Optimization of Multiple Input Single Output Fuzzy Membership Functions Using Clonal Selection Algorithm

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Abstract: - A clonal selection algorithm (CLONALG) inspires from Clonal Selection Principle used to explain the basic features of an adaptive immune response to an antigenic stimulus. In this study, a new method is proposed for optimization of the Multiple Input Single Output (MISO) fuzzy membership functions using CLONALG. The most appropriate placement of membership functions with respect to fuzzy variables can be determined using our method for a fuzzy system whose rules table and shape of membership functions were given previously. Also, how the membership functions compute as a parameter optimization problem using CLONALG is described for MISO fuzzy system on an illustrative example.

Key-Words: - Multiple Input Single Output Fuzzy Membership Functions, Optimization, Clonal Selection Algorithm.

1 Introduction

Fuzzy logic is used to solve a lot of problems related wide range of area because of providing flexible solutions. Designing fuzzy system contains fuzzy sets which are defined by rule table and membership functions. When the fuzzy sets have been established, how best to determine the membership function is the first question that has to be tackled.

For most fuzzy logic control problems, the membership functions are assumed to be linear and usually triangular in shape. So, the issues to be determined are the parameters that define the triangles. Because of this, the membership optimization problem can be reduced to parameter optimization problem. These parameters are usually based on the control engineer's experience and/or are generated automatically. To improve behavior of this parameter optimization problem some methods such as genetic algorithms (GA), self-organizing feature maps (SOFM), tabu search (TS) etc. can be used.

GA was used by Karr [1] in determination of membership functions. Karr applies GA to design of fuzzy logic controller (FLC) for the cart pole problem. Meredith et al. [2] have applied GA to the fine tuning of membership functions in a FLC for a helicopter. Lee and Takagi [3] also tackle the cart problem. They take a holistic approach by using GA to design the whole system. Cheng et al. [4] have chosen the image threshold via minimizing the measure of fuzziness. For selecting the bandwidth of fuzzy membership functions, they use peak locations which are chosen from the histogram using the peak selection criterion. Bağış [5] presents a method for the determination of the membership functions based on the use of Tabu Search algorithm. Cerrada et al. [6] proposed an approach permits incorporate the temporal behavior of the system variables into the fuzzy membership functions. Simon [7] employed $H_\infty$ state estimation theory for the membership function parameter optimization. He made some modifications on the $H$ filter with addition of state constraints so that the resulting membership functions are sum normal. Yang and Bose [8] described a method for generating a fuzzy membership functions with unsupervised learning using self-organizing feature map. They applied this method to pattern recognition.

In our previous work, a new method was proposed to determine the membership functions of a single input and output fuzzy system whose shape was triangular [9]. The suggested method is relevant for any membership function whose mathematical model is known. The method focused on finding the optimum values of the parameter of this membership function model. This problem solved using an artificial immune system algorithm: CLONALG. This algorithm is inspired from Clonal Selection Principle used to explain the basic features of an adaptive immune response to an antigenic stimulus. In this study, we optimized the multiple input-single output (MISO) fuzzy system whose rule base is complete using the adaptation of the method mentioned above.

This paper is organized as follow. In Section 2, CLONALG are explained. In Section 3, how the MISO membership functions is optimized using CLONALG on an illustrative example. The experimental results and the
discussion of them have been given in Section 4. Finally, in Section 5, we present our conclusions.

2 Clonal Selection Principle & CLONALG

Clonal selection is the theory used to explain the basic properties of an adaptive immune response to an antigenic stimulus. It establishes the idea that only those cells capable of recognizing an antigenic stimulus will proliferate and differentiate into effectors cells, thus being selected against those that do not. Mainly features of the clonal selection principle are affinity proportional reproduction and mutation. The higher affinity, the higher number of offspring generated. The mutation suffered by each immune cell during reproduction is inversely proportional to the affinity of the cell receptor with the antigen. The standard genetic algorithm doesn’t account for these immune properties [10].

De Castro & Von Zuben proposed a Clonal selection algorithm named CLONALG, to fulfill these basic processes involved in clonal selection principle. CLONALG will be initially proposed to perform machine-learning and pattern recognition tasks, and then adapted to be applied to optimization problems [11]. The algorithm of the CLONALG for the optimization task is given below.

1. Generate \( j \) antibodies randomly.
2. Repeat a predetermined number of times:
   a. Determine the affinity of each antibody (Ab). This affinity corresponds to the evaluation of the objective function.
   b. Select the \( n \) highest affinity antibodies.
   c. The \( n \) selected antibodies will be cloned proportionally to their affinities, generating a repertory \( C \) of clones: the higher affinity the higher number of clones and vice versa.
   d. The clones from \( C \) are subject to hypermutation process inversely proportional to their antigenic affinity. The higher affinity, the smaller mutation, and vice versa.
   e. Determine the affinity of the mutated clones \( C \).
   f. From this set \( C \) of clones and antibodies, select the \( j \) highest affinity clones to compose the new antibodies’ population.
   g. Replace the \( d \) lowest affinity antibodies by new individuals generated at random.
3. End repeat [12].

3 Computation of the MISO membership functions using CLONALG

In this section, we described how the membership functions compute as a parameter optimization problem using CLONALG for MISO fuzzy system on an illustrative example. In our example we have a fuzzy system with two inputs and one output whose shapes are triangular and each of them consist of three membership functions. They are denoted \( x_1, x_2 \) and \( y \), respectively. The ranges of the variables \( x_1, x_2 \) and \( y \) are \([0,7]\), \([0,80]\) and \([0,50]\). Each of them use \( \text{low, medium, high} \) linguistic terms. In this case, the linguistic rules are as follows:

- **Rule 1**: If \( x_1 \) is low and \( x_2 \) is high then \( y \) is medium.
- **Rule 2**: If \( x_1 \) is medium and \( x_2 \) is low then \( y \) is low.
- **Rule 3**: If \( x_1 \) is medium and \( x_2 \) is medium then \( y \) is medium.
- **Rule 4**: If \( x_1 \) is medium and \( x_2 \) is high then \( y \) is high.
- **Rule 5**: If \( x_1 \) is high then \( y \) is high.

Some reference values must be measured beforehand for this system. These are the output values obtained corresponding to specific input values:

1: If \( x_1=1 \) and \( x_2=80 \) then \( y=20 \).
2: If \( x_1=3 \) and \( x_2=20 \) then \( y=16 \).
3: If \( x_1=4 \) and \( x_2=50 \) then \( y=26 \).
4: If \( x_1=5 \) and \( x_2=80 \) then \( y=36 \).
5: If \( x_1=7 \) and \( x_2=60 \) then \( y=40 \).

The membership functions of fuzzy system for input and output variables will be as shown in Fig. 1.

![Fig.1: A sample membership functions for the MISO fuzzy system](image1)

Expected from a CLONALG is to find the base lengths of right triangles and intersection points of triangles.
corresponding to the reference data. Its second goal is to obtain the appropriate input values among definite ranges of system. If the base length of each membership function is represented by 6-bit, the maximum value each base can take is $2^6 - 1 = 128$. The domain intervals for input and output variables are [0, 7], [0, 80] and [0, 50], respectively. The base values are reflected under these values. This is formulated as follows:

$$X_i = X_{\text{min}} + \frac{d}{(2^L - 1)} (X_{\text{max}} - X_{\text{min}})$$

$L$ is the length of related variable (in this case, it is Antibody-Ab) in bits, $d$ is the decimal value of this variable, $X_{\text{min}}$ is the minimum value of region to be transformed, and $X_{\text{max}}$ is the maximum value of this region, then $X_i$ is the transformed figure of that variable.

The affinity function of this procedure is calculated as follows:

$$\text{Affinity} = \text{MaxError} - \text{TotalError}$$

$$\text{TotalError} = \sum_{i=1}^{n} (y_{\text{CLONALG}} - y_i)$$

$$\text{MaxError} = \sum_{i=1}^{n} \max[(y_{\text{CLONALG}} - y_{\text{min}}), (y_{\text{max}} - y_{\text{CLONALG}})]^2$$

where $y_i$ is the output of $i^{th}$ reference input; $y_{\text{CLONALG}}$ obtained by CLONALG, is output for $i^{th}$ reference input, and $n$ is the number of input-output data pairs given.

The aim is to approximate the TotalError to zero as close as possible. Because of that affinity function is converted to MaxError-TotalError, in this way; minimization process is also converted to maximization process. In order to prevent affinity function from getting negative values, the maximum number 5628 is used and this number is also maximum error at the same time. How this number acquired has been showed in the below.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$\max[(y_{\text{CLONALG}} - y_{\text{min}}), (y_{\text{max}} - y_{\text{CLONALG}})]^2$</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\max[(20-0),(50-0)]^2=30^2$</td>
<td>900</td>
</tr>
<tr>
<td>2</td>
<td>$\max[(16-0),(50-16)]^2=34^2$</td>
<td>1156</td>
</tr>
<tr>
<td>3</td>
<td>$\max[(26-0),(50-26)]^2=26^2$</td>
<td>676</td>
</tr>
<tr>
<td>4</td>
<td>$\max[(36-0),(50-36)]^2=36^2$</td>
<td>1296</td>
</tr>
<tr>
<td>5</td>
<td>$\max[(40-0),(50-40)]^2=40^2$</td>
<td>1600</td>
</tr>
</tbody>
</table>

$$\sum_{i=1}^{n} \max[(y_{\text{CLONALG}} - y_{\text{min}}), (y_{\text{max}} - y_{\text{CLONALG}})]^2 = 5628$$

Let solve an example for understanding how the output value has been computed regarding to given base values. The base values are taken 2 for $x_1$ and 45 for $x_2$. Firstly the membership functions must be found for these variables. Then, it is explored if two membership functions can be put in a rule or not. If they can, the grades of membership functions are calculated and analyzed to determine if the rule contains AND or OR. If these two membership functions are ANDed, outputs are determined from lower grades of membership functions, but if two membership functions are ORed, outputs are determined from higher grades of membership functions.

$$y^* = \frac{\sum_{i=1}^{n} \mu(y_i) y_i}{\sum_{i=1}^{n} \mu(y_i)}$$

This example is described in Fig. 2. The given values are shown as $x_{1i}$ and $x_{2i}$. $x_{1i}$ has intersected with low and medium and $x_{2i}$ has intersected with medium and high. Firstly, the grades of membership are determined for each membership function from the Fig.2. Then the rules which are triggered according to these values are detected. In our example first, third and fourth rules are triggered. For the first rule $\mu_{\text{low}}$ is 0.7 and $\mu_{\text{medium}}$ is 0.4.

![Fig. 2: Finding the appropriate outputs of CLONALG for given values for the MISO fuzzy system](image-url)

We choose the $\mu_{\text{medium}} = 0.4$ because the rule involves AND. The same process has repeated for third and fourth rule. In the third rule $\mu_{\text{medium}} = 0.2$ and in the forth rule $\mu_{\text{high}} = 0.2$ are taken. The acquired outputs corresponding to rules are 18, 16 and 23, respectively. We defuzzificate the output value using equation 5 and obtained 18.75 as result.
4 Experimental Results
In this section, CLONALG used to optimize the MISO fuzzy membership functions is implemented by Matlab 7.1 R14. After 25 iteration completed, the individual whose affinity is maximum has been accepted as optimal solution. The affinity value accepted as the optimum solution is 5625.9 and its membership function shape is depicted in Fig. 3.

Fig.3: The membership function shape of the optimal solution for the MISO fuzzy system

For showing the results weren’t obtained by coincidentally, 20 different groups were generated. A group consisted of 10 different initial populations and a population consisted of 10 Ab individuals. The algorithm was set to run for 25 generations on each population in the groups, respectively. Then the maximum affinities values of each group are depicted in the Fig. 5.

Fig.5: The maximum affinity values of each groups

Results are compared group by group. As shown in the figure, at least one Ab converged to the optimum solution in each group. Than it was straightforward to say the CLONALG represented a good performance while finding the optimum base distance of a fuzzy membership functions.

5 Conclusion
The most appropriate placement of membership functions with respect to fuzzy variables can be found for a fuzzy system whose rules table and shape of membership functions were given previously. There are no restrictions for shape of membership functions. They can be generally used but it can be mathematical function whose model is known. Then, the issues to be determined are the parameters that define the model. Because of this, the membership optimization problem can be reduced to parameter optimization problem. How the membership functions compute as a parameter optimization problem using CLONALG is described for Multiple Input Single Output (MISO) fuzzy system in this work.

CLONALG used to optimize the fuzzy membership functions was implemented by Matlab 7.1 R14. Firstly, 20 different groups were generated for showing the results weren’t obtained by coincidentally. A group consisted of 10 different initial populations and a population consisted of 10 Ab individuals. The algorithms were set to run for 25 generations on each population in the groups, respectively. Then the acquired results are examined. It was seen that, at least one individual Ab converged to the optimum solution in each
group. As well, it could be said that according to results, CLONALG converged to optimum solution quickly because of the own cloning and hyper mutation mechanisms. Also, the stability of the affinity values was provided by the CLONALG algorithm. Than it was straightforward to say the CLONALG represented a good performance while finding the optimum base distance of a fuzzy membership functions of a MISO fuzzy system.

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