The simulation of the adaptive systems using the MIT rule

COMAN ADRIAN, AXENTE CORNELIU, BOSCOIANU MIRCEA,
Military Technical Academy
81-83 George Cosbuc Street, Bucharest
Romania

Abstract: - In order to simulate the adaptive systems, we've chosen to simulate the adaptive control with reference model, also known as Model Reference Adaptive Control – MRAC. The general idea behind Model Reference Adaptive Control (MRAC, also know as an MRAS or Model Reference Adaptive System) is to create a closed loop controller with parameters that can be updated to change the response of the system. The output of the system is compared to a desired response from a reference model. The control parameters are updated based on this error. The goal is for the parameters to converge to ideal values that cause the plant response to match the response of the reference model.

Key-Words: MIT rule, adaptive control

1. Introduction

The design of a controller that can alter or modify the behavior and response of an unknown plant to meet certain performance requirements can be a tedious and challenging problem in many control applications. By plant, we mean any process characterized by a certain number of inputs *u* and outputs *y*, as shown in Figure 1.

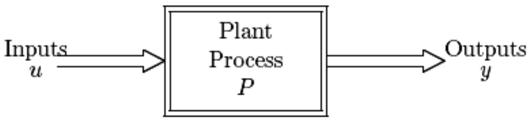

Fig. 1 - Plant representation

The plant inputs *u* are processed to produce several plant outputs *y* that represent the measured output response of the plant. The control design task is to choose the input *u* so that the output response $y(t)$ satisfies certain given performance requirements. Because the plant process is usually complex, i.e., it may consist of various mechanical, electronic, hydraulic parts, etc. The appropriate choice of *u* is in general not straightforward.

2. Theoretical considerations

The idea behind Model Reference Adaptive Control is to create a closed loop controller with parameters that can be updated to change the response of the system to match a desired model. Model reference adaptive control (MRAC) is derived from the model following problem or model reference control (MRC) problem. In MRC, a good understanding of the plant and the performance requirements it has to meet allow the designer to come up with a model, referred to as the *reference model*, that describes the desired I/O properties of the closed-loop plant.

The objective of MRC is to find the feedback control law that changes the structure and dynamics of the plant so that its I/O properties are exactly the same as those of the reference model. The structure of an MRC scheme for a LTI, SISO plant is shown in Fig. 2. The transfer function $W_m(s)$ of the reference model is designed so that for a given reference input signal $r(t)$ the output $y_m(t)$ of the reference model represents the desired response the plant output $y(t)$ should follow. The feedback controller denoted by $C(\theta_c^*)$ is designed so that all signals are bounded and the closed-loop plan transfer function from *r* to *y* is equal to $W_m(s)$. This transfer function matching guarantees that for any given reference input $r(t)$, the tracking error $e_1 \triangleq y - y_m$, which represents the deviation of the plant output from the desired trajectory y_m, converges to zero with time. The transfer function matching is achieved by canceling the zeros of the plant transfer function $G(s)$ and replacing them with those of $W_m(s)$ through the use of the feedback controller $C(\theta_c^*)$. The cancellation of the plant zeros puts a restriction on the plant to be minimum phase, i.e., have stable zeros. If any plant zero is unstable, its cancellation may easily lead to unbounded signals.

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The design of $C(\theta_c^*)$ requires the knowledge of the coefficients of the plant transfer function $G(s)$. If θ_c^* is a vector containing all the coefficients of $G(s) = G(s, \theta_c^*)$, then the parameter vector θ_c^* may be computed by solving an algebraic equation of the form

$$\theta_c^* = F(\theta^*) \quad (1)$$

It is, therefore, clear that for the MRC objective to be achieved the plant model has to be minimum phase and its parameter vector θ_c^* has to be known exactly.

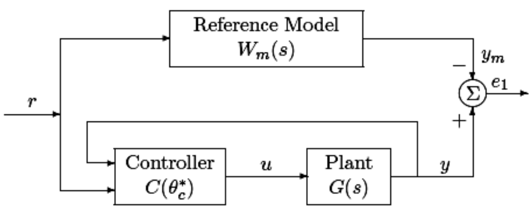

Fig. 2 - Model reference control

There are many different methods for designing such a controller. When designing an MRAC using the MIT rule, the designer chooses: the reference model, the controller structure and the tuning gains for the adjustment mechanism.

MRAC begins by defining the tracking error, e. This is simply the difference between the plant output and the reference model output:

$$e = y_{sistem} - y_{model} \qquad (2)$$

From this error a cost function of theta (J(theta)) can be formed. J is given as a function of theta, with theta being the parameter that will be adapted inside the controller. The choice of this cost function will later determine how the parameters are updated. Below, a typical cost function is displayed.

$$J(\theta) = \frac{1}{2} e^2(\theta) \qquad (2)$$

To find out how to update the parameter theta, an equation needs to be formed for the change in theta. If the goal is to minimize this cost related to the error, it is sensible to move in the direction of the negative gradient of J. This change in J is assumed to be proportional to the change in theta. Thus, the derivative of theta is equal to the negative change in J. The result for the cost function chosen above is:

$$\frac{d\theta}{dt} = -\gamma \frac{\delta J}{\delta \theta} = -\gamma e \frac{\delta e}{\delta \theta} \qquad (4)$$

This relationship between the change in theta and the cost function is known as the MIT rule. The MIT rule is central to adaptive nature of the controller. Note the term pointed out in the equation above labeled "sensitivity derivative". This term is the partial derivative of the error with respect to theta. This determines how the parameter theta will be updated. A controller may contain several different parameters that require updating. Some may be acting n the input. Others may be acting on the output. The sensitivity derivative would need to be calculated for each of these parameters. The choice above results in all of the sensitivity derivatives being multiplied by the error. Another example is shown below to contrast the effect of the choice of cost function:

$$J(\theta) = |e(\theta)|$$

$$\frac{d\theta}{dt} = -\gamma \frac{\delta e}{\delta \theta_c} sign(e) \qquad (5)$$

where, $sign(e) = \begin{cases} 1, & e > 0 \\ 0, & e = 0 \\ -1, & e < 0 \end{cases}$

To see how the MIT rule can be used to form an adaptive controller, consider a system with an adaptive feed forward gain. The block diagram is given below:

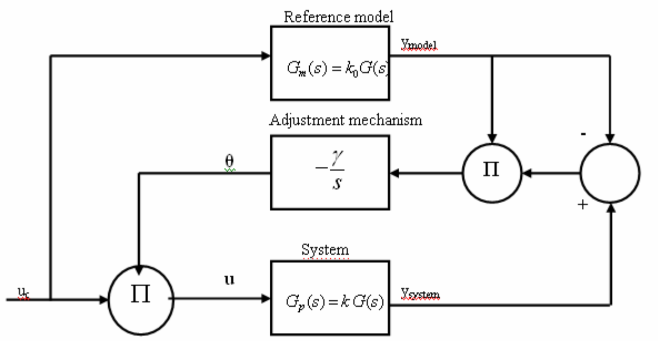

Fig. 3 – The system model for the adaptive control according to the MIT rule

$$\frac{Y(s)}{U(s)} = kG(s) \qquad (6)$$

The constant k for this plant is unknown. However, a reference model can be formed with a desired value of k, and through adaptation of a feed forward

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gain, the response of the plant can be made to match this model. The reference model is therefore chosen as the plant multiplied by a desired constant k_0:

$$\frac{Y(s)}{U_C(s)} = k_0 G(s) \qquad (7)$$

The same cost function as above is chosen and the derivative is shown:

$$J(\theta) = \frac{1}{2} e^2(\theta) \longrightarrow \frac{\mathrm{d}\theta}{dt} = -\gamma e \frac{\delta e}{\delta \theta} \qquad (8)$$

The error is then restated in terms of the transfer functions multiplied by their inputs.

$$e = y - y_m = kGU - G_m U_c = kG\theta U_c - k_0 G U_c \qquad (9)$$

As can be seen, this expression for the error contains the parameter theta which is to be updated. To determine the update rule, the sensitivity derivative is calculated and restated in terms of the model output:

$$\frac{\delta e}{\delta \theta} = kGU_c = \frac{k}{k_o} y_m \qquad (10)$$

Finally, the MIT rule is applied to give an expression for updating theta. The constants k and k_o are combined into gamma.

$$\frac{d\theta}{dt} = -\gamma' \frac{k}{k_o} y_m e = -\gamma y_m e \qquad (11)$$

The block diagram for this system is the same as the diagram given at the beginning of this example. To tune this system, the values of ko and gamma can be varied.

3. An analysis of the results for different simulations

In order to study the adaptive control, the following longitudinal dynamics was considered as a reference system:

$$G_m(s) = \frac{3.476(s+0.0292)(s+0.883)}{(s^2+0.019s+0.01)(s^2+0.841s+5.29)}$$

To analyze the behavior of the adaptive control the following model was designed in Matlab/Simulink:

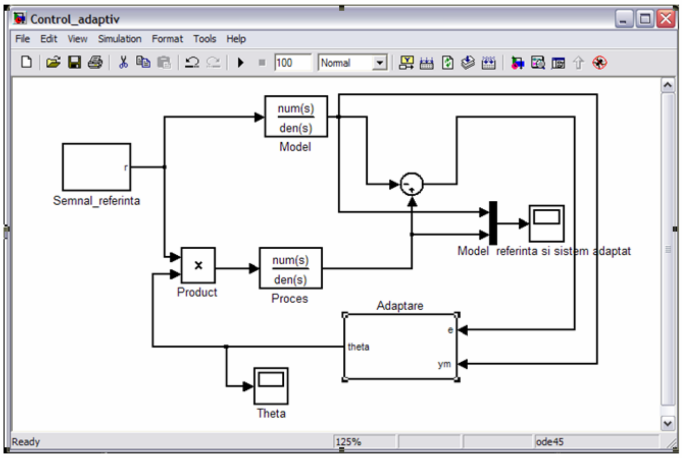
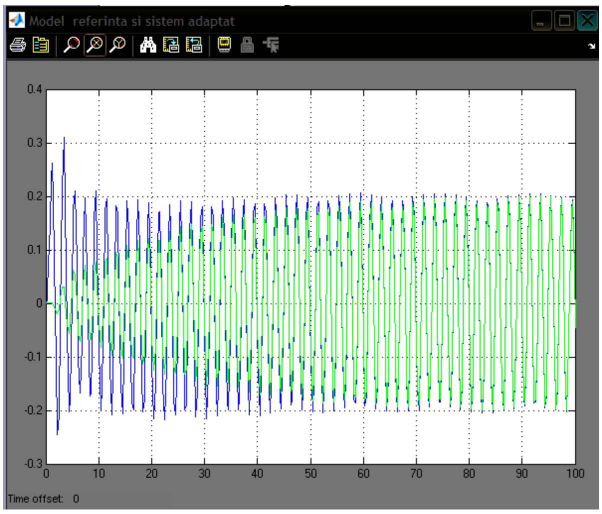


Fig. 4 – The implemented model in Simulink

The reference signal is a sine wave, which has the amplitude A=1 and the frequency f=0.5Hz, with an initial offset equal with 0 for k_0=0.5 and k=1. The *Adaptare* model executes the operation $\frac{d\theta}{dt} = -\gamma' \frac{k}{k_o} y_m e = -\gamma y_m e$. The system response and theta θ will be analyzed for different amplitude and offset values of the reference signal and *gamma* – γ.

a) offset = 0, γ = 1, A = 1



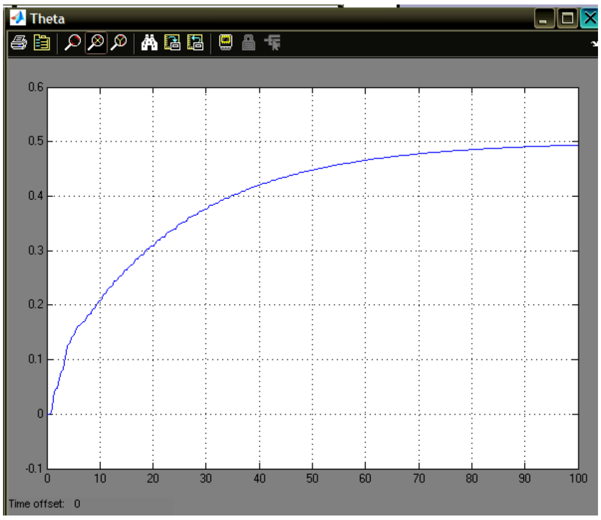
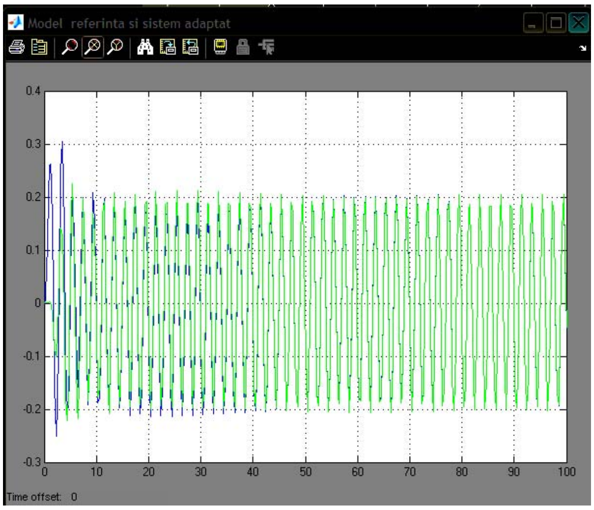


Fig. 5a) – The response for the reference model and adapted system and the evolution of the theta parameter

The response of the adapted system (green) becomes identical with response of the reference model (blue). The same interval of time is required for the theta parameter to stabilize itself around the value of 0.5.

b) offset=0, γ=5, A=1



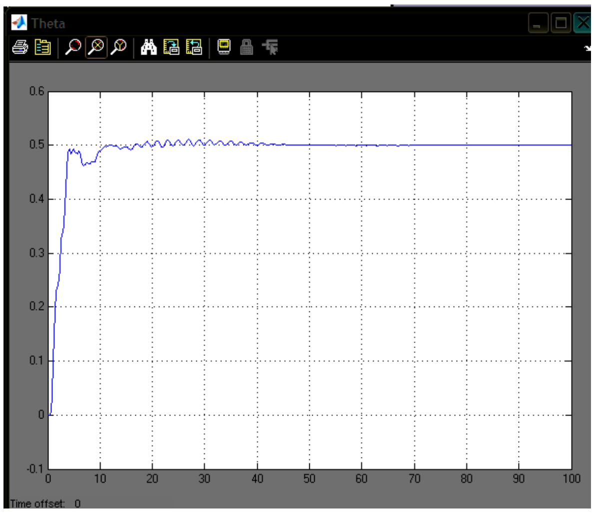


Fig. 5b) - The response for the reference model and adapted system and the evolution of the theta parameter

The period for stabilization is decreased if the *gamma* parameter is increased.

c) offset=0.5, γ=5, A=1

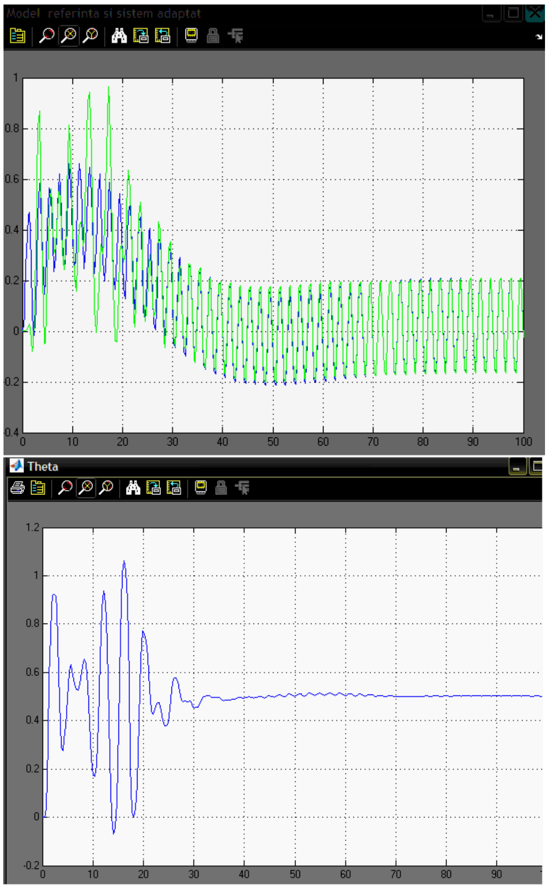


Fig. 5c) - The response for the reference model and adapted system and the evolution of the theta parameter

As the offset value is increased, some transitory conditions appear for a period of 40 seconds. Arbitrary oscillations which lead to instability are expected for values greater than 1 of the offset.

d) offset=0, γ=5, A=2.5

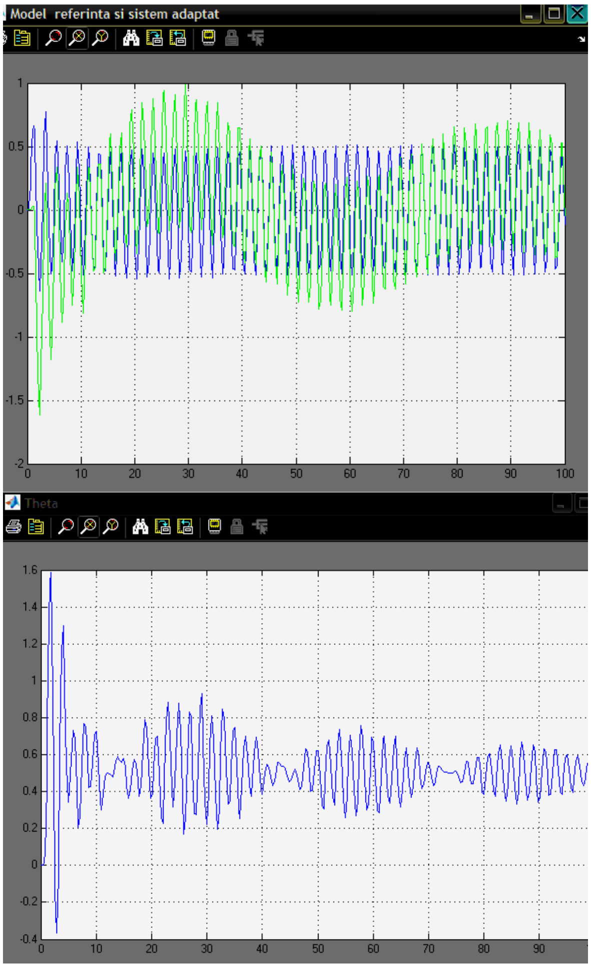


Fig. 5d) - The response for the reference model and adapted system and the evolution of the theta parameter

It can be seen that the adaptive system response doesn't follow anymore the reference system, the oscillations of the *theta* parameter have a slow damping period, but the adaptation is not done in useful time, so the system behavior is unsatisfactory.

4. Conclusions

The MIT rule by itself does not guarantee convergence or stability. An MRAC designed using the MIT rule is very sensitive to the amplitudes of the signals. For the studied model, the MIT rule provides satisfactory results for signal amplitudes which are in the [-1,1] domain. Divergence is obtained for negative values of *gamma*, and instability is obtained for values greater than 5. This parameter has to have small values. For greater values in absolute value of the offset leads also to instability.

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