KINETIC MODEL OF A SKID STEERED ROBOT

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Abstract: - The paper introduces mathematical models of a skid steered mobile platform for robotics. The platform consists of a rectangular steel construction with four wheels. Two banks of two drive wheels on each side are linked to two DC electrical motors via chain on sprocket drive and gearing. The two drive assemblies for the left and right banks are identical but they operate independently to steer the vehicle by skid. Wheels of the vehicle consist of rims and shallow tread pneumatic tyres. Dynamic and quasi-kinematic models are introduced.

Key-Words: - robot**,** skid steering, modelling, dynamic model, kinematic model,

1 Introduction

Skid steered robots are often used in robotics applications due to their simple and robust construction. The main application of these robots is in outdoor space [1], but due to low cost they are used in indoor applications as well. Typical for indoor application are simple four wheeled robots, see Fig.1.

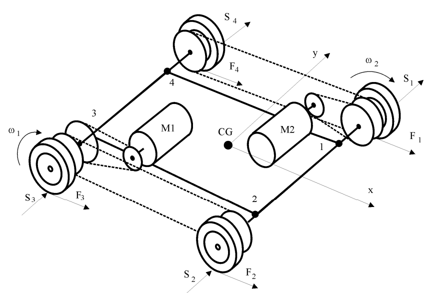

Fig.1. Skid steered wheeled robot.

One of big disadvantages of skid steered robots in indoor applications is absence of good model of robot's motion. The main problem in modelling of skid steering is complexity of modelling of tyre forces [2]. There are existing several models of skid steered robots e.g. [3],[4]. The following article describes relatively simple model that can be used for trajectory planning and control of indoor skid steering.

2 Modelling of the skid steered platform

To find a reasonable model of the platform motion one must accept many simplifying assumptions. We assume that the vehicle moves in flat horizontal plane only and that the wheels are in permanent contact with the plane. We also assume that slip and skid forces between the tyres and flat surface do not change with position of the platform and they are the only significant external forces acting on. Thus we neglect forces due to rolling of tyres, drag forces etc. These assumptions are quite acceptable for indoor applications e.g in factory shops, labs etc.

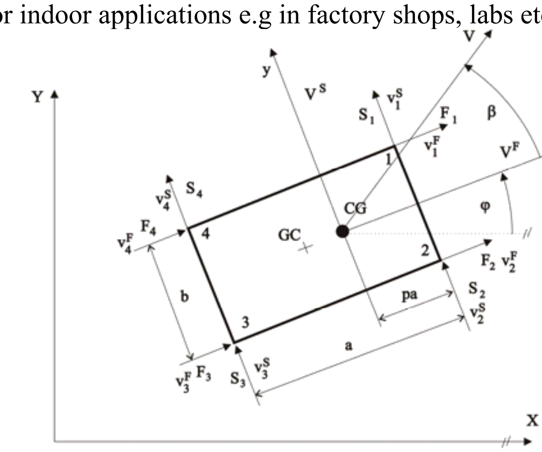

Fig.2. Geometry of the platform.

The platform is considered as system of four wheels and a rigid body see Fig. 1. Motion of the platform is calculated with respect to inertial frame XY. For modelling purpose we suppose that the platform is geometrically symmetric but centre of gravity CG is shifted forward or backward with respect to geometric centre GC of the platform. Position of the vehicle with respect to the inertial frame is given by x y co-ordinates of CG and by angle of course φ, see Fig.2. The platform which is modelled in the article is a rectangular steel construction. Two banks of two drive wheels are each linked to two motors via chain on sprocket drive and gearing. The two drive assemblies for the left and right banks are identical but they operate independently to steer the vehicle. The motors can be driven in either directions, thus causing the vehicle to move forward, backward, right or left. The motors are equipped with incremental encoders and are controlled in velocity loop. Wheels of the vehicle consist of a rims and shallow tread pneumatic tyres.

Individual symbols in figures and subsequent equations represent:

- F longitudinal force between the tyre and surface due to slip
- S lateral force between the tyre and surface due to skid
- $v_{1,2}$ circumferential velocity of right or left wheels respectively
- φ angle of course of the vehicle
- r radius of the wheel
- v^F longitudinal velocity of a wheel axle
- v^S lateral velocity of a wheel axle
- a wheel track
- b wheel base
- p $p \in <0,1>$ specifies position of CG with respect to GC
- M mass of the vehicle
- W weight of the vehicle
- J moment of inertia with respect to CG

2.1 Dynamic model

Contrary to nonholonomic wheeled platforms skid steered platforms cannot be described by simple kinematic model. The reason is substantial slipping and skidding that is present during skid steering.

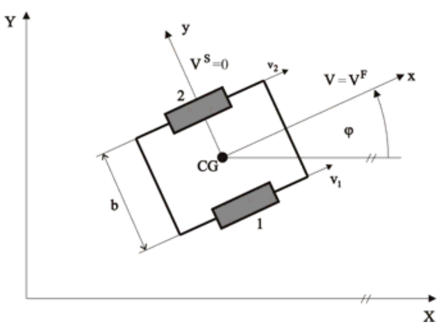

Fig.3. Geometry of the nonholonomic platform.

Kinematic model of nonholonomic differentially steered platform which is geometrically equivalent to the investigated skid steered platform is described by equations (1),

$$V^F = \frac{v_1 + v_2}{2}$$
$$V^S = 0 \qquad (1)$$
$$\dot{\varphi} = \frac{v_1 - v_2}{b}$$

where V^F and V^S are local velocities of the platform. They can be easily recalculated to global velocities of the CG of the platform

$$\begin{aligned} \dot{x} &= V^F \cos\varphi - V^S \sin\varphi \\ \dot{y} &= V^F \sin\varphi + V^S \cos\varphi \end{aligned} \qquad (2)$$

After integration of velocities we obtain pose of the platform see Fig.4.

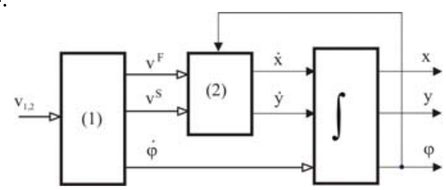

Fig.4. Simulation scheme of the nonholonomic platform.

Model of skid steered vehicle must consider slipping and skidding that can be taken into account only with help of forces and masses. Thus resulting model is of dynamic type. Basic equations of movement of the CG of the platform are

$$\begin{aligned} M\ddot{x} &= \sum_{i=1}^{i=4} F_1 \ cos\,\varphi - \sum_{i=1}^{i=4} S_i \ sin\,\varphi \\ M\ddot{y} &= \sum_{i=1}^{i=4} F_1 \ sin\,\varphi + \sum_{i=1}^{i=4} S_i \ cos\,\varphi \\ J\ddot{\varphi} &= (-F_1 + F_2 + F_3 - F_1)\frac{b}{2} + \\ &+ (S_1 + S_2)pa - (S_3 + S_4)(1-p)a \end{aligned} \qquad (3)$$

Should we know slip and skid forces F, S we would be able to calculate accelerations from equations (2), then after integration we would be able to calculate velocity and finally position of the platform. Thus a good model of tyre – ground forces is crucial part of the complete model of the platform. Firstly we used model described in [5], but the worked well for tyre cornering angles in range <-85; 85> deg. This range is not sufficient for description of motion of the skid steered robot. That is why we have proposed another model which works in any regime of robot motion. The model is based on Coulomb model of friction [6]. The basic idea of the model is to calculate the force of adhesion in direction of the tyre motion and to calculate its longitudinal and lateral components. The following relations describe the model. Index of the wheel was omitted for simplicity.

$$F = P\frac{v_s}{v_f}$$
$$S = -P\frac{v^S}{v_f} \qquad (4)$$

Where

$v_s = v - v^F$ (5)

is the velocity of longitudinal slipping

$$\mathrm{v}_f = \sqrt{v_s^2 + (\mathrm{v}^S)^2} \qquad (6)$$

is the total velocity of slipping including lateral skid. Using Coulomb model of friction yields

$$\begin{aligned} P &= Cv_f && for \quad v_f \le \frac{\mu_0 W}{C} \\ &= \mu_0 W && for \quad v_f > \frac{\mu_0 W}{C} \end{aligned} \qquad (7)$$

where C is stiffness of the tyre and μ_0 is coefficient of friction. Model of tyres described by equations (3)-(6) proved to be qualitatively the same as the model in [5].
The last problem is to calculate longitudinal and lateral velocities of individual wheels.

$$\begin{aligned} v_1^F &= v_4^F = \dot{x}\cdot\cos\varphi + \dot{y}\cdot\sin\varphi - \frac{b}{2}\dot{\varphi} \\ v_2^F &= v_3^F = \dot{x}\cdot\cos\varphi + \dot{y}\cdot\sin\varphi + \frac{b}{2}\dot{\varphi} \\ v_1^S &= v_2^S = -\dot{x}\cdot\sin\varphi + \dot{y}\cdot\cos\varphi + pa\dot{\varphi} \\ v_3^S &= v_4^S = -\dot{x}\cdot\sin\varphi + \dot{y}\cdot\cos\varphi - (1-p)a\dot{\varphi} \end{aligned} \qquad (8)$$

Simulation scheme of the model is shown on Fig.5

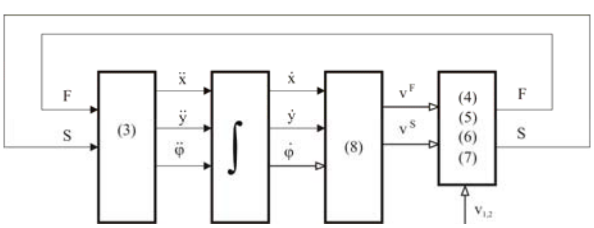

Fig.5. Simulation scheme of the skid steered platform.

2.2 Quasi-kinematic model

When we compare simulation schemes of nonholonomic and skid steered platform we can see the difference. The relation between velocities in (1) is a simple mapping from $\mathbf{R}^2$ to $\mathbf{R}^3$ there is no dynamics in that relation. This is not true for the skid steer. But we can find static relation between input velocities v_1,v_2 and local velocities of the skid steer in steady state by simulation.

$$\begin{aligned} V^F &= f_1(v_1, v_2) \\ V^S &= f_2(v_1, v_2) \\ \dot{\varphi} &= f_3(v_1, v_2) \end{aligned} \qquad (9)$$

Provided that dynamic transients during the robot motion are short, model described by (9) will be quite acceptable. Of course the model will be valid only for one specific configuration of the platform and one specific homogenous and even floor. The following table represents the last equation (9) for robot that was used in our simulation tests. Data of the robot are described in the following chapter.

Table 1. $\dot{\varphi} = f_3(v_1, v_2)$

v2/v1	-0,50	-0,40	-0,30	-0,20	-0,10	0,00	0,10	0,20	0,30	0,40	0,50
0,50	-1,24	-1,11	-0,99	-0,86	-0,74	-0,61	-0,49	-0,37	-0,25	-0,12	0,00
0,40	-1,12	-0,99	-0,87	-0,74	-0,62	-0,49	-0,37	-0,25	-0,12	0,00	0,12
0,30	-1,00	-0,87	-0,74	-0,62	-0,49	-0,37	-0,25	-0,12	0,00	0,12	0,25
0,20	-0,88	-0,75	-0,62	-0,50	-0,37	-0,25	-0,12	0,00	0,12	0,25	0,37
0,10	-0,75	-0,62	-0,50	-0,37	-0,25	-0,12	0,00	0,12	0,25	0,37	0,49
0,00	-0,63	-0,50	-0,37	-0,25	-0,12	0,00	0,12	0,25	0,37	0,49	0,61
-0,10	-0,50	-0,37	-0,25	-0,12	0,00	0,12	0,25	0,37	0,49	0,62	0,74
-0,20	-0,38	-0,25	-0,12	0,00	0,12	0,25	0,37	0,50	0,62	0,74	0,86
-0,30	-0,25	-0,12	0,00	0,12	0,25	0,37	0,50	0,62	0,74	0,87	0,99
-0,40	-0,12	0,00	0,12	0,25	0,37	0,50	0,62	0,75	0,87	0,99	1,11
-0,50	0,00	0,12	0,25	0,38	0,50	0,63	0,75	0,88	1,00	1,12	1,24

2.3 Verification of the model

Quasi-kinematic model can be useful only if there exists reasonably good equivalence between skid steered model and real system. Here are some results gained. Photo of the real robot is on Fig. 6.

Fig.6. Skid steered robot U.T.A.R.

Measured parameters of the robot are:

Wheel track a=0.5m, wheel base b=0.4m, weight W=59kg, position CG-GC p=0.54. Coefficient of friction on concrete floor $\mu 0=0.61$.

Calculated and estimated parameters of the robot are:

Moment of inertia J=2kgm2. Stiffness of the tyres C=5000Nms-1.

Results of simulation using skid steer dynamic model are shown on the following figures. Initial pose of the robot was always **0**.

1st simulation run was done with v_1=-0.12ms^{-1} and v_2=0.12ms^{-1}. Platform moved with velocity of COG V=0.007ms^{-1}, $\dot{\varphi} = -0.30 rad/s$ and radius of turn R=0.025m.

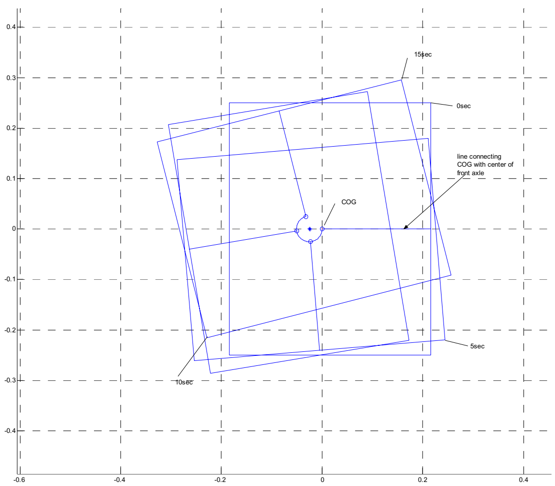

Fig.6. Stroboscopic view of the 1st simulation run

2nd simulation run was done with v_1=0ms^{-1} and v_2=0.12ms^{-1}. Platform moved with velocity of COG V=0.06ms^{-1}, $\dot{\varphi} = -0.14 rad/s$ and radius of turn R=0.4m.

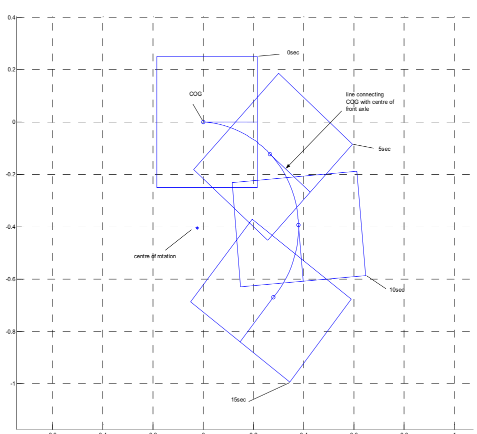

Fig.7. Stroboscopic view of the 2nd simulation run

Real results of the 1st run are:

Velocity of COG V=0ms^{-1}, $\dot{\varphi} = -0.31 rad/s$ and radius of turn R=0m.

Real results of the 2nd run are:

Velocity of COG V=0.07ms^{-1}, $\dot{\varphi} = -0.15 rad/s$ and radius of turn R=0.45m.

Equipment used during the real tests is shown on Fig.8. The robot position was captured by a camera that was positioned above the robot. Robot was equipped with coloured markers that allowed us to identify COG and orientation of the platform.

Fig.8. Real tests of skid steered robot

3 Conclusion

It is not possible to develop a classical kinematic model of a skid steered platform. Only a quasi-kinematic dynamic model that is similar to nonholonomic differentially steered platform can be obtained. This type of model can be derived from a dynamic model of the skid steered platform by simulation. We developed the dynamic model that showed a good correspondence with behaviour of the real robot.

Acknowledgement

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