Abstract: Euler rotation series is determined to describe the rotation of the shoulder joint which can be represented through direction cosines tables used to construct the kinematic equation of shoulder movement.

Key-Words: Euler rotation, Direction cosine, Shoulder joint.

1 Introduction
Many body movements involve turning and rotating motions. All linear movements originated from the lever actions of joint articulations. The shoulder is one of the most mobile joints in a human body and moves in a complex 3-dimensional pattern. Shoulder joint is divided into four separate articulations namely glenohumeral joint, sternoclavicular joint, acromioclavicular joint and coracoclavicular joint. The glenohumeral joint is an articulation between the humerus head and the glenoid fossa of the scapula. It is typically considered as the major part of the shoulder joint. It is called a ball-socket joint and it has three rotations and no translation in terms of movement. Clinically, the shoulder motions are defined as flexion/extension, abduction/adduction, horizontal abduction/adduction and external rotation/internal rotation. The first three rotations are not independent as only two of them are needed to determine the rotation of humerus about its long axis. Shoulder abduction can be defined as the angle between the humerus and the inferior direction of the trunk in the trunk’s frontal plane. The shoulder horizontal abduction can be defined as the angle between the humerus and a line connecting two shoulder marker in the trunk’s transverse plane while shoulder external rotation can be described as rotation of the upper arm about its own long axis [2].

Any desired orientation of a body can be obtained by performing rotations about three axes in sequence. There is, however, a multitude of possible ways of performing three such rotations. The famous mathematician Euler recognized that a series of three rotations could be used to uniquely define the orientation of a rigid body in three-dimensional space. Though there are many sets of three sequential rotations which lead to a unique orientation of a rigid body, the most commonly used are Euler rotations [6]. With Euler rotations, the axes of rotation are fixed in the object to be rotated. This means that as the object rotates, the orthogonal axes of rotation also rotate. Therefore, the objective of the current study is to determine the Euler rotation series on shoulder joint which then used to construct the kinematic equation for the shoulder joint.

2 Literature Review
A few researches have been done either in biomechanics field or other fields using Euler angle rotations in determining orientation of an object. Allen & Hahn performed a study in determining the kinematic and joint coordination changes necessary to cast lines of different length. Documenting the movement patterns needed to cast varying lengths of line should allow for greater understanding of the underlying mechanisms of upper extremity pathologies. It was hypothesized that kinematic casting parameters would increase in order to cast lines of
different length, and that time between peak joint velocities would decrease as length of line increased. The shoulder motion was calculated with respect to the trunk using Euler angle rotation in sagittal, frontal and transverse plane [1]. A recent study performed by McCue et al. Revealed that shoulder and elbow pathologies associated with repetitive, high velocity, overhand movements common to fly-casting [3].

Another research on Euler angle rotation, which focused on satellite orientation, has been done by Ramachandran. He determined two-axes Euler rotation on satellite orientation that is usually expressed in terms of Euler rotation about the three principle axes. According to him, the orientation is determined using the measurement of at least two known vectors. The Euler rotations are with respect to the three principal axes namely $x$, $y$, and $z$. This is used in control of the satellite where the known vectors are observed by a vector sensor like star sensor. He has constructed the algorithm from the mathematical derivation of Euler rotation by knowing only one pair of vectors and one angle. The input vector may be from the sun-sensor or vector-magnetometer while one axis rotation may be available from an earth sensor. This algorithm enables the knowledge of the three axes rotation to control the satellite [5].

Mustafa et al. studied a biologically-inspired anthropocentric shoulder joint rehabilitator: workspace analysis & optimization. They presented the design of a biologically inspired anthropocentric 7-DOF wearable robotic arm for the purpose of stroke rehabilitation. The proposed arm rehabilitator utilizes the human arm structure to combine with under deterministic cable-driven parallel mechanisms so as to form completely-deterministic structures. It adopted an anthropocentric design concept, thereby offering the advantages of being lightweight, having high dexterity and conforming to the human anatomical structure. The paper mainly focused on the workspace analysis of the 3-DOF shoulder module, with respect to the shoulder joint motion range. Workspace parameterization is based on a modified Z-Y-Z Euler Angles approach and utilizing cylindrical coordinates to determine the workspace volume, while workspace evaluation is carried out based on the shoulder joint motion limits and the cable-tension analysis. An effective cable-tension analysis method was also proposed based on the duality between force-closed multi-fingered grasping and cable-driven mechanisms. Finally, the workspace of the mechanism was optimized to match with that of the human shoulder and to obtain a set of dimensions for the shoulder module prototype development [4].

3 Mathematical Formulation

The first step to model a movement of a shoulder is to construct direction cosines tables which then used to construct the kinematic equation. The direction cosine table for the shoulder is constructed based on the Euler rotation series that determine the orientation of a rigid body in three-dimensional space. The rotation of the humerus relative to a fixed scapula is considered as shown in Figure 1.
Initially, the scapula reference frame N is considered to be coincident with the right humeral reference frame A, oriented such that $\hat{a}_1$ points anteriorly, $\hat{a}_2$ points laterally and $\hat{a}_3$ points inferiorly. This orientation is chosen because the resulting Euler rotations will roughly correspond to physiological rotation angles. Each rotation is a simple rotation, and brings the N frame into an intermediate orientation. Figure 1b shows the scapula in its final orientation after the three rotations have been performed. These three rotations are described in sequential order through the representation of direction cosines tables (i, ii, iii) below.

(i) Rotation about the common $\hat{n}_1 \hat{a}_1$, axis.

Table 1. Direction cosines table for reference frame N with respect to first orientation A'.

<table>
<thead>
<tr>
<th>$N_{R} A'$</th>
<th>$\hat{a}_1'$</th>
<th>$\hat{a}_2'$</th>
<th>$\hat{a}_3'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{n}_1$</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\hat{n}_2$</td>
<td>0</td>
<td>$c_1$</td>
<td>$-s_1$</td>
</tr>
<tr>
<td>$\hat{n}_3$</td>
<td>0</td>
<td>$s_1$</td>
<td>$c_1$</td>
</tr>
</tbody>
</table>
(ii) Rotation about the common \( \hat{a}_2 \hat{a}_2 \), axis.

Table 2. Direction cosines table for orientation \( A' \) with respect to second orientation \( A'' \).

\[
\begin{array}{c|ccc}
A' \rightarrow A'' & \hat{a}_1' & \hat{a}_2' & \hat{a}_3' \\
\hat{a}_1' & c_2 & 0 & s_2 \\
\hat{a}_2' & 0 & 1 & 0 \\
\hat{a}_3' & -s_2 & 0 & c_2 \\
\end{array}
\]

(iii) Rotation about the common \( \hat{a}_3 \hat{a}_3 \), axis.

Table 3. Direction cosines table for orientation \( A'' \) with respect to humerus orientation \( A \).

\[
\begin{array}{c|ccc}
A'' \rightarrow A & \hat{a}_1'' & \hat{a}_2'' & \hat{a}_3'' \\
\hat{a}_1'' & c_3 & -s_3 & 0 \\
\hat{a}_2'' & s_3 & c_3 & 0 \\
\hat{a}_3'' & 0 & 0 & 1 \\
\end{array}
\]

The table of direction cosines relating the scapula (N) and humerus (A) reference frames is obtained most simply via matrix multiplication,

\[ N \rightarrow A = (N \rightarrow A')(A' \rightarrow A'')(A'' \rightarrow A) \]  \hspace{1cm} (1)

which yields the rotation matrix \( N \rightarrow A \) and its corresponding table of direction cosines, Table 4.

Table 4. Direction cosines table for reference frame of scapula (N) with respect to humerus (A).

\[
\begin{array}{c|ccc}
N \rightarrow A & \hat{a}_1 & \hat{a}_2 & \hat{a}_3 \\
\hat{n}_1 & c_2 c_3 & -c_2 c_3 & s_2 \\
\hat{n}_2 & s_1 s_2 c_3 + c_1 c_3 & -s_1 s_2 s_3 + c_1 c_3 & -s_1 c_2 \\
\hat{n}_3 & -c_1 s_2 c_3 + s_1 s_3 & c_1 s_2 s_3 + s_1 c_3 & c_1 c_2 \\
\end{array}
\]

4 Conclusions

Direction cosines for virtually any compound rotation can be found easily using Euler rotation series. One needs only to define the sequence of simple rotations comprising the compound rotation and to perform the matrix multiplication correctly.

References: