

Profit Analysis and Simulation in Motor Insurance

NORISZURA ISMAIL & ABDUL AZIZ JEMAIN

School of Mathematical Sciences

Universiti Kebangsaan Malaysia

43600 UKM Bangi, Selangor

MALAYSIA

Abstract: - This study suggests the use of profit analysis and simulation for assessing the price of motor insurance. Utilization of profit analysis and simulation on insurance pricing has several advantages; they may identify the distributions that agree or disagree with the actual experience, they may predict the performance of pricing models under alternative scenarios, and they may be applied to assess the performance of pricing models by allowing random variabilities in the underlying distribution of such models.

Key-Words: - Profit analysis, simulation, motor insurance, claim frequency, claim severity, premium.

1 Introduction

The pricing of motor, fire and workmen's compensation insurances in Malaysia is governed by their respective tariffs formulated by Persatuan Insurans Am Malaysia (PIAM) (Malaysian Insurance Institute [1]). Section 144 (Part XII) of Insurance Act 1996 states that no licensed general insurer shall adopt a tariff of premium rates, or a tariff of policy terms and conditions, for a description of a general policy which is obligatorily applicable to a licensed general insurer, except with the prior written approval of Bank Negara Malaysia (Malaysia [2]). Thus, implementation of statistical modelling on motor insurance pricing is rarely practiced by the domestic insurers.

Statistical pricing in non-life insurance system requires estimates made of two important elements; the probabilities associated with the occurrence of insured events namely claim frequency and the magnitude of such events namely claim severity. Estimates of claim frequency and severity are usually estimated through the use of past experience of groups of similar risk characteristics known as risk classification.

In addition to frequency and severity modelling, insurance pricing should also take into account several important considerations such as statistical distributional assumptions, marketing goals, competition, legal restrictions, financial constraints and economic environment. The output of pricing, which is generally presented in a premium table, is therefore insufficient in providing information on the feasibility of insurance price under alternative economic and marketing scenarios. The objective of this study is to suggest the use of profit analysis and simulation to assess the price of motor insurance.

Simulation is a technique involving random numbers and probabilities in solving problems. It is, more precisely, a method which iteratively evaluates a deterministic or stochastic model by using sets of random numbers as inputs. This method is often used when the model is complex, nonlinear, or involves more than just a couple of uncertain parameters. Historically, studies on simulation have been afforded a relatively fair attention in the actuarial literature. Although simulation method has been applied by actuaries in solving problem not soluble via traditional means, primary emphasis has been given upon non-simulation pricing and reserving procedures. Nevertheless, several studies in simulation have been carried out on non-life insurance businesses. For instance, Arata [3] discussed the application of Monte Carlo simulation on the evaluation of full credibility standards and the pricing of new or unique or catastrophic exposures. Herzog and Lord [4] explained the application of simulation on the operation of a two-stage model of property-casualty insurance. Fu and Moncher [5] applied Monte Carlo simulation to examine the unbiasedness and stability of the distributions of Gamma, Lognormal and Normal on severity classification relativities.

In the actuarial literature, profit analysis has been suggested for assessing the price of insurance. Studies in profit analysis include those of Coutts [6] who explained the application of profit analysis on one of the UK motor insurance businesses, and Goford [7] who described the implementation of profit analysis in practice and the use of profit analysis in controlling the operation of an insurer.

2 Premium Estimation

The estimation of premium is based on a sample of data of 170,000 private motor insurance policies issued by an insurer in Malaysia in 1998-2000 consisting of three claim types namely Own Damage (OD), Third Party Property Damage (TPPD) and Third Party Bodily Injury (TPBI). The data is provided by Insurance Services Malaysia Berhad (ISM).

The risk premium in each rating class is estimated as the product of expected claim frequency and severity (Brockman & Wright [8]; Renshaw [9]; Haberman & Renshaw [10]).

The claim frequencies are estimated using Poisson and Negative Binomial regression models whereby the dependent variable, independent variables and weight respectively are represented by the claim count, rating factors and exposure. The main advantage of comparing between the Poisson and the Negative Binomial is that the variance of Negative Binomial is larger than the mean and hence, allowing for overdispersion in the claim count data (Lawless [11]; Nelder & Lee [12]; Ismail & Jemain [13]; Ismail & Jemain [14]). Likelihood ratio tests are performed to choose a better model between the Poisson and the Negative Binomial. There are five rating factors to be considered for the claim frequency data:

- Scope of coverage (comprehensive and non-comprehensive)
- Vehicle make (local and foreign)
- Vehicle use and driver's gender (private-male, private-female and business)
- Vehicle year (0-1 year, 2-3 years, 4-5 years and 6+ years)
- Vehicle location (central, north, east, south and East Malaysia).

Altogether, there are $2 \times 2 \times 3 \times 4 \times 5 = 240$ cross-classified rating classes of claim counts to be fitted. The fitted claim counts are estimated using a log-linear function.

The claim costs are estimated using Gamma regression model whereby the dependent variable, independent variables and weight respectively are represented by the average claim cost, rating factors and claim count (McCullagh & Nelder [15]; Renshaw [9]; Ismail & Jemain [16]). The rating factors considered in the claim cost model are similar to the ones employed in the claim frequency model. The fitted claim costs are estimated using an inverse linear function (McCullagh & Nelder [15]).

Finally, the gross premium in each rating class is calculated as the sum of risk premium, fixed expense, variable expense, profit and contingency (Booth et al. [17]; McClenahan [18]).

3 Profit Analysis

3.1 Profit Analysis of Each Estimate

The basic equation for projecting future profit in each rating class may be written as,

$$\begin{aligned} \text{projected profit} &= \text{premiums} - \text{claims} - \text{expenses} \\ &= (1 - v)e_i \hat{G}_i - \sum_k e_{ik} \hat{f}_{ik} \hat{c}_{ik} - e_i F \end{aligned} \quad (1)$$

where $i = 1, 2, \dots, 240$ denotes the rating classes, $k = 1, 2, 3$ the claim categories, e_{ik} the assumed exposure, \hat{G}_i the estimated gross premium, \hat{f}_{ik} the estimated claim frequency, \hat{c}_{ik} the estimated claim cost, v the assumed variable expense factor and F the assumed fixed expense amount.

The difference between actual and projected profit may be calculated as,

$$\begin{aligned} \text{profit difference} &= (1 - v)e_i G_i - (1 - v)e_i \hat{G}_i - \\ &\quad \sum_k e_{ik} \hat{f}_{ik} \hat{c}_{ik} + \sum_k e_{ik} f_{ik} c_{ik} \end{aligned} \quad (2)$$

where the hat symbol, $\hat{}$, is used to differentiate between estimated and actual values.

The equation for profit difference may be derived directly from equation (2). As an example, the difference in profit due to the estimate of premium is equal to,

$$(G_i - \hat{G}_i)(1 - v)e_i \quad (3)$$

Table 1 shows the equations of profit difference due to the estimates of premium, frequency and cost.

Table 1 Equations of profit difference

Estimates	Profit difference	Fixed values
Premium	$(G_i - \hat{G}_i)(1 - v)e_i$	v, e_i
Claim frequency	$-\sum_k (f_{ik} - \hat{f}_{ik})e_{ik}c_{ik}$	e_{ik}, c_{ik}
Claim cost	$-\sum_k (c_{ik} - \hat{c}_{ik})e_{ik}f_{ik}$	e_{ik}, f_{ik}

Table 2 shows the results of profit difference (per policy) of each rating class. It should be noted that the information on actual premiums are also provided in the same data. Since the premiums for motor insurance in Malaysia are governed by the Motor Tariff, the premiums are not calculated based on statistical methods. The results in Table 2 indicate that profit analysis may be utilized to identify classes expected to produce positive profit difference (or gain) and negative profit difference (or loss). As an example, the estimated and actual premiums in the first rating class, $i = 1$, are

$\hat{G}_1 = \text{RM}1,458$ and $G_1 = \text{RM}1,102$. If the variable expense factor, ν , is assumed to be 9% of premium, the difference in profit (per policy) due to the estimate of premium is equal to

$$(\text{RM}1,102 - \text{RM}1,458)(1 - 0.09) = -\text{RM}324.$$

Table 2 Profit difference of each class

Rating class	Profit difference per policy (RM)		
	Estimated premium	Estimated frequency	Estimated cost
1	-324	-563	184
2	-664	-84	43
3	-581	-645	534
4	-467	-425	92
5	-136	63	92
6	-544	-479	-233
7	-750	-470	-80
8	-665	-535	85
9	-602	-482	-179
10	-460	-415	151
^	^	^	^
240	-39	0	0

Table 3 shows the total profit difference and the overall profit ratio between actual and projected profits. The estimates of premium indicate a total profit difference of -RM26.9 million and an overall profit ratio of 0.6050. Based on these figures, the actual premiums, i.e. the premiums that are mainly influenced by the Malaysian Motor Tariff, are significantly low in most classes and if the estimated premiums are to be corrected, the estimated premium in each rating class is suggested to be multiplied by a correction factor of 0.6050. The estimates of claim frequency and cost in Table 3 provide total profit differences and overall profit ratios of -RM24.1 million, -RM5.6 million, 1.3929 and 1.0754 respectively. The estimated claim frequency and cost in each rating class are suggested to be multiplied by the correction factors of 1.3929 and 1.0754 respectively.

3.2 Profit Analysis of Each Scenario

Profit analysis may also be carried out to investigate and select the scenario that produces the best projected profit. As an example, the scenarios considered are as follows:

- Scenario 1

The provision for fixed expense is lowered from RM95 to RM60 per policy, and the provision for variable expense is lowered from 9% to 7% of gross premium. This scenario is created to observe the movement of profit if the provision of expense is

lowered.

- Scenario 2

The counts in all claim types are estimated using Poisson regression model. This scenario is generated to study the effects on profit if the frequency model which allows for overdispersion, i.e. Negative Binomial model, is replaced by the Poisson model.

Table 3 Total profit difference and overall profit ratio

Estimates	Total profit difference (RM)	Overall profit ratio
Premium	$\sum_i (G_i - \hat{G}_i)(1 - \hat{\nu})e_i$ = -RM26,853,842	$\frac{\sum_i G_i(1 - \hat{\nu})e_i}{\sum_i \hat{G}_i(1 - \hat{\nu})e_i} = 0.6050$
Claim frequency	$-\sum_i \sum_k (f_{ik} - \hat{f}_{ik})e_{ik}c_{ik}$ = -RM24,051,188	$-\frac{\sum_i \sum_k f_{ik}e_{ik}c_{ik}}{\sum_i \sum_k \hat{f}_{ik}e_{ik}c_{ik}} = 1.3929$
Average claim cost	$-\sum_i \sum_k (c_{ik} - \hat{c}_{ik})e_{ik}f_{ik}$ = -RM5,554,459	$-\frac{\sum_i \sum_k c_{ik}e_{ik}f_{ik}}{\sum_i \sum_k \hat{c}_{ik}e_{ik}f_{ik}} = 1.0754$

Table 4 shows the results of profit analysis for Scenario 1 and Scenario 2.

Table 4 Profit analysis for several scenarios

Scenarios	Estimates	Profit difference & ratio	
Original scenario	Premium	Difference	-RM26,853,842
		Ratio	0.6050
	Frequency	Difference	-RM24,051,188
		Ratio	1.3929
	Cost	Difference	-RM5,554,459
		Ratio	1.0754
Scenario 1	Premium	Difference	-RM23,881,040
		Ratio	0.6377
		Performance	Better
	Frequency	Difference	-RM24,051,188
		Ratio	1.3929
		Performance	Same
	Cost	Difference	-RM5,554,459
		Ratio	1.0754
		Performance	Same
Scenario 2	Premium	Difference	-RM32,505,583
		Ratio	0.5586
		Performance	Worse
	Frequency	Difference	-RM17,402,699
		Ratio	1.2537
		Performance	Better
	Cost	Difference	-RM5,554,459
		Ratio	1.0754
		Performance	Same

The results of scenario 1, the scenario that shows a situation where the provisions for fixed and variable expenses are lowered, indicate that the estimates of premium provide an improved profit. In scenario 2, the counts in all claim types are estimated using Poisson regression model. Two contradictory results are provided; frequency estimates provide an improved profit whereas premium estimates provide a deteriorated profit.

4 Simulation

4.1 Simulation of Claim Frequency, Claim Cost and Profit Difference

This section extends the technique presented in the previous section by including considerations for the movement of premium and profit, and at the same time, allowing for random variabilities in the underlying distribution of claim frequency and cost models. In particular, simulations are applied to compare the projected profits if the claim counts are simulated based on Negative Binomial or Poisson distributions, and the claim costs are simulated based on Gamma distribution.

The probability of observing y_i claim count in the i th rating class for Negative Binomial distribution is (Lawless [13]),

$$\Pr(Y_i = y_i) = \frac{\Gamma(y_i + a^{-1})}{\Gamma(y_i + 1)\Gamma(a^{-1})} \left(\frac{1}{1 + a\lambda_i} \right)^{a^{-1}} \left(\frac{a\lambda_i}{1 + a\lambda_i} \right)^{y_i} \quad (4)$$

with mean $E(Y_i) = \lambda_i$ and variance $Var(Y_i) = \lambda_i(1 + a\lambda_i)$, where a denotes the dispersion parameter and $i = 1, 2, \dots, n$ the rating classes.

In the other hand, if the probability of observing y_i claim count in the i th rating class is Poisson distributed,

$$\Pr(Y_i = y_i) = \frac{\exp(-\lambda_i)\lambda_i^{y_i}}{y_i!} \quad (5)$$

where the mean and variance are equal to $E(Y_i) = Var(Y_i) = \lambda_i$. By comparing the variance of Poisson and Negative Binomial distributions, the variance for Negative Binomial distribution is larger than the mean and hence, allowing for overdispersion in the claim count data.

If the claim costs is assumed to be Gamma distributed, the probability density function of observing c_i claim cost in the i th rating class is (McCullagh & Nelder 1989),

$$h(c_i) = \frac{1}{c_i \Gamma(u)} \left(\frac{uc_i}{\mu_i} \right)^u \exp\left(-\frac{uc_i}{\mu_i}\right), \quad (6)$$

with mean $E(C_i) = \mu_i$ and variance $Var(C_i) = \mu_i^2 u^{-1}$, where u denotes the index parameter. If u is allowed to vary between classes, u_i may be written to be proportional to the claim count or weight, and inversely proportional to the variation coefficient of claim cost, $u_i = y_i \sigma^{-2}$. Therefore, the variance of Gamma distribution may be written as $Var(C_i) = \sigma^2 \mu_i^2 y_i^{-1}$ and the estimate of σ^2 may be determined by dividing the Pearson chi-squares with the degrees of freedom, $\hat{\sigma}^2 = \sum_i \frac{y_i (c_i - \mu_i)^2}{\mu_i^2 (n - p)}$, where n denotes the number of rating classes and p the number of regression parameters.

The claim count in the i th rating class and k th claim type is first simulated using Negative Binomial or Poisson distributions. If the claim count is simulated based on Poisson, the mean from fitting Poisson regression model on claim count data is used as input parameter. If the claim count is simulated based on Negative Binomial, the mean and the estimated dispersion parameter from fitting Negative Binomial regression model on claim count data are used as input parameters. Once the simulated claim count is obtained, the claim frequency is calculated by dividing the simulated claim count with the exposure.

After simulating the claim count, the claim costs are then simulated based on Gamma distribution. The mean and the estimated index parameter from fitting Gamma regression model on claim cost data are used as input parameters. The average claim cost is then calculated by dividing the total claim costs with the claim count.

The simulations of claim count and costs are repeated for each claim type, producing simulated frequencies and average costs of all claim types. The simulated values are then applied to calculate the premium.

The profit difference in the i th rating class is calculated by applying the equations of profit difference shown in Table 1, replacing estimated values with simulated values. The same procedure is then repeated for 10,000 times, producing 10,000 simulated values of claim frequencies, claim costs, premiums and profit differences. The mean and quantiles are calculated for further interpretation and discussion.

The same procedure may also be repeated for all rating classes, $i = 1, 2, \dots, 240$. If the simulation procedure is to be carried out using computer programming, two loops are required to complete the simulation of all classes; the first should loop for each

simulation trial and the second should loop for each rating class.

Two sets of simulations are carried out using SPLUS programming. The first involves the simulation of 10,000 trials of TPPD, TPBI and OD claim counts based on Negative Binomial, Poisson and Negative Binomial distributions respectively. Individual claim costs from Gamma distribution are also simulated in each trial. The second set is similar to the first, except that the counts for all claim types are simulated based on Poisson distribution.

The details of simulated scenarios are reported for the following two rating classes:

- Class 1

Class 1 refers to the rating class of $i=1$ representing the intercept of the regression model. In particular, the class consists of policyholders with comprehensive coverage, local vehicle, private use-male driver, 0-1 year old vehicle and central location.

- Class 2

Class 2 refers to the rating class of $i=76$ representing the class with large exposures. In particular, the class consists of policyholders with comprehensive coverage, foreign vehicle, private use-male driver, 6+ year old vehicle and central location.

4.2 Simulated Profits in Class 1

Table 5 shows the mean and quantiles of the simulated frequencies, costs, premiums and profits of class 1. The result shows that the premiums and profits are sensitive to the changes in claim frequency distribution. The choice of an appropriate set of distributions is of course subjective to the insurer. If the claim frequencies are simulated based on Negative Binomial distribution, the 90% confidence intervals of profit difference due to the premium and frequency assumptions are (-RM855, RM39) and (-RM848, RM64) respectively. The 90% confidence intervals are based on 5% and 95% quantiles of the distributions. On the contrary, if the claim frequencies are simulated based on Poisson distribution, the 90% confidence intervals of profit difference due to the same assumptions are (-RM941, -RM581) and (-RM339, -RM32) respectively. The quantiles indicate that the Poisson distribution provides improved frequencies but deteriorated premiums.

4.3 Simulated Profits in Class 2

Table 6 shows the mean and quantiles of the simulated frequencies, costs, premiums and profits of class 2. The result indicates that the premiums and profits are also

sensitive to the changes in claim frequency distribution. If the claim frequencies are simulated based on Negative Binomial distribution, the 90% confidence intervals of profit difference due to the premium and frequency assumptions are (-RM1481, -RM271) and (-RM1074, -RM717) respectively. On the contrary, if the claim frequencies are simulated based on Poisson distribution, the 90% confidence intervals of profit difference due to the same assumptions are (-RM746, RM847) and (-RM112, RM226) respectively. The quantiles also indicate that the Poisson distribution provides improved frequencies but deteriorated premiums. However, the magnitudes of improvement in frequencies and deterioration in premiums are larger in class 2 compared to class 1.

What conclusions can be drawn from the results of the simulated mean and quantiles in class 1 and class 2? Sanchez [19] suggested that simulation can be treated as a method for validating new processes or procedures. In this study, we wish to simulate profits by taking into account the random variabilities in the underlying distributions of claim frequency model. The results in class 1 and class 2 provide the average simulated values of claim frequency, claim cost, premium and profit difference, and also the likely or probable range of the output values, i.e. the quantiles. The lower and upper quantiles can be used as indicators by the actuaries to investigate the outcome of profit on the extent of the risks or distributions involved.

5 Conclusion

This study suggests the use of profit analysis and simulation for assessing the price of motor insurance. The equations for assessing profit difference of each rating class are shown in Table 1 whereas the equations for assessing total profit difference and overall profit ratio are shown in Table 2. The results of total profit difference and overall profit ratio on the Malaysian data show that the frequencies and costs are slightly underestimated, whereas the actual premiums are significantly low for most rating classes.

Profit analysis is also carried out to investigate and select the scenario that produces the best projected profit for the Malaysian data. If the provisions for fixed and variable expenses are lowered, the result of profit analysis indicates that the premium estimates are improved. If the claim frequencies are fitted using Poisson distribution, the frequency estimates are improved but the premium estimates are worsen. In terms of magnitude, the deterioration of premium estimates and the improvement of frequency estimates are almost similar.

Finally, the modeling of premium and profit are extended to allow for random variabilities in the underlying distribution of the pricing model. In particular, simulations are applied to compare the projected profits if the claim counts are simulated based on Negative Binomial and Poisson distributions, and the claim costs are simulated based on Gamma distribution. The details of simulated results for the Malaysian data are reported for two rating classes; class 1 which represents the intercept of the regression model, and class 2 which represents the class with large exposures. The mean and quantiles of both classes indicate that the Negative Binomial distribution resulted in improved premiums and deteriorated frequencies.

In most insurance cases, the event of losses or claims involve unknown parameters such as claim frequency and cost which are actually random variables whose value cannot be predicted, i.e. the models are stochastic. If the actuaries are interested to determine how sensitive the premium or the profit is to the variations in these parameters, simulation is one of the techniques that can be used for this purpose. As an example, this study shows that the premiums and profits in class 1 and class 2 are sensitive to the distribution of claim frequency. In addition, the simulation result shows that the premiums are deteriorated and the frequencies are improved if the claim counts are simulated using Poisson distribution. The mean and quantiles resulted from simulation in class 1 and class 2 do not only provide the average values but also the likely or probable range of the output values. The lower and upper quantiles can be used by the actuaries to investigate the outcome of profit on the extent of the risks or distributions involved.

It should be noted that the implementation of a more thorough and comprehensive profit analysis and simulation requires the fullest support and cooperation of each department of the insurer. Initially, the management has to determine the insurer's objectives, whether to obtain the best profit in a fixed period of months or years, or to accomplish a targeted volume of sale, or to lower the operational or management costs, and etc. The equation suggested for profit analysis in this study is fairly flexible. If there exist any additional or new or latest information that need to be taken into account in the insurer's profitability, additional parameters representing such information may be inserted.

Acknowledgements

The authors gratefully acknowledge the financial support received in the form of a research grant (Code: UKM-GUP-TMK-07-02-107) from the Ministry of Higher Education, Malaysia. The authors also thank the Insurance Services Malaysia for the data.

References:

- [1] Malaysian Insurance Institute, *The pre contract examination for insurance agents (study course)*, 7th edition, The Malaysian Insurance Institute, Kuala Lumpur, 2001.
- [2] Malaysia, *Insurance Act 1996 (Act 553) and Regulations and Orders (As at 25th July 2001)*, 2001.
- [3] Arata, D.A., Computer simulation and the actuary: a study in realizable potential. *Proceedings of the Casualty Actuarial Society*, Vol.68, 1981, No.129-130, pp.24-64.
- [4] Herzog, T.N. & Lord, G., Applications of simulation models in finance and insurance, *Proceedings of the 2003 Winter Simulation Conference*, 2003, pp.249-257.
- [5] Fu, L. & Moncher, R.B., Severity distributions for GLMs: Gamma or Lognormal? Evidence from Monte Carlo simulations, *Casualty Actuarial Society Discussion Paper Program*, 2004, pp.149-230.
- [6] Coutts, S.M., Motor insurance rating, an actuarial approach, *Journal of the Institute of Actuaries*, Vol.111, 1984, pp.87-148.
- [7] Goford, H., The control cycle, *Journal of the Institute of Actuaries Students' Society (SIAS)*, Vol.28, 1985, pp.99-114.
- [8] Brockman, M.H. & Wright, T.S., Statistical motor rating: making effective use of your data, *Journal of the Institute of Actuaries*, Vol.119, No.3, 1992, pp.457-543.
- [9] Renshaw, A.E., Modelling the claims process in the presence of covariates, *ASTIN Bulletin*, Vol.24, No.2, 1994, pp.265-285.
- [10] Haberman, S. & Renshaw, A.E., Generalized linear models and actuarial science, *The Statistician*, Vol.45, No.4, 1996, pp.407-436.
- [11] Lawless, J.F., Negative Binomial and mixed Poisson regression, *The Canadian Journal of Statistics*, Vol.15, No.3, 1984, pp.209-225.
- [12] Nelder, J.A. & Lee, Y., Likelihood, quasi-likelihood and pseudolikelihood: some comparisons, *Journal of the Royal Statistical Society (B)*, Vol.54, No.1, 1992, pp.273-284.
- [13] Ismail, N. & Jemain, A.A., Handling overdispersion with Negative Binomial and Generalized Poisson regression models, *Casualty Actuarial Society Forum*, Winter, 2007, pp.103-158.
- [14] Ismail, N. & Jemain, A.A., Rating factors identification using claim frequency approach: the Malaysian experience, *ICFAI Journal of Applied Economics*, Vol.6, No.2, 2007, pp.60-77.
- [15] McCullagh, P. & Nelder, J.A., *Generalized Linear Models*, Second edition, Chapman and Hall, London, 1989.

- [16] Ismail, N. & Jemain, A.A., A comparison of risk classification methods for claim severity data, *Journal of Modern Applied Statistical Methods*, Vol.5, No.2, 2006, pp.513-528.
- [17] Booth, P., Chadburn, R., Cooper, D., Haberman, S. & James, D., *Modern actuarial theory and practice*. Chapman and Hall, London, 1999.
- [18] McClenahan, C.L., Ratemaking, In Bass, I.K., Basson, S.D., Basline, D.T., Chazit, L.E., Gillam, W.R. & Lotkowski, E.P., *Foundations of casualty actuarial science*, pp.25-90, R&S Financial Printing, New York, 1990.
- [19] Sanchez, P.J., As simple as possible but not simpler. A gentle introduction to simulation modeling, *Proceedings of the 2006 Winter Simulation Conference*, 2006, pp.2-10.