Stabilization of the Fuzzy Control Systems, with Application at the Second Order Systems

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Abstract: - The paper presents a method to assure stability of the fuzzy control systems of the second order processes, based on a compensation of the input-output characteristic of the fuzzy blocks used in the PI fuzzy controllers. For this purpose is acting at the correction of the fuzzy control law by the action in parallel of a crisp block. Stability analysis of the control systems of the second order systems with fuzzy controllers is framed in the field of non-linear systems. The paper presents the way to prove the absolute stability in such a situation. The correction principle is theoretically substantiated and methodologically structured with application at the fuzzy control of the second order processes.

Key-Words: - Fuzzy control systems, control of second order processes, absolute internal stability, non-linear systems.

1 Introduction
1.1 Generalities
Generally, the literature "spares" the stability analysis of the control systems with fuzzy controllers. Even if there are a number of main papers [1, 2, 3, 4, 5, 6, 7] on this theme, in the most cases the remake of the results is very difficult. In the concrete cases the difficulties increase. This paper is based on a study made by the author [8], [9] and, in principal, it systematises the theoretical aspects for the problem of the fuzzy control of the second order processes. The term quasi-fuzzy is used for a hybrid fuzzy-crisp structure. The situation is reduced at a known case [2], for what a theoretical substantiation, based on the non-linear characteristic of the fuzzy block and circle's criterion is developed.

1.2 The Conventional Fuzzy Controller
The structure of the fuzzy speed control system is presented in fig. 1. A classical PI fuzzy controller with output integration is used. The incremental fuzzy controller RF is characterised by the utilisation of a fuzzy block BF, which has as input variables the control error e and its derivative de, and as output variable the increment uf of the control variable u. The fuzzy block BF has the classical structure with fuzzyfication, inference on a rule base and defuzzyfication. Different rule bases, inference methods and defuzzyfication may be used.

The controller commands a process P, and w represents the reference input of the control system and y the output.

Fig. 1 The diagram of a conventional fuzzy control system of a second order process

The fuzzy block BF is a non-linear element, non-inertial, with two inputs and one output. Some input-output characteristics may be used to describe the fuzzy block. To achieve system stabilisation the non-linear characteristic of the fuzzy block, transformed in a convenient manner, must fulfilled some sector conditions. Generally, the fuzzy block BF does not satisfy the sector conditions, (for example, due of the integrator block).

2 The Correction of the Fuzzy Block
The situation may be remedied using a correction block BC, which implements a crisp dependence uc(e,de,e,de) (fig. 2). This block, included in the new controller RFC, leads to a controller whose output variable uf has two components, one of them is fuzzy uf(e, de) and the other uc(e, de) is a crisp one. In this context is used the term quasi-fuzzy controller.
The fuzzy control system with the correction of the fuzzy block

The behaviour of the fuzzy block is determinate in principle by the rule base. But the membership function has a great roll in this. The characteristic of the fuzzy block is situated only in the first and the third quadrant. If the fuzzy block will be used only on the universes of discussion \([-1, 1]\) of his inputs the characteristics \(u_F(x_\Sigma; de)\) will be situated in a sector \([K_1, K_2]\), \(0<K_1<K_2\). But, introducing the saturation elements at the inputs of the fuzzy block the resultant non-linearity \(\tilde{N}\) is situated in a sector \([0, K]\).

To accomplish the sector condition necessary to assure then internal stability the correction is made by the summation at the output of the fuzzy block \(u_F\) the quantity \(u_c\) given by the following relation:

\[
u_c = K_c[(e-e) + (de-de)]
\]

Let \(x_1=e\) and \(x_2=de\) be two state variables. In the place of \(x_1\) it is introduced the combined state variable \(x_\Sigma = x_1 + x_2\). In consequences:

\[
u_{Fc} = u_F(e, de) + u_c(e, de) = u_F(x_\Sigma - x_1, x_2) + u_c(x_\Sigma - x_1, x_2)
\]

The particular situation when \(x_2=0 \Rightarrow u_{Fc}=0\) is interesting for that what follow. In this case \(u_c=0\), and the characteristics \(u(\Sigma)\) is in Fig. 3.

It is noticed they satisfied the sector relation \(0 \leq u_F(x_\Sigma) < K\), which does not suit. This sector property of the fuzzy block suggests the usage of the circle's criterion in the stability analysis of the fuzzy control system [2]. The same sector properties are valid for other fuzzy blocks, with a larger number of rules, with min-max, or sum-prod inference and for the fuzzy blocks that use different defuzzification methods, with the centre of gravity or the mean of maxima [8].

3 Absolute Stability

To apply circle's criterion the control system structure from Fig. 2 is transformed in a quasi-continual one, shown in Fig. 4, using the Pade transformation.

The quasi-continual diagram is then transformed in an adequate one, shown in Fig. 5, which matches the standard structure for stability analysis of non-linear systems from [2].
Comparing the structure from Fig. 4 and Fig. 5 it is easy to determine the composition of the linear part L. The non-linear part \( N_c \) is placed in a stabilisation feedback. The linear part has the input \( u_m \) and the output vector \( \tilde{y} = [\tilde{y}_1, \tilde{y}_2]^T = [\tilde{e} \; \tilde{de}]^T \). The input vector of the fuzzy block is \( [e \; de]^T \). The correction coefficient \( K_c \) is chosen as the characteristic of the non-linearity \( N_c \) to be situated in the stability sector \([K_{\text{min}}, K_{\text{max}}]\). Two examples are presented in Fig. 6.

\[
\begin{align*}
\| f_N (y_1) - f_N (y_2) \| & \leq \\
\leq \mu \| y_1 - y_2 \| & \mu > 0 \\
y_1 = [e_1 \; de_1], \; y_2 = [e_2 \; de_2]
\end{align*}
\]

The fuzzy control system has the following state-space model:

\[
\begin{align*}
\dot{x}_{a1} &= \frac{1}{h} u_F \\
\dot{x}_{a2} &= \frac{2}{h} w - \frac{2}{h} y - \frac{2}{h} x_{a2} \\
\dot{x}_{a3} &= \frac{K_P}{T_1} x_{a1} + \frac{K_P}{2T_1} u_F - \frac{1}{T_1} x_{a3} \\
y &= \frac{1}{T_2} x_{a3} - \frac{1}{T_2} y
\end{align*}
\]

where

\[
\begin{align*}
x_{a1} &= u - \frac{1}{2} u_F \\
x_{a2} &= e_w - \frac{h}{2} de_w
\end{align*}
\]

and

\[
\begin{align*}
\dot{e} &= c_e e_w \\
\dot{de} &= \frac{2c_{de}}{h} (x_{a2} - e_w) \\
u_m &= -f_{N_c} (\dot{y}) = -c_{de} u_{Fc}
\end{align*}
\]

The no linearity \( u_{Fc} = f_{N_c}(\dot{y}) \) satisfies the sector condition:

\[
\begin{align*}
[f_{N_c} (\dot{y})(e, de) - K_{\text{m}} [1 \; 1] \dot{y}] & \leq 0, \\
[f_{N_c} (\dot{y})(e, de) - K_{\text{M}} [1 \; 1] \dot{y}] & \leq 0, \\
\forall t \geq 0, \forall y \in R^2
\end{align*}
\]

Through the modification from fig. 2, when the component \( u_c \) appears, the characteristics from fig. 6 are obtained. It is important to notice that the value of \( K_c \) modifies the inferior limit \( K_{\text{m}} \) and the superior limit \( K_{\text{M}} \) of the sector. The result is synthesised in Fig. 8.

The correction effect is limited. Values of \( K_c \geq 0.55 \) do not modify the value of parameter \( K_{\text{m}} \) \((K_{\text{m}} \approx 0.52)\), and \( K_{\text{M}} \) is constant \((K_{\text{M}} \approx 1.17)\).

Assume the parameters \( c_e \) and \( c_{de} \), of the interface of the linear block output with the fuzzy block, fixed. In these conditions for the complete designing of the control system it remains to determine the values of \( K_c \).
Fig. 8 The influence of $K_c$ on sector limits

Because the linear part is not Hurwitz the non-linear part $N_c$ must be situated in the sector:

$$[c_{du} K_m, c_{du} K_M] \subset [K_{\min}, K_{\max}]$$  \hspace{1cm} (9)

This stability sector will result after the applying of the circle's criterion. The increment $c_{du}$ is included in the structure of the non-linearity. The correction of the non-linearity is also made using this increment. The sector limits have the property:

$$[K_{\min}(K_c), K_M(K_c)] \subset [K_1, K_2] \subset [K_{10}, K_{1/\mu}]$$  \hspace{1cm} (10)

in which $K_1$ and $K_2$ are determinable with circle's method, and $[K_{10}, K_{1/\mu}]$ is the Hurwitz stability domain associated to the linear loop in Fig. 4. In the same time, the multitude of possible increments $\{c_{du} \{u_c\}\}$ must be included in the multitude of acceptable increments, resulted from technical grounds.

The problem of the internal stability is related to the continuity of the state transition, in the condition of a free system:

$$w = 0$$  \hspace{1cm} (11)

The equation of the equilibrium point are obtained from the equality:

$$x = 0$$  \hspace{1cm} (12)

In the equilibrium point for the fuzzy block there are the equations:

$$u_m = 0$$  \hspace{1cm} (13)

$$\dot{e} = \dot{e} = 0$$  \hspace{1cm} (14)

The control system is closed through the fuzzy block. The above conditions must be obtained only when:

$$e = \dot{e} = 0$$  \hspace{1cm} (15)

So, at the equilibrium:

$$x = 0$$

$$\dot{y} = 0$$  \hspace{1cm} (16)

$$u_m = u_F = u_{F_c} = u_F = 0$$

The control system is asymptotic stable if the error $e$ and his derivative $\dot{e}$ are zero at the limit. The no linearity introduced by the fuzzy block must assure these conditions. To assure the necessary condition for the permanent regime when $\dot{e}=0$, the corrected no linearity must give $u_F=0$ only if $e=0$. The fuzzy blocks, which use the defuzzification method with the centre of gravity, accomplishes this condition. The fuzzy blocks, which use the defuzzification with the method of the mean of maxima, don't accomplish this condition [8]. The relation between the control system and the rule base could be analysed in the state space. The 9 rules base from Fig. 9 assures trajectories for a stable control system.

<table>
<thead>
<tr>
<th>$u_F$</th>
<th>$e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>$\epsilon$</td>
</tr>
<tr>
<td>P</td>
<td>$\epsilon$</td>
</tr>
</tbody>
</table>

Fig. 9 Stable trajectories in the rule base

Some asymptotic stable trajectories are shown in Fig. 9.

### 3 Notice on The Linear Part

The linear part has the transfer matrix $H_0(s)$: $\tilde{y}(s) = H_0(s)u_m(s)$. It has a pole in the origin, so it is not a Hurwitz matrix. Using the main property of the fuzzy block the non-linear part $\tilde{f}_{N_c}(\tilde{y})$, with two inputs and one output becomes $c_{du} \tilde{f}_{Nc}(x \Sigma)$ with one input and one output. The transfer function from $u_m$ to $x \Sigma$ is:

$$H_{01}(s) = \frac{\tilde{y}(s)}{u_m(s)} = \left[1 \hspace{1cm} 1\right]H_0(s) = \begin{pmatrix} c_{de} + c_{de} \frac{s}{1+hs/2} \end{pmatrix} \frac{K_p(1+hs/2)}{hs(T_1s+1)(T_2s+1)}$$  \hspace{1cm} (17)

Because of the pole from the origin the linear part must be stabilised. The no linearity $\tilde{f}_{N_c}(\tilde{y})$ must have a sector condition $[K_{\min}, K_{\max}] \subset (\epsilon, K_{1/\mu})$, and the function

$$H(s) = \frac{H_{01}(s)}{1+K_{\min}H_0(s)}$$  \hspace{1cm} (18)

must be a Hurwitz one [2].
5 Internal Stability Analysis

A fictive transformation of the non-linear control system is made. It is shown in Fig. 10.

\[
\begin{align*}
    x &= (A - b_K K_{\text{min}} \begin{bmatrix} 1 & 1 \end{bmatrix}) x + b_u u_m + b_w w \\
    y &= C x + d w
\end{align*}
\]

(19)

The new system matrix:

\[
A = A - b_K K_{\text{min}} \begin{bmatrix} 1 & 1 \end{bmatrix}
\]

is a Hurwitz one. And the nonlinearity part:

\[
f_{\text{Nm}}(\tilde{y}) = f_{\text{Nc}}(\tilde{y}) - K_{\text{min}} \begin{bmatrix} 1 & 1 \end{bmatrix} \tilde{y}
\]

(21)

is situated in the sector \([0, K]\):

\[
f_{\text{Nm}}(\tilde{y}) f_{\text{Nm}}(\tilde{y}) - K_{\text{min}} \begin{bmatrix} 1 & 1 \end{bmatrix} \tilde{y} \leq 0,
\]

\[\forall t \geq 0, \forall \tilde{y} \in R^2
\]

with \(K = K_{\max} - K_{\min}\).

The circle's criterion for multivariable systems [2] may be used in stability analysis of this fuzzy control system.

Circle's criterion: The non-linear system with a linear part \(e\)-stable, with the transfer function \(H_0(s)\) and the non-linear part with the sector condition is absolute global stable is \(H_1(s)\) is a Hurwitz matrix and is strictly positive real.

\[
F(s) = \frac{1 + K_{\text{max}} H_0(s)}{1 + K_{\text{min}} H_0(s)} = \frac{1 + K_{\text{max}} \begin{bmatrix} 1 & 1 \end{bmatrix} H_0(s)}{1 + K_{\text{min}} \begin{bmatrix} 1 & 1 \end{bmatrix} H_0(s)}
\]

(23)

It is easy to notice that the transfer function associated to the system is a minimal one. Consequently, agreed to Kalman-Yakubovitch-Popov lemma [2], the achievement of condition is equivalent to the real positively of function \(F(s)\), valuable by the circle's method. It is noticed that the time variation of dependence \(f(x)\) is implicit.

6 Some Indications to Correct the Non-Linear Part

To impose an adequate dynamical behaviour of the closed-loop fuzzy control system the values of the following coefficients may be modified: \(c_e, c_{de}, c_{du}\).

Also, to correct the non-linear part the value of \(K_c\) may be modified. Considering the values of \(c_e, c_{de}\) chosen it is possible to choose the values of \(c_{du}\) and \(K_c\).

\#1. For a certain fuzzy block the values of \(K_{\min M}\) and \(K_{M}\) are chosen from the characteristics \(f(x_\Sigma; \tilde{de})\), \(u_{\text{ref}} = f(x_\Sigma; \tilde{de})\) and \(K_{\text{M}} = f(K_c)\).

\#2. The value of the command increment is limited at \(c_{du M}\).

\#3. The values of \(c_{du}^*\) and \(K_c\) may be chosen to assure the condition:

\[
K_{\min M} \leq c_{du}^* K_{M} \leq c_{du} M \leq c_{du}^* K_{M} \leq K_{\max M} K_{M} < K_{H}
\]

(24)

where \(K_{H}\) is the limit of the Hurwitz sector.

\#4. The choosing of \(c_{du}\) is made taking account of the value \(K_{M}\) of the concrete non-linearity given by a certain fuzzy block BF.

\#5. The choosing of \(K_c\) is made taken account of the \(K_{\min M} K_{\max M}\) and of the characteristic \(K_{\min M} = f(K_c)\).

The characteristic \(f_{\text{Nc}}(x_\Sigma)\) must be situated in a stability sector with a security coefficient, such as at the variation of the process parameters it must remain in the stability sector.
7 Conclusion
The stability analysis of a control system with an incremental PI fuzzy controller leads to the conclusion of the theoretical necessity of the foreseen of a parallel correction block. It is demonstrating that the structure is absolute stable if the resultant quasi-fuzzy controller with the help of the correction block fulfills some sector conditions.

A lot of fuzzy blocks have the sector property that makes the characteristics of the fuzzy block to be situated in the first and the third quadrant. This property of the fuzzy block allows using of the circle's criterion for internal stability analysis of the speed fuzzy control systems.

To assure the stability of the fuzzy control system the non-linear part must be corrected from a sector \([0, K]\) into a sector \([K_{\text{min}}, K_{\text{max}}]\). A method to correct the non-linear characteristics is presented.

The way to use the circle's criterion is presented, with application at the second order process.

The stability sector is imposed by the hodograph of the linear part of the speed control system and also by the characteristic of the non-linear part, which includes the fuzzy block.

Some indications to choose the limits of the stability sector are given.

Also, some indications to choose the correction coefficients of the speed control system are given.

References: