ij} s) \quad (1)$$

where “retarded” refers to the fact that the highest s -power is not affected by exponentials. Quasipolynomial (1) is stable iff it owns no finite zero s_0 such that $\text{Re} \{s_0\} \geq 0$. In other words, a term in R_{MS} ring is analytic in the right half complex plane. Stability can be verified by the Michailov stability criterion, see e.g. in [14]. The numerator $y(s)$ of an element in R_{MS} can be factorized in the form $y(s) = \tilde{y}(s) \exp(-\tau s)$, where $\tau > 0$ and $\tilde{y}(s)$ is a retarded quasipolynomial of degree l

$$\tilde{y}(s) = s^l + \sum_{i=0}^{l-1} \sum_{j=1}^h \tilde{y}_{ij} s^i \exp(-\tau_{ij} s) \quad (2)$$

The quasipolynomial fraction is called proper iff $l \leq n$.

2.1 Integrating Delayed Plant in R_{MS}

R_{MS} ring can be naturally utilized for description of systems with delays in both left and right sides of appropriate differential equation. The transfer function of the plant or the controller is then expressed as a ratio of two terms in R_{MS} ring. This contribution deals with the integrating time delay systems inscribed with the transfer function

$$G(s) = \frac{K \exp(-\tau s)}{s} = \frac{m_0(s)}{s} = \frac{B(s)}{A(s)}; A(s), B(s) \in R_{MS} \quad (3)$$

where $m_0(s)$ is an appropriate quasipolynomial of degree one. The suitable form of $m_0(s)$ is discussed in the Section 4 where controller design for inner feedback loop is described.

3 Control System with Two Controllers

Controller design described in Section 4 have been implemented for a simple feedback loop or internal model structure (IMC) up to this day. In this paper, the control system structure with two controllers is utilized, see Fig.1.

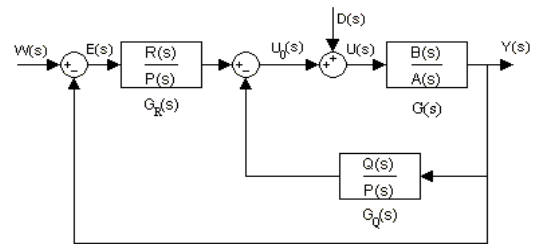


Fig.1 - Proposed control structure with two controllers.

In the scheme, $W(s)$ is the reference signal, $D(s)$ is the load disturbance, $E(s)$ is the control error, $U_0(s)$ is the controller output, $U(s)$ is the plant input, and $Y(s)$ is the plant output (controlled value) in the Laplace transform. The plant transfer function is depicted as $G(s)$, the “inner” feedback controller is $G_Q(s)$, and $G_R(s)$ represents “outer” controller.

The following basic transfer function can be derived in the control system in general:

$$\begin{aligned} G_{WY}(s) &= \frac{Y(s)}{W(s)} = \frac{B(s)R(s)}{M(s)}; \quad G_{DY}(s) = \frac{Y(s)}{D(s)} = \frac{B(s)P(s)}{M(s)} \\ G_{WE}(s) &= \frac{E(s)}{W(s)} = \frac{A(s)P(s) + B(s)Q(s)}{M(s)} \\ G_{DE}(s) &= \frac{E(s)}{D(s)} = -\frac{B(s)P(s)}{M(s)} \end{aligned} \quad (4)$$

where

$$G_R(s) = \frac{R(s)}{P(s)}; \quad G_Q(s) = \frac{Q(s)}{P(s)} \quad (5)$$

$$M(s) = A(s)P(s) + B(s)[R(s) + Q(s)] \quad (6)$$

and $R(s)$, $Q(s)$ and $P(s)$ are from R_{MS} and the numerator of $M(s)$ corresponds to the characteristic quasipolynomial of the closed loop.

4 Algebraic controller design in R_{MS} ring

The algebraic controller design presented in this paper supposes that all transfer functions and signals in the control systems are in the form of a ratio of terms in R_{MS} .

The control system scheme in Fig. 1 can be grasped either as the whole system corresponding to transfer functions (4) or as an inner feedback loop with controller G_Q and outer loop with controller G_R . Algebraic control design for the former was shown in [12]. This contribution concerns in the latter idea.

Basic requirements on the control systems are closed-loop stability, asymptotical reference tracking and load disturbance attenuation. To avoid the presence of input disturbance in the inner feedback for controllers design, let the control system scheme be rearranged as in Fig.2. All transfer functions in (4) are still valid; however, controllers design for the inner loop excludes the disturbance. The idea is that inner feedback pre-stabilizes the controlled process, i.e. zero pole is moved to the left, and the outer feedback controller ensures mentioned requirements for pre-stabilized system G_0 .

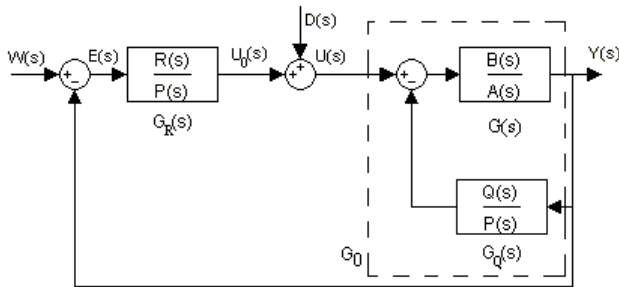


Fig.2 - Reconfigured control scheme

4.1 Inner loop pre-stabilization

Let an integrating delayed plant be pre-stabilized using a proportional controller $G_Q = q_0$. The condition for closed-loop stability is given by Diophantine equation

$$\frac{s}{m_0(s)} q_0 + \frac{K \exp(-\tau s)}{m_0(s)} = 1 \quad (7)$$

The natural task is to find a suitable stable quasipolynomial $m_0(s)$. The solution of (6) gives

$$q_0 = \frac{m_0(s) - s}{K \exp(-\tau s)} \quad (8)$$

The requirement is q_0 to be real; therefore the simplest $m_0(s)$ has to be of the form

$$m_0(s) = q_0 K \exp(-\tau s) + s \quad (9)$$

Stability of $m_0(s)$ can be studied e.g. using Michajlov criterion, which results in

$$q_0 = \frac{1}{A_m} \frac{\pi}{2K\tau} \quad (10)$$

where $A_m > 1$ is the gain margin ($A_m = 1$ correspondences to stability border).

Thus, transfer function of the inner feedback loop is

$$G_0(s) = \frac{K \exp(-\tau s)}{s + q_0 K \exp(-\tau s)} \quad (11)$$

4.2 System stabilization

Now the task is to control G_0 using a simple feedback with controller G_R . However, the numerator and denominator in (10) are not from R_{MS} and thus the transfer function must be factorized as

$$G_0(s) = \frac{\frac{K \exp(-\tau s)}{m_1(s)}}{s + q_0 K \exp(-\tau s)} \quad (12)$$

where $m_1(s)$ is a stable (quasi)polynomial. In order to have (12) as simple as possible, let

$$m_1(s) = s + \lambda \quad (13)$$

where $\lambda > 0$ is a selectable real parameter which brings an additional degree of freedom. Naturally one can take another $m_1(s)$; an example is presented in Section 4.4.

Closed loop stability is ensured by a solution of Diophantine equation

$$\frac{s + q_0 K \exp(-\tau s)}{s + \lambda} R(s) + \frac{K \exp(-\tau s)}{s + \lambda} P(s) = 1 \quad (14)$$

A particular solution

$$R_0(s) = 1, \quad P_0(s) = \frac{s + \lambda - K \exp(-\tau s)}{s + q_0 K \exp(-\tau s)} \quad (15)$$

it is possible to parameterize using the Youla-Kučera parameterization

$$\begin{aligned} R(s) &= R_0(s) + A(s)Z(s) = 1 + \frac{s + q_0 K \exp(-\tau s)}{s + \lambda} Z(s) \\ P(s) &= P_0(s) - B(s)Z(s) \\ &= \frac{s + \lambda - K \exp(-\tau s)}{s + q_0 K \exp(-\tau s)} - \frac{K \exp(-\tau s)}{s + \lambda} Z(s); \quad Z(s) \in R_{MS} \end{aligned} \quad (16)$$

to fulfill other control requirements via appropriate choice of $Z(s)$.

4.3 Reference Tracking and Disturbance Rejection

Parameterization (15) enables to find the solution of (13), so that requirements of reference tracking and disturbance rejection are accomplished. Both inputs are considered as step functions

$$W(s) = \frac{H_W(s)}{F_W(s)} = \frac{\frac{w_0}{m_w(s)}}{\frac{s}{m_w(s)}}; \quad D(s) = \frac{H_D(s)}{F_D(s)} = \frac{\frac{d_0}{m_d(s)}}{\frac{s}{m_d(s)}} \quad (17)$$

where $m_w(s)$ and $m_d(s)$ are arbitrary stable (quasi)polynomials of degree one and $H_W(s)$, $H_D(s)$, $F_W(s)$, $F_D(s) \in R_{MS}$.

The requirement is both $F_W(s)$ and $F_D(s)$ divide $P(s)$; in other words control error, $E(s)$, must be from R_{MS} . It means that the numerator of $P(s)$ contains at least one zero root. Let

$$Z(s) = \frac{\left(\frac{\lambda}{K} - 1\right)(s + \lambda)}{s + q_0 K \exp(-\tau)} \quad (18)$$

Then the feedback controller reads

$$G_R(s) = \frac{R(s)}{P(s)} = \frac{\frac{\lambda}{K}[s + q_0 K \exp(-\tau)]}{\frac{s + q_0 K \exp(-\tau)}{s + \lambda[1 - \exp(-\tau)]}} \quad (19)$$

$$= \frac{\lambda[s + K \exp(-\tau)]}{q_0 K[s + \lambda[1 - \exp(-\tau)]]}$$

Note that this controller is of so-called anisochronic structure [14], i.e. it owns delays in the feedback.

Recall that the inner-feedback controller is proportional, $G_Q = q_0$; however, in terms of algebraic philosophy and Fig. 1 it can be written also as

$$G_Q(s) = \frac{Q(s)}{P(s)} = \frac{\frac{q_0[s + \lambda[1 - \exp(-\tau)]]}{s + q_0 K \exp(-\tau)}}{\frac{s + \lambda[1 - \exp(-\tau)]}{s + q_0 K \exp(-\tau)}} \quad (20)$$

4.4 An Alternative Solution

As was mentioned in Section 4.1, stable (quasi)polynomial $m_1(s)$ can be chosen other than in (13). Another natural choice is

$$m_1(s) = m_0(s) = s + q_0 K \exp(-\tau) \quad (21)$$

which gives the inner-feedback transfer function

$$G_0(s) = \frac{\frac{K \exp(-\tau)}{s + q_0 K \exp(-\tau)}}{\frac{s + q_0 K \exp(-\tau)}{s + q_0 K \exp(-\tau)}} = \frac{K \exp(-\tau)}{1} \quad (22)$$

In this case, stabilizing Diophantine equation

$$P(s) + \frac{K \exp(-\tau)}{s + q_0 K \exp(-\tau)} R(s) = 1 \quad (23)$$

has one of particular solutions

$$R_0(s) = 1, \quad P_0(s) = \frac{s + q_0 K \exp(-\tau) - K \exp(-\tau)}{s + q_0 K \exp(-\tau)} \quad (24)$$

and choosing $Z(s) = q_0 - 1$ in the Youla-Kučera parameterization, the final outer-feedback controller structure ensuring reference tracking and disturbance rejection is then

$$G_R(s) = \frac{q_0[s + q_0 K \exp(-\tau)]}{s} \quad (25)$$

which is generalized (delayed) PI controller. The inner-feedback controller is $G_Q = q_0$ again.

4.5 Comparison to direct controller design

Control system in Fig. 1 can be conceived also as one system without separation of inner feedback loop. Algebraic controller design in R_{MS} for this philosophy was proposed in [12] where the following two sets of (alternative) controllers were derived

$$G_R(s) = \frac{R(s)}{P(s)} = \frac{\gamma \left(\alpha + \frac{\bar{\lambda}}{K} \right) s + \alpha \bar{\lambda}}{s + \bar{\lambda} [1 - \exp(-\tau)]} \quad (26)$$

$$G_Q(s) = \frac{Q(s)}{P(s)} = \frac{(1 - \gamma) \left(\alpha + \frac{\bar{\lambda}}{K} \right) s}{s + \bar{\lambda} [1 - \exp(-\tau)]}$$

$$G_R(s) = \frac{R(s)}{P(s)} = \frac{\gamma 2\alpha s + K \exp(-\tau)}{s} \quad (27)$$

$$G_Q(s) = \frac{Q(s)}{P(s)} = (1 - \gamma) 2\alpha$$

where α is a selectable real parameter constrained in the same way as q_0 in (10), another selectable real parameter $\gamma \in \langle 0, 1 \rangle$ represents the distribution of the solution, see details in [12], and $\bar{\lambda}$ has analogous function to λ .

5 Controllers Tuning

The final sets of controllers, (19), (20), (25) – (27), still own unknown parameters that have to be set properly. There are naturally plenty of approaches solving the problem of controller tuning.

The well applicable and relatively simple tuning method was described e.g. in [13]. This method enables to set the desired dominant poles of the closed loop, the maximum number of which is given by the number, k , of unknown parameters in the characteristic quasipolynomial. If the dominant poles are denoted as σ_i , $i = 1 \dots k$, the characteristic equation as $m(s)$, and a vector of unknown parameters as \mathbf{v} , then the following system of k linear equations is obtained

$$m(\sigma_i, \mathbf{v}) = 0; \quad i = 1 \dots k \quad (28)$$

For complex poles, one root from each complex pair is taken and (28) is divided into two equations of the form

$$\begin{aligned} \operatorname{Re}\{m(\sigma_i, \mathbf{v})\} &= 0 \\ \operatorname{Im}\{m(\sigma_i, \mathbf{v})\} &= 0 \end{aligned} \quad (29)$$

The significant feature is that set (28) or (29) is linear with respect to unknown parameters, which makes the solution easy to find.

In the case of delayed integrator, the system given by outer controller (25) has the characteristic quasipolynomial

$$m(s) = (s + q_0 K \exp(-\tau s))^2 \quad (30)$$

with unknown parameter q_0 .

Since there is a single parameter to be found, q_0 , the only double real dominant root or a conjugate pair of complex roots can be prescribed. Moreover, stability condition (10) must be considered. In case of real poles, the optimal choice is the prescription of leftmost dominant roots. It can be proved that this optimal double real pole is ensured by the option

$$q_0 = \frac{1}{K \tau} \quad (31)$$

which corresponds with dominant pole $\sigma_{1,2} = -\tau^{-1}$

In case of conjugate poles, the solution of (29) gives the same real q_0 if a complex root satisfies

$$\operatorname{Re}\{\sigma_i\} = -\operatorname{Im}\{\sigma_i\} / \tan(\operatorname{Im}\{\sigma_i\} \tau) \quad (32)$$

These roots (real or complex) must be chosen carefully because some attempts to place them excessively to the left in the complex plane can lead into the following situation. Due to the fact that the quasipolynomial (30) has the infinity number of roots, the chain of complex roots can move to the right near to the stability border (imaginary axis) and thus these roots can take over the role of dominant poles of the system.

The characteristic of the quasipolynomial feedback system with outer controller (19) is

$$m(s) = [s + q_0 K \exp(-\tau s)](s + \lambda) \quad (33)$$

In this case, two parameters are to be set. Again, condition (10) must be fulfilled and $\lambda > 0$. The task of pole placement can be solved separately for each of factors in (33). In [8], the suggestion for the choice of λ is

$$\lambda = \frac{2}{\tau} \quad (34)$$

which, in comparison with (31), does not allow for the dominant pole.

Coefficient γ in (26) and (27) influences the feedback system behavior as well because it appears in the numerators of closed loop transfer functions. However, it does not impact the spectrum of the system. In this paper, various values of γ are set randomly.

6 Illustrative example

This simulation example composed in Matlab-Simulink environment demonstrates the usability of the proposed controller design method in R_{MS} . For the sake of limit space, simulation shows output only and the control action values are commented if necessary.

Hence, let $K = 1$ and $\tau = 5$. Assume controllers (19) and (20) first. The highest value of q_0 according to (31) when a double real root is chosen is $q_0 = 0.0736$ (for $\sigma_{1,2} = -0.2$). This choice corresponding to gain margin $A_m = 4.27$ is confronted with the dominant pole $\sigma = -0.18 \pm 0.1j$, i.e. $q_0 = 0.0835$, $A_m = 3.76$. Parameter λ is chosen according to (34) as $\lambda = 0.4$. The control performance of this controller is compared with controllers (19) and (25), with the same setting, in Fig. 3.

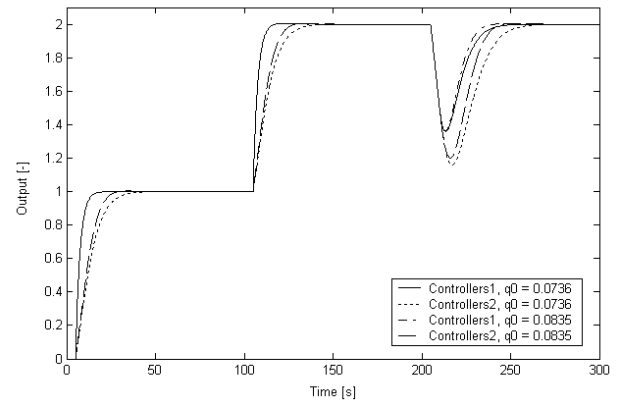


Fig.3 - Step setpoint and load disturbance responses – Controllers1: (19), (20); Controllers2: (19), (25); $K = 1$, $\tau = 5$, $\lambda = 0.4$, $d = -0.1$.

As can be seen in Fig.3, a change of q_0 for controllers (19) and (20) does not affect setpoint response. Disturbance response for these controllers is much better than for (19) and (25); however, this performance is afflicted by higher control action peaks.

Finally, the response using (19) and (20) is compared with responses of the feedback system containing controllers (26) and (27) obtained via “direct” controller design in Fig.4. These controllers have the consistent setting with controllers (19) and (20), i.e. $\lambda = \bar{\lambda} = 0.4$, $q_0 = \alpha = 0.0736$, and equable distribution of the solution, i.e. $\gamma = 0.5$. Fig.4 clearly indicates that controllers (19)-(20) and (26) causes identical disturbance response. The advantage of the latter controllers insists in the possibility of setting the distribution of the solution using γ . However, proposed alternative approach via consideration of the inner feedback loop gives comparable simulation results.

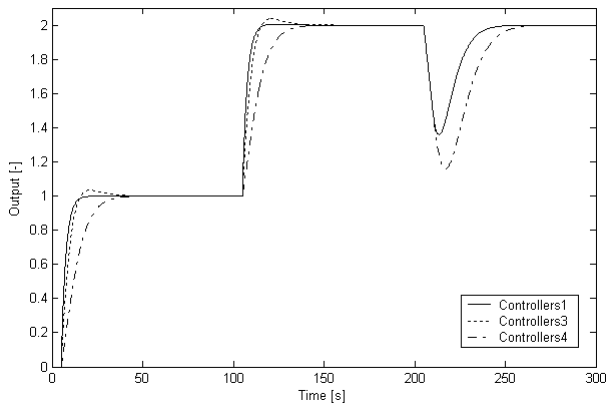


Fig.4 - Step setpoint and load disturbance responses – Controllers1: (19), (20); Controllers3: (26); Controllers4: (27); $K = 1$, $\tau = 5$, $\lambda = \bar{\lambda} = 0.4$, $q_0 = \alpha = 0.0736$, $\gamma = 0.5$, $d = -0.1$.

7 Conclusion

In this contribution, the problem of algebraic control design in the ring of stable and proper RQ meromorphic functions for integrating time delay processes has been investigated. The proposed method does not involve the delay approximation. The controller structure is derived through the solution of the Bézout equation with Youla-Kučera parameterization. The methodology enables to find various controllers that satisfy requirements on closed loop stability, step reference tracking and step load disturbance attenuation. The control system combines conventional 1DOF and 2DOF schemes and it is conceived as a inner (pre-stabilizing) feedback loop plus outer loop. The final controllers are tuned using dominant pole assignment method. The efficiency and usability of the proposed method is verified on a simulation example.

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