Musical Microtonal Scales based in the quantitative measurement of the sensory dissonance of Sethares

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Abstract: - William A. Sethares of the University of Wisconsin in Madison, has developed a quantitative theory of the dissonance from the experimental curves of dissonance/consonance for two sinusoidal sounds of different frequency obtained in the sixties from R. Plomp and W. J. M. Levelt. The theory proposes that the intervals more consonants for a given instrument depend on the structure of his timbre. This document explains how you can capture a correct timbre with a computer then to apply the dissonance algorithm to create a musical microtonal scale with criteria.

Key words: Musical Microtonal Scales, Pitch, Consonance, Dissonance, Timbre, Spectrum.

1. Introduction

The Pitch is related to the frequency of a simple tone (sinusoidal) and to the fundamental frequency of a complex tone (when the harmonics happens to frequencies that they aren't multiple of the fundamental frequency). The physical correlate that underlies the loudness of a tone is intensity, usually expressed as sound pressure level in decibels. Timbre is, after pitch and loudness, the third attribute of the subjective experience of musical tones [1]. Subjectively, timbre is often coded as the function of the sound source or of the meaning of the sound.

The simultaneous sounding of several tones may be pleasant or euphonious to various degrees. The pleasant sound is called consonant; the unpleasant or rough one, dissonant. Consonance is perceived when two harmonic sounds having one or more component frequencies in common are sounded together. Perception of dissonance is caused by near misses between frequency components of the two sounds, producing rapid but perceptible beating [2].

The terms consonance and dissonance have been used here in a perceptual or sensory sense. Musical consonance has its root in perceptual consonance, of course, but is dependent on the rules of music theory, which, to a certain extent, can operate independently from perception.

The consonance is based on an explicit parameterization of Plomp and Levelt's consonance curves. It explains the relationship between spectrums of a sound (its timbre) and a tuning (or scale) in which the timbre will appear most consonant.

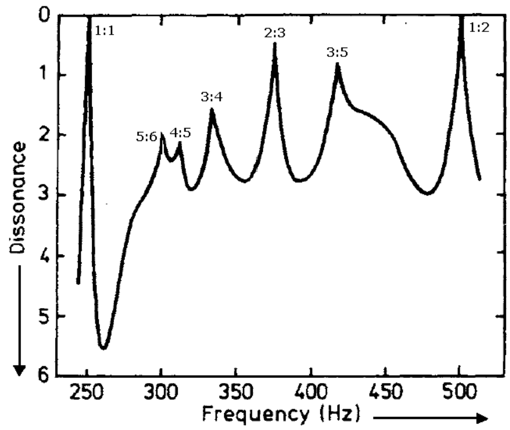


Fig.1 Consonance of an interval consisting of two complex tones (with six harmonics). The lower tone (D has a fundamental frequency of 250 Hz; the fundamental frequency of the higher tone is the variable along the horizontal axis. The consonance /dissonance values are predictions from the model of Plomp and Levelt (1965) Reproduced from [1].

Plomp and Levelt examined consonance experimentally, by generating pairs of sine waves and asking volunteers to rate them in terms of their relative consonance [3]. Despite considerable variability among responses, there was a simple and clear trend. At unison, the consonance was at maximum. As the interval increased, it was judged less and less consonant until at some point a minimum was reached. After this, the consonance increased up towards, but never quite reached the consonance of the unison.

This relationship is defined in terms of the local minima of a family of dissonance curves. Sethares points out that, for certain timbres with simple spectral configurations, dissonance curves can be completely characterized, and bounds are provided on the number and location of points of local consonance [4]. Computational techniques are used for how finding microtonal scales for harmonic or non-harmonic timbres.

2. Algorithm

The goal is how should be processed the input signal in the Continuous Time for search the dissonance curve in the Discrete Time and finally to program the MIDI with the microtonal scale.

Timbre has two recommendations: first, if the timbre is very simple, the spectrum has a dissonance curve with modest minimum points and second, if the timbre is very complex, the spectrum should have too much partial to work with, it's impractical.

Before validation of the resolution, it has to be caring taken, that the input signal doesn't have clip, although it can be normalized. The objective is to work with all the signal information without distortion.

For the resolution validation, we need three parameters: the fundamental frequency, the sample frequency and the length of the signal.

The fundamental frequency is looked in agreement to two procedures, the autocorrelation method (2) and the power spectral method (1), being the sample frequency 44.1 KHz.

Power Spectral Density

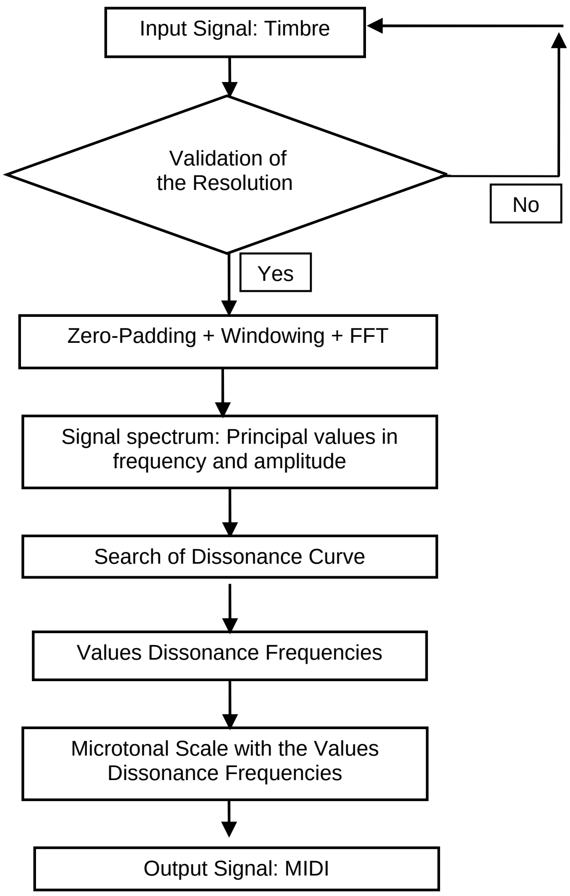
$$K_x^F = \int_{-\infty}^{\infty} Kx1(\mathrm{T}) e^{-j\omega \mathrm{T}} \delta \mathrm{T} \qquad \textbf{(1)}$$

Autocorrelation

$$r_{11}(l) = \sum_{n=-\infty}^{\infty} x_1(n) x_1(n-l) \qquad \textbf{(2)}$$

The best resolution is the one that gets closer to 0.01, it's just a comma, a small difference that the ear can appreciate between two sounds. It's the ninth part of the halftone. The value of the comma temperate is 1.0129 [5], the value of the halftone is $2^{1/12}$ = 10.6, and (1 halftone)$^{1/9}$ = 1.0129.

To increase the frequency resolution for spectrum analysis, we simply take more samples for the FFT algorithm with the Zero-Padding. There is a remark: The FFT requires that the number of samples be a power of two or some highly composite number.



The windowing is done with Hamming window. It's used with the purpose of avoiding discontinuities of high frequency components of signal's spectrum, which were introduced when analyzing a fraction of the signal or when samples with value zero have been added.

Hamming Window

$$\omega_B(n) = 0.42 - 0.5\cos(2\Pi n / N) + 0.08\cos(4\Pi n / N),$$

$$\omega_H(n) = 0.54\cos(2\Pi n / N)$$

$$n = 0,1,\ldots, N-1$$

$$N \rightarrow samples\ by\ the\ window\ \ function$$

$$x_\omega = w(n).x(n)\ with \quad 0 \le n \le N-1$$

Spectrum analysis via the Fast Fourier Transform (FFT) algorithm is used for locate the most representative values of the harmonics in the processed signal.

DFT

$$x(k) = DFT\left[x(n)\right] = \sum_{n=0}^{N-1} x(n) e^{-j2\Pi nk/N}$$

$$k = 0,1,\ldots, N-1$$

To explain perceptions of musical intervals, Plomp and Levelt note that most traditional musical tones have a spectrum consisting of a root or fundamental frequency, and a series of sine wave partials that occur at integer multiples of the fundamental.
If the timbre is sounded at various intervals, the dissonance of the intervals can be calculated by adding up all of the dissonances between all pairs of partials. Carrying out this calculation for a range of intervals leads to the dissonance curve.
The Plomp and Levelt curves of Fig. 2 can be conveniently parameterized by a model of the form

$$d(x) = e^{-ax} - e^{-bx}$$

Where x represents the difference in frequency between two sinusoids, and a y b determine the rates at which the function rises and falls. Using a gradient minimization of the squared error between the (average) data and the curve $d(x)$ gives values of a and b. The dissonance function $d(x)$ can also be scaled so that the curves for different base frequencies and with different amplitudes are represented conveniently. If the point of maximum dissonance occurs at d*, the dissonance between sinusoids at frequency f_1 with amplitude v_1 and at frequency f_2 with amplitude v_2 (for $f_1 < f_2$) is

$$d(f_1, f_2, v_1, v_2) = v_{12}(e^{-as(f_2 - f_1)} - e^{-bs(f_2 - f_1)})$$

where

$$s = d^*(s_1 f_1 + s_2)$$

$$v_{12} = v_1 v_2$$

More generally, a (complex) timbre F with base frequency f_1 is a collection of n sine waves with frequencies $f_1 < f_2 <, \ldots, < f_n$ and amplitudes v_j for $j=1,2,\ldots,n$. The dissonance of any timbre F can be calculated as the sum of the dissonances of all pairs of partials

$$D_F = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} d(f_i, f_j, v_i, v_j)$$

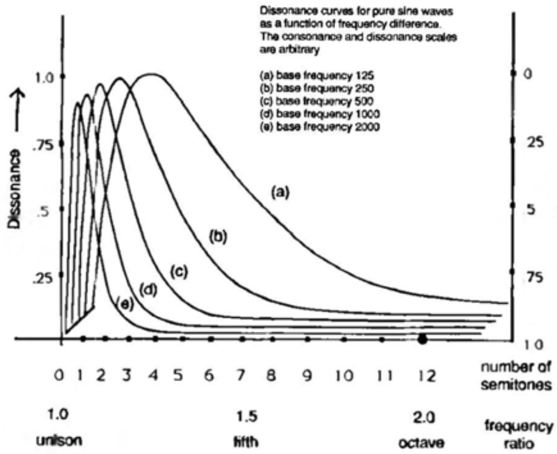


Fig.2 Dissonance curves for pure sine waves as a function of frequency difference. The consonance and dissonance scales are arbitraries. Reproduced from [4]

For more information, this page is recommended
http://eceserv0.ece.wisc.edu/~sethares/prelude.html

3. The Sethares Method

The principle of local consonance describes a relationship between the timbre of a sound and a tuning (or scale) in which the timbre will sound most consonant [4]. The goal is, given an arbitrary timbre T, that we can draw the dissonance curve generated by T. Their local minimums happen on the values that are good candidates for notes of the scale = local points of minimal dissonance = maximum consonance, therefore timbre generates a dissonance curve in which the local minimal happen in positions of the scale.

The perceptual consonance of an interval consisting of two simple tones depends directly on the frequency difference between the tones, not on the frequency ratio (or musical interval). If the frequency separation is very small, less than a semitone, or larger than critical bandwidth, the two tones together sound consonant.

In the first case, tones fuse to a single one; in the second case, tones do not interfere with each other. Dissonance occurs if the frequency separation is less than a critical bandwidth.

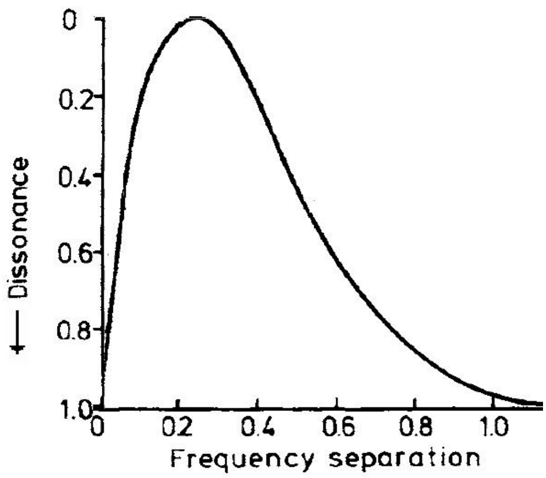


Fig.3 Consonance of an interval consisting of two simple tones as a function of frequency separation, measured relative to critical bandwidth. (Based on Plomp & Levelt, 1965) Reproduced from [3].

The most dissonant interval arises with a separation of about a quarter of the critical bandwidth: about 20 Hz in low-frequency regions, about 4% (a little less than a semitone) in the higher regions (Figure 1). The frequency separation of the minor third (20%), major third (25%), fourth (33%), fifth (50%), and so on, is usually enough to give consonant combination of simple tones. However, if the frequencies are low, the frequency separation of thirds (and eventually also fifths) is less than critical bandwidth so that even these intervals cause a dissonant beating. For this reason, these consonant intervals are not used in the bass register in musical compositions.

Basically we are looking for encapsulation of the Plomp and Levelt curve in pure sine waves and the total dissonance is the sum of all the individual dissonances of each possible combination of them.

For the trivial case: a spectrum with 3 harmonic, the dissonance is calculated for two harmonics every time, until finishing with all, where each proportional dissonance to the amplitude of each of harmonic couples considered and average by the curve of experimental dissonance of agreement to the frequency relative separation of both partial ones under consideration.

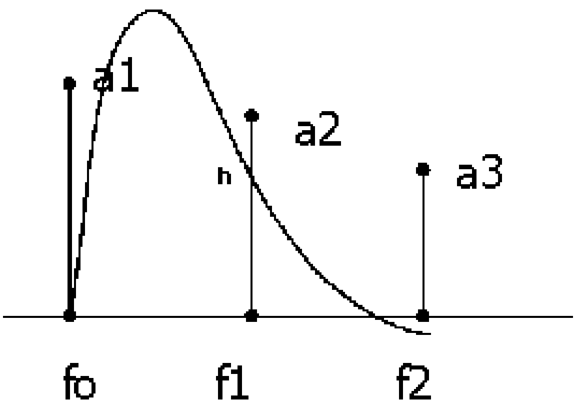


Fig.4 Calculation of the dissonance for harmonic a1 and a2

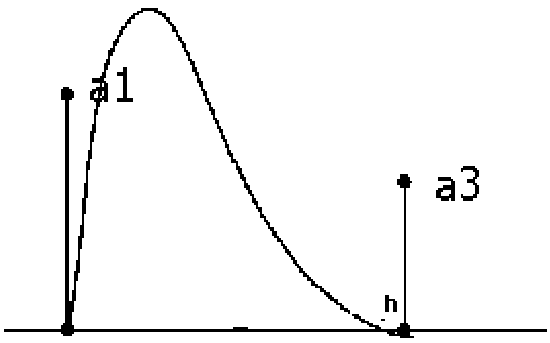


Fig.5 Calculation of the dissonance for harmonic a1 and a3

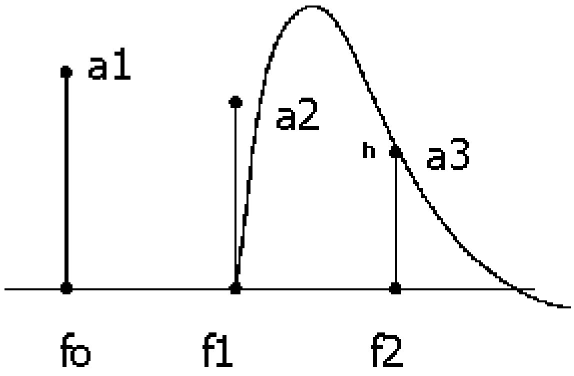


Fig. 6 Calculation of the dissonance for harmonic a2 and a3

The total dissonance is the sum of all possible dissonances between all harmonics.

The kind of music which is called tonal, appears to prove that the human auditory system possesses a sense for certain special frequency tone intervals. These particular intervals are usually called musical or harmonic intervals. They are described by frequency ratios of small integers as 1: 2 (octave), 2: 3 (fifth), 3: 4 (fourth). [6]

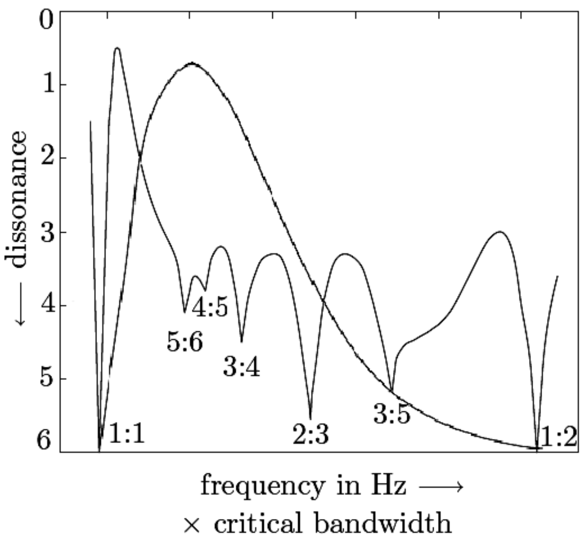


Fig.7 Dissonance Graphic for a classic instrument. The minimums exactly do not happen to passages of scale of 12 tones of the *temperada* scale but in "the next" frequency ratios 1:1, 2:1, 3:2, 4:3, 5:4, 5:3 respectively, which are the locations of notes in the intoned scale right. Reproduced from [4]

4. Experiments

The algorithm interface was created with MATLAB, where the user chooses dissonance points (ginput function) from which he wants to form his scale.

Fig. 8 Dissonance Curve for a bell

With the dissonance points chosen (frequency ratio), the developed software converts from frequency ratios of r:1 to cents, where the value in cents is:

$$1200\log_2(r) = 1200\ln(r)/\ln(2)$$

For figure 8, frequency ratios: 1.3599, 1.5394, 1.8227, 2.1004, 2.3423 and 2.4707, and frequencies are: 598.356, 677.336; 801.988, 924.17, 1030.612 and 1087,108 respectively.

If the fundamental frequency for the scale is LA (440 Hz), the ultimate scale is: La, Sib + 32 cents (fourth ratio), Do + 21 cents (fifth ratio), Do# + 37 cents (seventh ratio), Re -15 cents (ninth Bemol ratio), Mi -26 Cents (tenth minor ratio) and Sol -34 Cents (tenth major ratio).

With the microtonal scale, finally the MIDI programming is done and the scale can be divided within the keyboard, deactivating or activating keys in order to form the amount of new microtonal scales inside the keyboard.

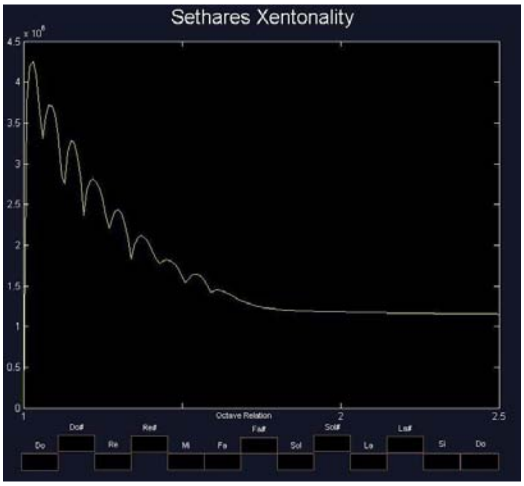


Fig. 9 Dissonance Curve for a Resonance of piano

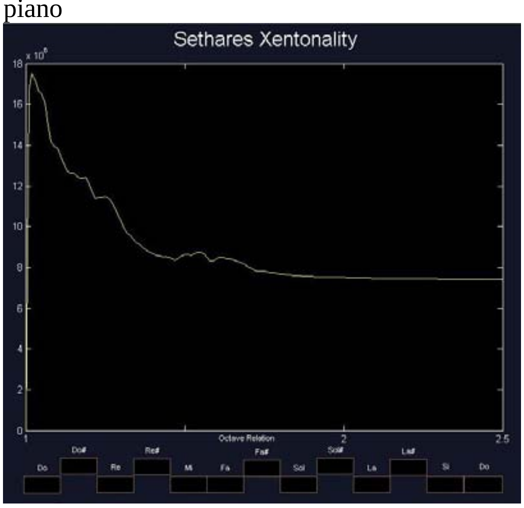


Fig. 10 Dissonance Curve for a distortion guitar

5. Conclusions

The dissonance measure ability is a critical component in different audio devices and accurate methods of music analysis.

Dissonance is a timbre sound function as source or interval music that has important implications to understand: western music, atonal music, experimental compositions and the design of electronic musical instruments.

The composition should do it with xenharmonics scales; the composer has a new composition form and new criterion when he decides the microtonal scale.

Parameter frequency elements generation tool allows an appropriate relation between the sound element and the generational theory element: a direct relation amid mind creation and the real sound.

Dissonance creates a new space of composition because the composer must not necessarily use western scales.

It predicts that timbre classes are appropriate in a given musical context and that this musical context is appropriate for a given timbre.

6. Future work

Our tasks include: improving the interface in combination with actual composers, optimizing detection algorithms of harmonics and fundamental frequency measurement, and programming an ample range of electronic instruments.

ACKNOLEDGMENT

The author wishes to thank Prof. Carlos Edmundo Murillo Sanchez, Ph.D (Cornell University (U.S.A.) and - Autónoma University (Manizales - Colombia) for all the support in this subject and for the dedicated engineering and music education.

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