Upscaling Groundwater Models for inclusion in Management Models - Case Study of California’s Sacramento Valley

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Abstract: This article describes and evaluates the upscaling of a calibrated groundwater model of California’s Sacramento Valley for the purpose of incorporating it into general management (optimization) models. Upscaling is performed in two stages, by vertical integration, than by combining grid elements into isolated lumped subbasins and switching from a distributed to a simplified lumped subbasin model. Simulation results of the upscaled models are compared to those of the initial spatially distributed model. Because anthropogenic stresses (pumping and recharge) outweigh interbasin flows, the fully upscaled formulation estimates groundwater levels very close to those of the vertically integrated model. An optimization formulation based on the upscaled model efficiently retains almost all relevant management information contained in more detailed models without recourse to expensive distributed embedding methods.

Key-Words: Groundwater, groundwater management models, integrated, simulation, optimization, model upscaling

1. Introduction

Many detailed spatially distributed groundwater models exist at the regional scale to investigate and make predictions about complex hydrogeologic settings. Models with a broad outlook such as economic [1, 2] or “integrated” [3] water resource management models may wish to represent groundwater but cannot afford to include a full spatial model. Upscaling and simplification of the groundwater physics are often deemed appropriate or even necessary for inclusion into a management model. This may lead to concerns whether important features of the hydrology have been lost through upscaling and simplification of groundwater physics. This paper follows a two-step upscaling process that allows progressive verification of how much information and resolution is lost. This strategy is relevant to models that must represent multiple distinct aquifers based on detailed local models.

Section 2 briefly describes the hydrogeology of California’s northern Central Valley and section 3 summarizes the CVRASA1 model, the initial distributed groundwater model. Section 4 details the two step spatial upscaling (vertical integration then regrouping of grid cells into subbasins) and the simplification of groundwater physical modeling (from finite-differences to a lumped storage coefficient equation approach). After each change, simulation results are compared with the more detailed initial model. In section 5 the simplified model is incorporated into a preliminary optimization formulation to test the numerical stability of its solutions. Section 6 discusses the findings and the last section presents conclusions.

Figure 1: The Sacramento Valley lies in the Northern portion of California's Central Valley [4].
2. Groundwater in the Central Valley

The Central Valley is an elongated topographic basin (640 km long, 80 km average width) surrounded by several mountain ranges. It is drained by two major rivers, the Sacramento River in the North (Figure 1) and the San Joaquin River in the South. The valley is underlain by a heterogeneous alluvial-fill aquifer system consisting of discontinuous beds of clay, silt, sand, and gravel. The thickness of these deposits averages about 730 m and increases from north to south, with a maximum thickness of more than 2700 m near Bakersfield. Deposits are continuous except for the Sutter Butte volcanic plug in the Sacramento Valley. Groundwater pumping has drastically changed natural recharge and discharge rates to the aquifer system. In the 1960’s the recharge rate increased 5-fold from percolated irrigation water [5]. Rather than being discharged by evapotranspiration and baseflow as occurred before extensive well pumping, most discharge now occurs through pumping. Total flow through the system has increased to more than 6 times pre-stressed conditions to more than 15 km³ (per year) [5]. Vertical movement of groundwater has also been greatly enhanced by more than 100,000 irrigation wells perforated at various levels.

3. CVRASA1 Groundwater Model

The Central Valley Regional Aquifer-System Analysis (CVRASA1) groundwater model was built by the US. Geological Survey during the 1980’s [6, 7]. The model uses a finite-difference scheme for spatial and temporal discretization of the groundwater flow equation. The latest version of CVRASA1 was implemented in MODFLOW [8].

The quasi-three-dimensional regional model has four layers and at most 529 square grid cells per layer; each cell has a length of 6 miles (9.66 km). The vertical dimensions of blocks vary; their depths are incorporated into depth-averaged aquifer parameters. Given the large size of each block (almost 100 km²) only broad regional hydrogeologic characteristics are described. Transmissivity values are time-invariant in all model cells (modeled as a confined aquifer).

Layers 1 (bottom) through 4 (top layer) encompass different hydrologic conditions (Figure 2). All pumping occurs in topmost layers 3 and 4; the boundary between them is where most wells are no longer perforated. When present, the Corcoran Clay Member of the Tulare Formation also serves as a boundary between the two top layers. Layer 4 represents the mostly unconfined superficial aquifer which is in contact with surface water bodies and rivers. Sixty one percent of pumping occurs in this top layer in CVRASA1. Layer 3 represents deeper pumping in mostly confined zones. The bottom layer (number 1) consists of the continental deposits below the depth penetrated by any production wells.

The Central Valley model is surrounded by no flow boundary conditions except at the Sacramento-San Joaquin Delta where there are 3 specified head cells in layer (Figure 3). Transmissivity values were calculated for each layer using data on horizontal hydraulic conductivity (from pump tests and driller’s logs) and model block thickness. Storage coefficients are much higher in the unconfined layer 4 where they are equivalent to specific yield. This is reflected somewhat in model results; piezometric levels show less seasonal variation in the top unconfined layer than in lower confined layers.

The CVRASA1 model simulates 16.5 years of historical flow between 1961 and 1977 with a 6 month time step. CVRASA1 uses 2 stress terms: the first is a constant calibration term and the second is a time-varying net stress term based on historic water budget calculations summarized in the Appendix A of Williamson et al. [6].

4. Two Stage Upscaling

The upscaled model of the Sacramento Valley is built using the northern-most 170 cells of CVRASA1. The first upscaling phase converts from three-dimensions to two and the time-step is changed from 6-months to one. The relevant grid
with its three specified head cells (at the Sacramento-San Joaquin River Delta) is displayed in Figure 3. The second upscaling includes a coarsening of the grid and a change from a finite-difference formulation to modeling head variations in isolated lumped subbasins due to stresses (and not to groundwater flow).

### 4.1. Upscaling from 3D to 2D

Converting the model from 3D to 2D involved combining the 4 superimposed layers of CVRASA1. Because both transmissivity and storage coefficients are depth-averaged parameters, their value for the combined 1-4 layer is the sum of the parameter in each layer. Leakage coefficients between layers ($T_k$) are not used in the 2D version. Average, minimum and maximum values for $T$ and $S$ for the combined-layer SVGM are compared to individual CVRASA1 layers in Table 1. Most importantly, the storage coefficient of the combined layer is similar to the specific yield of the unconfined layer (layer 4). This means the combined layer will essentially be modeling the top unconfined layer; this is confirmed in simulation results presented later in this section.

Concerns with upscaling include losing accuracy in representing groundwater heads and generally misrepresenting the system. For example, well pumping from the confined layer 3 should have lower and more variable heads due to the layer’s smaller storage coefficient. This is relevant since 40% of CVRASA1 pumpage occurs in layer 3. Another concern is that stream-aquifer interaction cannot be considered if the upper unconfined layer is not modeled separately. Both concerns are valid and are partially sacrificed to a trade-off for gains in efficiency management. However, in reality piezometric head levels in several CVRASA1 cells show that seasonal variability differs relatively little between different layers (see cells 21, 31 or even 61 of Figure 5). If confined layer 3 was essential for the management model, a two-layer model could be envisaged, with a leakance term between layers. This was not considered here because of the similarity between confined and unconfined layers in CVRASA1.

The Sacramento Valley model is run using custom MATLAB code implementing an implicit scheme finite-difference groundwater model, analogous to MODFLOW [8]. Unconfined aquifers are modeled as linear confined aquifers (with constant transmissivity) so that associated management models can use linear programming. This is valid at the regional scale for deep unconfined aquifers where changes in head due to stresses are small compared to the saturated thickness of the aquifer [9].

Figure 5 shows how combined layers 1-4 simulated head levels compare to CVRASA1 individual layers at selected cells. A wide variety of behaviors exists; cells 61 and 97 follow the expected pattern of a relatively steady unconfined layer 4 with deeper and more variable water levels in confined layers 1-3. Many cells however show layer 4 varying more widely than lower confined layers (e.g. 1, 48, 56, 107); in other cells all layers show similar levels and seasonal variation is comparable (e.g. 31).

Groundwater levels at individual grid cells show that the combined layer 1-4 resemble those of unconfined layer 4. The 2D upscaled model basically represents the top unconfined layer. Modeling the aquifer as confined is acceptable considering seasonal head variations rarely exceed 5 m while the saturated aquifer thickness averages 730m. Because 60% of pumping in the Sacramento Valley occurs in this superficial layer (according to CVRASA1 data) and because heads of all layers behave reasonably similarly, we tentatively state that the 2D model has effectively simplified the system with relatively little loss of information.

### 4.2. Upscaling to isolated subbasins

The target management model will use average regional groundwater levels over groundwater subbasins (i.e. delineated portions of the aquifer – see Figure 5) to track groundwater quantities extracted and extraction costs. The storage
coefficient equation can be used to quantify piezometric head levels in single- or multi-cell aquifer models. The storage coefficient relates the volume of water released (or absorbed) from (into) storage per unit surface area of aquifer per unit change in hydraulic head in a confined aquifer. For gravity drainage in unconfined aquifers its equivalent is the specific yield.

\[
    h'_g = h'^{-1}_g + \frac{Q'_g}{sc_g \times area_g} \quad \forall g, t
\]

where:

- \( Q'_g \) = mean stress (net pumping, recharge term) in groundwater subbasin \( g \) at time \( t \)
- \( h'_g \) = mean hydraulic head in groundwater subbasin \( g \) at time \( t \)
- \( sc_g \) = mean storage coefficient of subbasin \( g \)
- \( area_g \) = surface area of subbasin \( g \)

This simplified model ignores an essential element of groundwater physics: flow. The key question for the management model is how much information is lost with this simplification in the context of a specific hydrogeology and set of hydrologic inputs.

The storage coefficient model represents groundwater subbasins 1 through 9 as single-cell aquifers without inter-connections. Storage coefficients were taken by summing the vertically integrated storage coefficients from the once upscaled 2D model. Figure 6 shows the storage coefficient method obtained results very similar to the CVRASA1 combined layer model.

5. Optimizing with the Upscaled Model

Will the storage coefficient equation applied to groundwater basins be stable and solvable in an optimization model? To answer this question we pose embed the formulation into a linear program constraint set of a problem to which we already know the answer: maximizing pumping subject to historical heads should reproduce both historical heads and extractions. The model is written:

\[
    \begin{align*}
    \text{Max} & \sum_{(g,t)} \sum_{g} Q'_g \\
    \text{S.T.} \quad H'_g = H'^{-1}_g + \frac{Q'_g}{sc_g \times area_g} \quad \forall g, t \\
    H'_g & \geq \text{min}h'_g \quad \forall g, t
    \end{align*}
\]

where \( \text{min}h'_g \) = minimum allowed hydraulic heads at subbasin \( g \) at time \( t \) (this time-series is the SVGM simulated historical record averaged per subbasin). Resulting heads exactly match historical head constraints (Figure 7) showing that in these conditions the formulation is stable.

Comparing historical stresses with optimized stresses solved for by the simplified model constrained to match historical head levels will also reveal how well the formulation is working. Differences between optimized and historical monthly stresses ranged from 0% to 36%, with an average 5.3% difference. In addition to this relative good accuracy, the model is also small (3,565 equations and variables) and fast (0.003 seconds to build and solve the LP model with the GAMS software).

6. Discussion

The storage coefficient equation can be used to dynamically model regional changes in piezometric head when the error due to neglecting flow between subbasins is small. This occurs here because piezometric levels are dominated by the influence of fluxes (pumpage, recharge) while flows between basins have comparatively small influence. Inclusion of partial differential equations in optimization model constraint sets has long been recognized as challenging and the field is under continual development [10]. Particularly with groundwater models, embedding the entire spatially discretized model into the management model constraint set has been known to sometimes experience numerical infeasibilities [11]. These factors encourage use of simpler upscaled formulations into management models. [12]
The storage coefficient method is efficient, adding only 1 equation per groundwater basin and per time step in both the simulation and optimization cases. In the case of the Sacramento Valley model examined here, regional piezometric heads from the upscaled model were almost indistinguishable from the results of the vertically integrated model, itself a good summary of the 4 layer CVRASA1 model. The disadvantage of the upscaled storage equation model is its lack of flexibility - model scale and scope are fixed and limited. Boundary conditions such as stream-aquifer interaction or specified heads can never be represented. Although flow between neighboring basins was not considered here, it could be modeled using the Darcy equation. In our case flow between subbasins doesn’t strongly affect head levels as there is little difference between the storage coefficient equations heads and thus from the 2D model.

7. Conclusions

CVRASA1 is an early USGS groundwater model of the Central Valley from 1961 to 1977 using 4 layers with 529 6x6 mile grid cells per layer. For the present study CVRASA1 was reduced to the northern Sacramento Valley and upscaled to 2D in order to be efficiently included into a management (optimization) model. The combined layer model is simulated and compared to CVRASA1 layer by layer results. Combining the four layers is almost equivalent to modeling the topmost unconfined aquifer. This was deemed acceptable since head levels of lower confined CVRASA1 layers and their seasonal variation are relatively similar to the top layer. A second phase of upscaling was performed (from grid to isolated subbasins) coupled with a simplification of the physical model (finite difference to storage coefficient equation). The storage equation formulation reproduced almost the same regional head levels as the 2D modeled averaged by subbasins. The simplified model was deemed appropriate for inclusion in any further management (optimization) models as it was used in an optimization formulation without unstabilities or infeasabilities in the results.

References


Figure 2: Cross-sectional view of Central Valley in CVRASA1 showing 4 layers and general patterns of recharge, discharge, and ground-water flow (Williamson et al. 1989).

Figure 3: Sacramento Valley Groundwater Model grid with its 167 variable head 10 x 10 km cells and 3 specified head cells. The first upscaling results in a 2D version of the northern portion of the USGS CVRASA1 groundwater model.

Table 1: Aquifer characteristics for the Sacramento Valley (cells 1-170) from CVRASA1 and Sacramento Valley Groundwater Model (SVGM)

<table>
<thead>
<tr>
<th>Sac. Valley</th>
<th>Transmissivity (m²/day)</th>
<th>Storage Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Avg</td>
<td>Max</td>
</tr>
<tr>
<td>Layer 1-4</td>
<td>0.060</td>
<td>0.27</td>
</tr>
<tr>
<td>Layer 4</td>
<td>0.013</td>
<td>0.05</td>
</tr>
<tr>
<td>Layer 3</td>
<td>0.015</td>
<td>0.06</td>
</tr>
<tr>
<td>Layer 2</td>
<td>0.011</td>
<td>0.09</td>
</tr>
<tr>
<td>Layer 1</td>
<td>0.026</td>
<td>0.11</td>
</tr>
</tbody>
</table>
Figure 4: Semi-annual CVRASA1 individual cell comparisons: combined layers 1-4 and individual layers (cell number is last number in legend).
Figure 5: Groundwater subbasins whose boundaries coincide with the desired discretization of the management model.

Figure 6: Storage coefficient equation simulation results for selected groundwater subbasins 1, 2, 4 and 5. Groundwater subbasin results using the storage coefficient method are very close to the combined layer 1-4 CVRASA1 model.

Figure 7: Average head per subbasin in the SVGM-SC optimization model; SVGM historical head levels were used to contrain the model.
Figure 8: Stresses solved for in the SVGM-SC optimization model are similar to SVGM historical stresses.