# Total Electron Content (TEC) and Model Validation at an Equatorial Region

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*Abstract:* - The ionosphere has practical importance in GPS applications because it influences transionospheric radio wave propagation. The parameter of ionosphere that produces most of the effects on radio signals is Total Electron Content (TEC). By modelling this TEC parameter, the evaluation of the ionospheric error and the correction of these ionospheric errors for differential GPS can be done. A new approach in the determination of the differential ionospheric error to sub- centimeter accuracy is described in this paper utilizing a developed model. An ionospheric delay model was developed to accurately determine the difference in ionospheric delay expected over a short baseline so that a more accurate differential GPS correction could be made. An ionospheric error correction model should be made applicable to any location including the equatorial region. The results showed that the developed algorithm is a function of elevation angle and TEC from the reference station path to the satellite and could give differential ionospheric delay in sub-centimetre accuracy.

*Key-Words:* - GPS, TEC, ionosphere, baseline, differential GPS, transionospheric

## 1 Introduction

Scientists are now studying space weather with a wide range of tools to try to learn more about the physical and chemical processes taking place in the upper atmosphere and beyond. One of these tools is GPS. The signals from the GPS satellites travel trough the ionosphere on their way to receivers on or near the Earth's surface. The free electrons populating this region of the atmosphere affect the propagation of the signals, changing their speed and direction of travel. The ionosphere is that region of the Earth's atmosphere in which ionizing radiation causes electrons to exist in sufficient quantities to affect the propagation of radio waves. The ionosphere is prone to significant disturbances, which get considerably worse during periods of high solar activity, such as at solar maximum [1].

Ionospheric models are usually computed by determining the TEC in the direction of all GPS satellites in view from a ground GPS network. The simple but less accurate method is the Klobuchar model for single frequency where the required parameters are available from the ephemerides data [2]. In order to provide ionospheric corrections for positioning and navigation for single-frequency GPS receivers, the ionosphere needs to be mathematically described by a given ionospheric model. Meanwhile, precise ionospheric modelling is also important for other space-based observation systems as well as communication systems and space weather studies.

The purpose of this work is to develop an accurate ionospheric model that best suits the equatorial region and that could get differential ionospheric delay in sub-centimetre accuracy.

## 2 Total Electron Content (TEC)

As GPS signals propagate through the ionosphere, the propagation of the GPS signals are changed in proportion to the varying electron density along the line of sight between the receiver and the satellite. The integrated total electron content (TEC) from the receiver to the satellite is proportional to the accumulated effect by the time the signal arrives at the receiver. This affects the GPS range observables: a delay is added to the code measurements and advance to the phase measurements. To achieve very precise positions from GPS, this ionospheric delay or advance must be taken into account. A GPS operates on two different frequencies $f_1$ and $f_2$, which are derived from the fundamental frequency of = 10.23 MHz:

$$f_1 = 154.fo = 1575.42 \text{ MHz} \quad \text{and}$$
$$f_2 = 120.fo = 1227.60 \text{ MHz} \tag{1}$$

Both code and phase measurements are affected by the dispersive behaviour of the ionosphere, but with different leading signs, which the absolute value of the group delay can be written as:

$$I_{\phi,g} = \int \frac{X}{2} ds = \frac{80.6}{2f^2} \int N_e ds \tag{2}$$

This delay is characterized by the total electron content where $TEC = \int N_e ds$. Substituting TEC into (2) yields

$$I_{\phi,g} = \frac{40.3}{f^2} TEC \text{ [m]} \quad \text{or}$$

$$\Delta t = \frac{40.3}{cf^2} TEC \text{ [s]} \tag{3}$$

These delays can be expressed in units of distance or time delay by dividing the right hand side of the Eq. (3) that measured in metres by the velocity of light.

A dual-frequency GPS receiver can measure the difference in ionospheric delays between the L1 (1575.42 MHz) and L2 (1227.60 MHz) of the GPS frequencies, which are generally assumed to travel along the same path through the ionosphere. Thus, the group delay can be obtained from (3) as:

$$P_1 - P_2 = 40.3 TEC \left( \frac{1}{f_2^2} - \frac{1}{f_1^2} \right) \tag{4}$$

where

P1 and P2 are the group path lengths corresponding to the high GPS frequency ($f_1$=1575.42MHz) and the low GPS frequency ($f_2$ =1227.6 MHz), respectively.

By processing the data from a dual-frequency GPS receiver, it is actually possible to estimate just how many electrons encountered by the signal along its path – the total electron content (TEC) [3]. A dual-frequency GPS receiver measures pseudo ranges and carrier phases at L1/L2 and its observables are used to compute TEC. The "phase levelling" technique is used to compute precise phase-derived slant TEC for each tracked satellite at each observation epoch [4].

The GPS observables are biased on the instrumental delays and therefore, it is necessary to remove the differential instrumental biases from receivers and satellites for accurate estimation of the TEC [5]. The differential time delay measurements are used to remove the ambiguity term. Then by combining the phase and the code measurements for the same satellite pass, the absolute TEC is obtained with greater precision [6].

The line-of-sight TEC values were converted to vertical TEC values using a simple mapping function and were associated to an ionospheric pierce point latitude and longitude, assuming the ionosphere to be compressed into a thin shell at the peak ionospheric height of 350 km. This conversion introduces a few errors in the middle latitude where electron density is small. But it may result in obvious error at low latitude with large electron density and great gradient [7, 8].

## 3 Short Distance GPS Models

The ionospheric model used in the ray tracing is determined by fitting realistic ionospheric profiles (such IRI2001 and the NeQuick prediction model) with a number of exponential layers to arrive at minimum total electron content (TEC) residual as required [9]. The main parameter to be considered in the modelling is the variations of ionospheric delay.

To obtain the LOS, the receiver and satellite positions should be known, and there are several methods to obtain them. The difference in the delays (Δtd) between the paths can be found from the difference in delays between the reference and mobile stations.

$$\Delta td = td_{ref} - td_{mob} \quad (5)$$

The difference in LOS (ΔLOS) can be found from the difference in LOS between the reference and mobile station as eqn. (6). The real time satellite position is sufficient in this application and the precision of LOS is not so crucial compared to other parameters in the model.

$$\Delta LOS = LOS_{ref} - LOS_{m} \quad (6)$$

where

$LOS_{ref}$ : line of sight at reference station

$LOS_{m}$ : line of sight at mobile station

The relation between Δtd and the difference in true range (ΔLOS) for a given satellite position and their ratio as:

$$Ratio = \frac{\Delta LOS}{\Delta td} \quad (7)$$

have been determined for the baseline orientations as a function of elevation angle for a given ionospheric model.

# 4 Result

GPS data on 8 November 2005 from Wisma Tanah, Kuala Lumpur, KTPK (3˚ 10' 15.44"N; 101˚ 43' 03.35"E) as a reference station, and UPM Serdang, UPMS (2˚ 59' 36.22"N; 101˚ 43' 24.63"E) as a mobile station, was collected, processed and analysed. The baseline length is 20 km. The GPS data was recorded in universal time system, whereby the sampling interval was 15 seconds and the cut-off elevation mask was 10˚.

## 4.1 Ionospheric Model

The ratio for S-N direction was modelled for the range of β up to 60˚ by fitting the obtained relationships with polynomial functions, $f$ (β) as defined in eqn. (8). It should not be extrapolated outside this range to higher elevation angles (80 to 90˚). The baseline was 10 km length and it used 16 elevation angles.

$$f(\beta) = 8.1 \times 10^2\beta^{10} - 3.7 \times 10^3\beta^9 + 4.5 \times 10^3\beta^8 + 2.7 \times 10^2\beta^7 - 4.7 \times 10^2\beta^6 - 8.1 \times 10^3\beta^5 + 1.4 \times 10^4\beta^4 - 3.2 \times 10^4\beta^3 + 5.2 \times 10^4\beta^2 + 2 \times 10^5\beta + 4.8 \times 10^5 \quad (8)$$

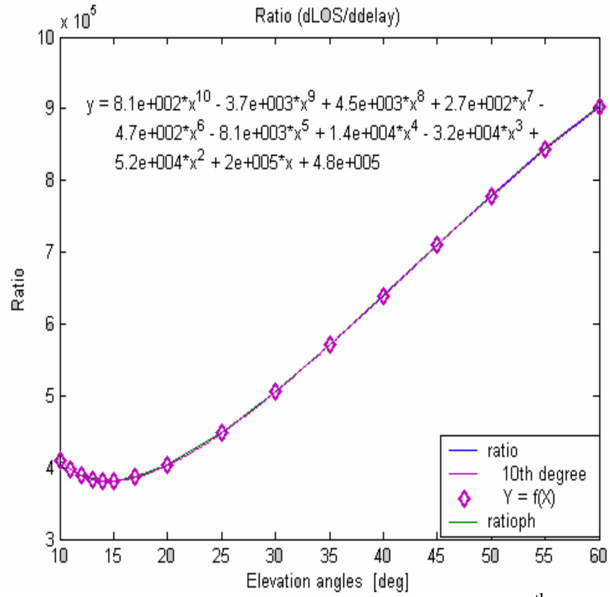


Fig. 1: The basic fitting for ratio using 10th order polynomial function.

Figure 1 shows the basic fitting using 10th order polynomial function. From the fitting, the coefficients and norm of residuals can be obtained. For this case, the norm of residuals is 37.345. The ratio is for the TEC 31 TECU. From (7) and (8), the difference in ionospheric delay at the mobile station compared with reference station is given as:

$$\Delta td = \frac{\Delta LOS}{f(\beta)} \quad (9)$$

ΔLOS must be known to determine the difference in delay. The delay at mobile station can be obtained by substituting (5) with (9) provided that the ionospheric delay at reference station is known. This is given as:

$$td,mob = td,ref - \frac{\Delta LOS}{f(\beta)} \quad (10)$$

In addition to the variation in elevation angles, the variation in TEC also affects the difference in delay between the two stations as:

$$f(TEC) = \frac{\text{TEC from satellite to receiver path}}{\text{TEC used in } f(\beta)} \quad (11)$$

Ionospheric models are usually computed by determining the TEC in the direction of all GPS satellites in view from a ground GPS network.

From this model, there are two main parameters that need to be determined: the difference in LOS and TEC. Apart from these, the ratio as a function of elevation angles was examined. The difference in ionospheric induced error between two stations can be expanded as:

$$\frac{1}{Ratio} = \frac{\Delta t_d(TEC)}{\Delta LOS} = f(TEC)$$

$$\Delta td\left(\beta, TEC\right) = \frac{\Delta LOS}{f(\beta)} f(TEC) \qquad (12)$$

where

*TEC* : total electron content
*ΔLOS* : differential in line of sight
*β* : elevation angle at reference station
*Δtd* : differential delay, in metre

### 4.2 Model Validation using Real GPS Data

Real GPS data was used for model validation. Figures 2 to 5 show representative cases of the different situations found in the analysis. The elevation angles for reference station at PRN 23 are 31˚ to 58˚ as shown in Fig. 2 as used in this study. A model for computing the effects of the ionosphere was presented. The differential delay model can be seen for 1 hour, which is from 3 to 4 (UT) for GPS satellite PRN 23.

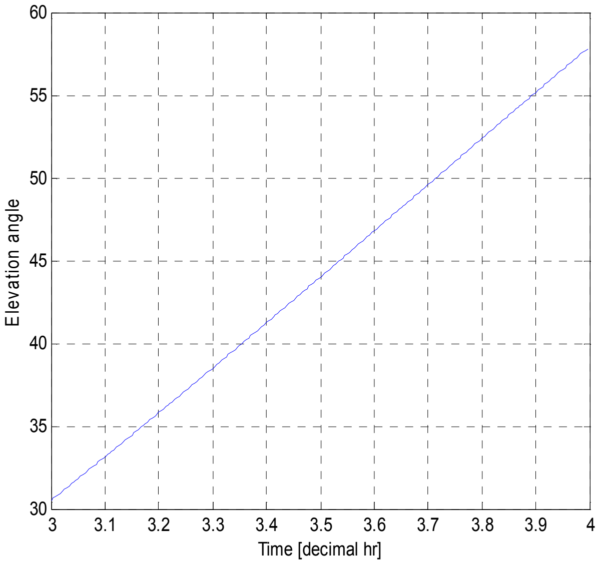


Fig. 2: Elevation angles for PRN23

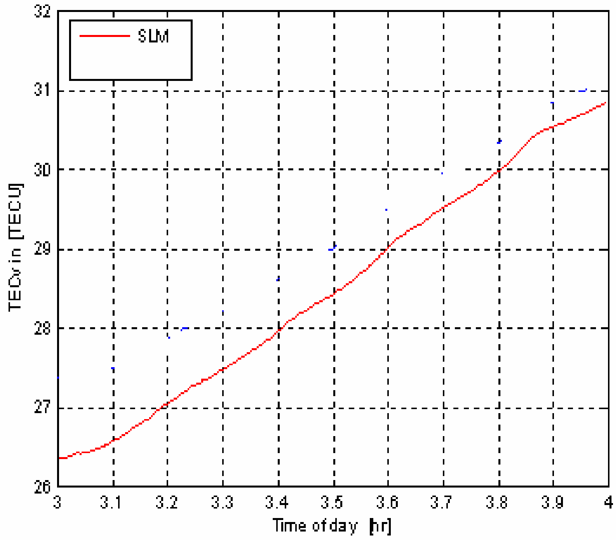


Fig. 3: TEC vertical for PRN 23

Figure 3 shows TEC vertical for GPS satellite PRN 23. Single Layer Model (SLM) was used to convert the slant TEC to vertical. This analysis at an equatorial region used SLM mapping function. The peak altitude ranges from 350 to 500 km at equatorial latitudes. The final TEC values are precise, accurate and without multipath.

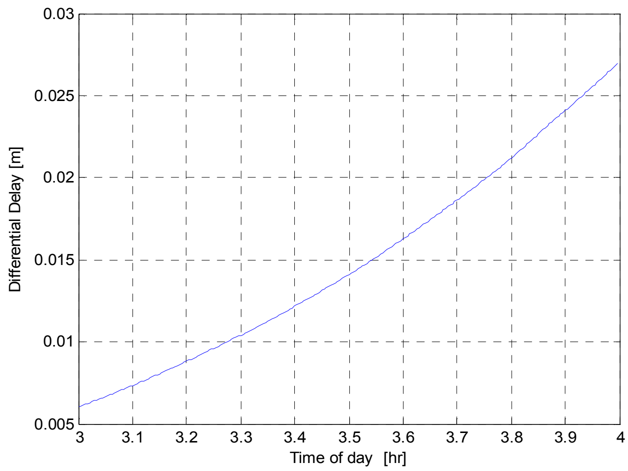


Fig. 4: Differential Delay Model for PRN 23

The differential ionospheric delay ($\Delta t_d$) calculated from the model is shown in Fig. 4. The correction model in (12) was used by substituting the above parameters at elevation angles roughly between 10˚ and up to 90˚, that were visible to the satellites for one hour with 15 second intervals. The accuracy was achieved at sub-cm level.

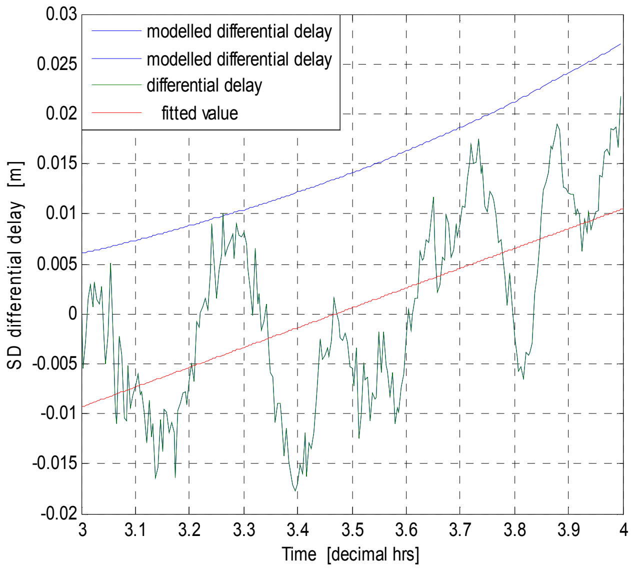


Fig. 5: Differential delay scaled to $L_1$ of PRN 23

Figure 5 shows the modelled differential delays. The computed values from $L_4$ residual (scaled to $L_1$) are shown in the same figure. From Fig. 5, the model and the $L_4$ residual show approximately the same trend which is the differential delay at one hour processing.

## 5 Conclusion

The work presented here has shown promising results based on the utilisation of carrier phase observation for precise positioning. The developed differential ionospheric correction model has been validated. From the validated model, it shows that the pattern is the same for both computed values from L4 residual and differential delay output. The developed algorithm is a function of elevation angle and TEC from the reference station path to the satellite.

The model is mostly suitable for short baseline. Simultaneously the model could also be preferably used among the single frequency users. From the model we can get differential ionospheric delay in sub-centimetre accuracy.

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