Design Method for Robust PI-like Process Fuzzy Logic Controller

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Abstract: - The industrial applications of fuzzy logic controllers (FLC) for complex, nonlinear, time varying processes with time delay mark a considerable progress. It is inspired by the recent theoretical achievements in the FLC’s design and tuning related to system stability and robustness. The design and the FLC simplicity are of crucial importance. There exists an easy design and tuning procedure for a simple single input fuzzy controller (SI FC) that ensures fuzzy system stability and robustness. The SI FC, however, loses some of the richness of the behaviour of the ordinary FLC. The aim of the present investigation is to develop a simple design method for incremental PI-like ordinary FLCs on the basis of the SI FC approach, ensuring stability and robustness in the closed loop system for the control of industrial plants with time delay and model uncertainty. The main contributions are: 1) a design method for FLC, based on modification of the SI FC stability and robustness approach; 2) FLC tuning algorithm. The results are applied for the stabilisation of the air temperature in a furnace using MATLAB™. The advantages of the designed SI FC system are assessed in comparison with SI FC control system via simulations.

Key-Words: - frequency domain incremental PI-like process FLC design, MATLAB, robust stability and performance, temperature control, time delay, tuning algorithm

1 Introduction to Problem Area

A remarkable progress has been recently noticed in the industrial application of fuzzy logic controllers (FLCs) for controlling of complex inertial processes under uncertainties – lack of reliable plant model, nonlinearity, time-variance of parameters, etc. It is stimulated mainly by the recent developments in the theory of the FLC’s design and tuning, resulting in various solutions to the basic problems of FLC system - stability and robustness. Being intrinsically nonlinear the FLC system stability is successfully approached from the viewpoint of the nonlinear system stability theory employing basically the bounded input-output technique, the time-domain Lyapunov direct method, the frequency-domain Popov or the describing functions (DF) approaches [1-6]. Some solutions lead to an increased design and/or FLC complexity. Others do not consider plants with time delay. Often various FLC nonlinear models (multilevel relay, piece-wise linear, sector bounded, waveforms, etc.) and linear, linearised or nonlinear plant mathematical models, mostly in state space, are assumed. Therefore, only few approaches are suitable for use in the process FLC design. In [6] is suggested rule base generation out of stability considerations on the basis of a linguistic transformation of the Lyapunov function using fuzzy arithmetic and empiric knowledge on how plant state variables are related to required control action.

Recently attempts have been made to combine stability with robustness considerations [7-9]. The general mathematical framework of robust stability and robust performance - the $H_2$ or the $H_{\infty}$-optimisation is, however, inapplicable in the engineering practice because it lacks rules for weighting functions selection, produces high-order controllers, offers approximate non-unique solutions, does not consider industrial plants with time delay [10]. The DF stability approach is employed in [9] accounting also for the plant model uncertainty due to the discarded higher harmonics. The FLC redesign algorithm on the basis of the conicity criterion and small-gain in [7] shapes of the output fuzzy sets to reconcile nominal system performance and robust stability. A common drawback is the controllers’ and the design complexity.

For simplification of the FLC design and tuning various incremental PI-like FLCs have been developed [11, 12] easing the rule base building. The normalisation of the universes of discourse of the LVs and the introduction of adjustable scaling factors unifies the FLC design and also facilitates its tuning. The problems that still remain are controllers’ complexity, trial-and-error design and
tuning. The most popular techniques to preserve design simplicity and enhance its adaptability and tuning are to employ supervisory fuzzy controllers [2, 5, 13] for objective autotuning or neuro-fuzzy synergism [4, 14] for simplifying of the FLC’s structure and enhancing its adaptability. The first increases the controller complexity, while the second requires sufficient and rich in amplitudes and frequencies free of noise data samples for network training, which is difficult to provide.

The suggested in [1, 15] single input FLC (SI FC) ensures both system stability and robustness on one hand, and simplicity of the controller’s structure and design on the other. Its main advantages are based on the reduced by times rule base, which can be objectively designed, and the sector-bounded control curve instead of the ordinary FLC control surface, which eases the design and tuning from stability and robustness considerations. The SI FC, however, loses some properties, characterising the complex and rich behaviour of the ordinary FLC as a nonlinear controller that deals also with linguistically expressed perceptions rather than with measurements and numerical data, and can perform better than the classic linear controllers.

The aim of the present investigation is to develop a simple and general design and tuning procedure for a process incremental PI-like ordinary FLC, based on the SI FC robust stability and robust performance design for the control of industrial plant with time delay and model uncertainty.

An approximate simple LTI stable Ziegler-Nichols nominal plant model and a multiplicative plant model uncertainty [15, 16], both based on expert assessment, describe the industrial plant. The plant model uncertainty reflects the nominal model imperfectness and the process time-varying, inertial and nonlinear properties [1, 10, 15, 16]. The frequency domain approach to robust stability and robust performance in [15], suitable for plants with time delay and SI FC with sector bounded static nonlinearity, is adapted to the design and tuning of an ordinary incremental PI-like FLC.

The organisation of the paper is the following. Section 2 defines the problems addressed in the investigation and refers to the robust stability and robust performance design of SI FC systems. This design technique is further adapted in Section 3 for the development and tuning of an ordinary incremental PI-like FLC and a design algorithm is suggested, employing combined Popov and Morari approaches to the projection of the control surface. In Section 4 a FLC for the control of the air temperature in a furnace is designed according to the algorithm of Section 3 using the Fuzzy Logic Toolbox of MATLAB™ [17]. Then the simulated in Simulink step responses of the closed loop FLC and SI FC systems are compared.

### 2 Problem Formulation

The block diagram of a SI FC system is depicted in Fig.1. It consists of a LTI plant with transfer function \( P(s) \) and a SI FC. The SI FC is built on a pre-processing unit \( W_1(s) = K_{ds}[1 + W_{d}(s)] \) that computes the dynamic components of the PI-like algorithm in the single input to the fuzzy unit (FU), a FU – a static element with sector bounded nonlinearity and gain \( C(d_s) = \frac{W(d_s)}{d_s} \) and a post-processing integrating unit \( W_2(s) = K_a/s \), where \( K_a = K_{ua}K_i \) unites the normalisation factor \( K_{ua} \) and the integrator gain \( K_i \). A one time-step memory element or an integrating actuator performs the integration. The system error \( e = y_r - y \) is the difference between reference \( y_r \) and measured plant output \( y \), the scaled derivative of error \( \lambda \dot{e} \) is the output of a differentiator \( W_d(s) \), the normalised in the range \([-1, 1]\) signed distance \( d = e + \lambda \dot{e} \) by the help of the scaling factor \( K_{ds} \) is the single input to the FU. The incremental controller output \( \Delta u = \dot{u} \) is uniquely determined from the requirement \( \Delta u = -d \) and is also normalised in the range \([-1, 1]\).

The robust stability of the SI FC system is derived in [15] given the nominal plant model

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![Fig.1. SI FC system](image-url)
Fig. 2. Illustration of SI FC system robust stability. 

$P^o(s)$ and the plant model multiplicative uncertainty $l(s) = |P(s) - P^o(s)|/P^o(s)$ using Fig.2. The modified plant Nyquist plot for a nominal plant $P_m^o(j\omega)$ is defined on the basis of a stabilised plant $P_m(j\omega)$ as $P_m(j\omega) = ReP_m(j\omega) + j\omega ImP_m(j\omega)$, which appears after stabilisation of the marginally stable linear part $W_1(j\omega), W_2(j\omega), P(j\omega)$ by means of a local feedback with gain $r$, yielding

$$P(j\omega) = \frac{W_1(j\omega)W_2(j\omega)}{1 + W_1(j\omega)W_2(j\omega)},$$

where for a typical first order differentiator $W_d(j\omega) = K_dT_d\omega/(\omega^2 + 1)$,

$$W_d(j\omega)C_r(j\omega) = C_r(j\omega)[1/(\omega^2 + 1) + 1/T_s]$$

a PI controller with gain $K_p = K_aT_a K_b K_s$ and integral action time $T_i = K_s T_d$ and $P_i(j\omega) = C_p(j\omega)P(j\omega)/(1 + rC_p(j\omega)P(j\omega))$. The Popov line in Fig.2 is defined by its cross point with the abscissa (-1/K1, j0) and a slope 1/q, where $K = K_T > 0$ is determined by the angular coefficients of the sector lines $l_1$ and $l_2$, bounding the SI FC control curve as shown in Fig.3.

It is assumed that the initial plant model uncertainty $l(j\omega)$ leads to a modified plant model uncertainty $l_m(j\omega)$ in the form of disks around the modified plant model Nyquist plot with radiiuses $r_l(\omega) = |P_m(j\omega)| - |P_m^o(j\omega)| = \Delta P_m(j\omega) = |l_m(j\omega)P_m^o(j\omega)|$. In terms of robust stability [15, 16] according to Fig.2 for a modified plant with no poles with positive real part the following vector magnitude inequality should hold

$$1/K_1 + P_m^o(\omega) > |l_m(j\omega)P_m^o(\omega)|$$

for all nonnegative frequencies. After division by $1/K_1 + P_m^o(\omega)$ it is obtained:

$$|l_m(j\omega)| < 1, \forall \omega \geq 0$$

or $|\Phi_m(j\omega)| < 1, \forall \omega \geq 0$, where $|\Phi_m(j\omega)| = |P_m(j\omega)K_1|/(1 + P_m(j\omega)K_1)$ is the magnitude of the frequency response of an equivalent linear closed loop system with plant – the modified plant and PI controller with gain $K_1$, $|l_m(j\omega)| \leq s_{max} = \sup_{\omega} |l_m(j\omega)|$

and $l_m(j\omega) = \frac{l(j\omega)}{1 + P_m(j\omega)[1 + l(j\omega)])} = \frac{l(j\omega)}{1 + l(j\omega)\sup R_P(j\omega)}$ is the multiplicative stabilised plant model uncertainty.

Thus the SI FC system is robustly stable for all plants of the family, defined by $[P^o(s), l(s)]$, with SI FC with sector bounded nonlinear control curve and tuning parameters $p^1 = [T_a K_a K_s r]$, which ensures stability of the system with nominal plant and fulfillment of the robust stability criterion (1). Graphically this means that the Nyquist plot $P_m(j\omega)$ in Fig.2 with all the disks on it should be located below and on the right from the Popov line.

The robust performance requirement is derived after linearisation of the SI FC system with stabilised plant and applying Morari approach [16] in the form:

$$S_m(j\omega)W_m(j\omega) + |\Phi_m(j\omega)|l_m(j\omega) < 1, \forall \omega \geq 0$$

where $|\Phi_m(j\omega)| = |P_m(j\omega)K_1|/(1 + P_m(j\omega)K_1)$ is the frequency response of the closed loop system with nominal stabilised plant $P_m^o(s)$ and a linearised FU, $S_m^o(j\omega) = [1 + (K - r)P_m^o(j\omega)]$ is the SI FC system sensitivity function for $\gamma = 0$, linearised FU and nominal plant, $W_m(j\omega)$ is the shaping filter for the disturbance $d - d_s = W_m(j\omega)1$. $|W_m(j\omega)| = 0.3 \pm 0.9$.

The SI FC system robust performance condition (2) includes as a second term the robust stability (1), modified for a linearised SI FC system, so it sets stronger requirements. Either of (1) and (2) can be used for tuning of the SI FC parameters $K$ and $p^1 = [T_a K_a K_s r]$ of the nonlinear static FU and the pre- and post-processing linear dynamic units in order to preserve system stability or moreover, system performance for given plant model uncertainties [15].

The system considered in the present work differs from the depicted in Fig.1 by the type of the
fuzzy controller. It uses the shown in Fig.4 ordinary incremental PI-like controller with two inputs – \( e \) and \( \dot{e} \) at the output of the differentiator \( W_d(s) \), both normalised in the range [-1, 1] by means of the scaling factors \( K_c \) and \( K_{de} \) respectively. The FU output \( \Delta u \) in the range [-1, 1] is further denormalised and integrated in the post-processing unit.

The plant to be controlled is approximately described by the couple \([P'(s), l(s)]\), where \( P'(s) = k'\exp(-c'e)T_s^{s+1} \) is a comprehensible, simple and well-studied model commonly used in the engineering practice to describe a wide range of industrial plants with a few generalised parameters with clear physical interpretation. The nominal values of the gain \( K' \), the time constant \( T' \) and the time delay \( c' \) are easily estimated by experts for the most typical operating conditions. The pure time delay describes the general effect of plant inertia, transport delay, high order and parameter distribution. Robustness as a control objective saves the need for more precise and sophisticated high-order plant models. The multiplicative plant model uncertainty \( l(s) \), accounted for, stems from the changes in the operation conditions and modes and the set point shifting along nonlinear characteristics as a result of disturbances, aging of elements, etc. It is estimated from the greatest possible increase of the plant model gain and time delay and decrease of the time constant, and the impact of the unmodelled dynamics. All these simultaneous deviations from the nominal plant model characterise the most unfavourable case of perturbations with respect to system stability and time domain specifications that can take place over the whole operating range.

The input signals to the closed loop FLC system - the reference \( y_r \) and the disturbance \( d \), are assumed 2-norm bounded and in stability studies – absolutely vanishing functions \((y_r(t) \to 0, d(t) \to 0\) for \( t \to \infty \)).

The control objectives include:

1) High performance of the nominal and the varied FLC system, estimated in terms of static and dynamic accuracy and short settling time;

2) Robust stability and performance, i.e. preservation of system stability and enclosing of system time response within a narrow envelope around the nominal response \( y^0(t) \) at bounded within known ranges signal and plant model uncertainties.

So, this work addresses the following problems. Given a family of possible plants \([P'(s), l(s)]\) and incremental PI-like FLC with tuning parameters, related to the static nonlinearity - the FU control surface (parameters of the membership functions (MFs), inference and defuzzification methods, etc.) and the pre- and post-processing units (scaling, differentiator and actuator gains)

(I) Develop a robust stability and robust performance design method for the incremental PI-like FLC, based on the SI FC design approach

(II) Develop a tuning algorithm for the FLC

(III) Apply (I-II) for the design of a FLC system of an industrial plant and assess its advantages comparing with SI FC system.

3 Robust Stability and Robust Performance Design of FLC

The incremental PI-like FLC design is based on the relationship between the FU control surface of the FLC, shown in Fig.5, and the FU control curve of a corresponding SI FC. The FU of the SI FC is obtained from the FU of the FLC by dropping off the second input variable – the derivative of error, preserving the same parameters – MFs of the input (normalised \( d \), for the SIFC and normalised \( e \) for the FLC) and MFs of the normalised output \( \Delta u \), the same inference and fuzzification methods. The number of rules in the rule base of the SI FC is reduced to the number of input MFs, which predetermines the same number of output MFs as the rules are derived from the requirement \( d_{\Delta u} \). The derived in this way FU of the SI FC from the FU of the FLC has a control curve that envelopes from below and above the \( e-\Delta u \) projection of FU control surface, as illustrated in Fig.6. The \( e-\Delta u \) projection of FLC control surface can be considered a fuzzified control curve. The SI FC curve is bounded in the sector, enclosed by the abscissa and the line with angular coefficient \( K_e \), which also is a measure for the projection surface greatest slope. The projection surface can be similarly considered bounded in a sector, determined by the lines with slopes \( K \) and \( r \) respectively with the exception of a small area.
Fig. 6. $e\Delta u$ projection of FU control surface of FLC around the origin, approximately described by a disk. The disk diameter $\delta$ can be determined from the deviation of $\Delta u$ of the projection surface from zero for $e=0$. This relationship is not influenced by changes of the number and parameters of the MFs of the second input $\dot{e}$ to the FLC.

Thus the established similarity between the nonlinear FLC and SI FC allows applying the SI FC robustness-based design algorithm to the tuning of the FLC’s parameters. The FLC design and tuning algorithm comprises the following steps.

1. Design the FU of the FLC with inputs $e$ and $\dot{e}$ and output $\Delta u$, all normalised in the ranges $[-1, 1]$.
2. Obtain the FLC control surface and its $e\Delta u$ projection.
3. Draw the sector lines that bound the projection surface (Fig. 5) is obtained and depicted in Fig. 6.
4. Use $K$, $r$, the greatest expected error $|e_{\text{max}}|$ and the data for the plant family $[P(s), I(s)]$ as input for the algorithm for tuning of the equivalent SI FC from robust stability (1) or robust performance (2) requirement [15] and obtain the parameters of the SI FC $p_{\text{SIRC}}=[T_d, K_d, K_a, K_a, r]$, $K_a=[0+K_e]|e_{\text{max}}|$ is calculated to normalise $e$, in the range $[-1, 1]$.

5. Read from the projection surface the disk diameter $\delta$ and calculate the tuning parameters of the FLC $p_{\text{FLC}}=[T_d, K_d, K_a, K_a, r]$, which differ from the SI FC parameters in the scaling factors $K_e=[|e_{\text{max}}|]^{-1}$ and $K_{de}=[K_e]|e_{\text{max}}|^{-1}$, and in $K_{a}=kK_a$. The scaling gain for $K_e$ $k=0.1/\delta$ is calculated to be in the range $[0, 1]$ for $\delta=0.1+2$ and to reduce the overall open-loop FLC system gain inversely to the deflection of $\Delta u$ from zero for error close to zero. This is to compensate the possible nonzero control action for $e=0$, depending on $\dot{e}$, which is equivalent to a higher gain of the FU of the FLC than $K$ in the $\delta$-surrounding of $(e, \Delta u)=(0, 0)$.

4 Performance Assessment

The developed FLC design approach is applied for the control of the temperature of the air inside a furnace [15]. The inputs to the FU are the system error $e$, expressed as the difference between the measured temperature $T$ and its reference $T_r$. The FLC output $u$ is passed via a pulse-width modulator to a relay that connects the electric heater to the voltage supply during the pulses. The disturbances $d$ and $f$ reflect respectively the variable load and heater characteristics at the plant input.

The experts estimate the nominal plant model parameters $k^0=0.8$ °C, $T^0=45$ min, $e^0=17$ min and the perturbed plant model parameters $k=2k^0$, $\tau=30$ min and $T=15$ min for the most unfavourable with respect to closed loop system stability case.

The control objectives are minimum possible settling time $t_s$ and overshoot $\sigma$ of the nominal step response and robust performance.

The required input data for the design of the FLC is: 1) nominal and perturbed plant model parameters; 2) $B=0.6$ for overshoot of $0$%; 3) $|\nu(t, 0)|=0.5$ and 4) $|e_{\text{max}}|=5$ °C. The FLC design and tuning follows the steps of the algorithm below.

1. The FU of the FLC has been designed employing Mamdani model with five MFs for both $e$ and $\Delta u$ and three for $\dot{e}$, as shown in Fig. 7, Mamdani inference and centroid defuzzification.
2. The $e\Delta u$ projection of the resulting control surface (Fig.5) is obtained and depicted in Fig. 6.
3. The angular coefficients of the sector lines that bound the projection surface are $K=1.4$ and $r=0.2$.
4. The corresponding SI FC is designed and tuned from robust performance requirement (2) using MATLAB. The computed SI FC parameters are $T_d=5$ min, $K_d=4$, $K_e=0.15$
5. From the projection surface in Fig. 6 is read disk diameter $\delta=0.4$ and the FLC parameters obtained are $T_d=5$ min, $K_d=4$, $K_e=0.2$, $K_{de}=0.05$, $K_{a}=0.04$.

This procedure is repeated for two more FUs, which differ in the allocation of the MFs for the input $e$ in the first FU2 and for output $\Delta u$ in the second FU3. The step responses of the three designed FLC and the corresponding SI FC systems with nominal and perturbed plant are simulated in Simulink.

The simulation framework is shown in Table 1, where the tuned parameters of the different FLCs and SI FCs as well as the assessed performance indices of the designed systems - settling time $t_s$ and maximal deviation between nominal and varied response $|\Delta y|_{\text{max}}$, are presented. The superscript “o” denotes for nominal plant. The simulated step responses are depicted in Fig.8, Fig.9 and Fig.10. The responses of the systems with perturbed plant
Fig. 7. Basic FU membership functions

Fig. 8. Simulated step responses of FLC₁ and SI FC₁ systems

Fig. 9. Different MFs for FU₁ and simulated step responses of FLC₂ and SI FC₂ systems

Fig. 10. Different MFs for FU₃ and simulated step responses of FLC₃ and SI FC₃ systems

Table 1. Comparison between FLC and SI FC systems

<table>
<thead>
<tr>
<th>Performance indices</th>
<th>FLC₁ (Kₐ₁=0.04, Kᵦ=0.15)</th>
<th>SI FC₁</th>
<th>FLC₂ (Kₐ₁=0.05, Kᵦ=0.19)</th>
<th>SI FC₂</th>
<th>FLC₃ (Kₐ₁=0.02, Kᵦ=0.11)</th>
<th>SI FC₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tₛ, min</td>
<td>780</td>
<td>600</td>
<td>780</td>
<td>780</td>
<td>850</td>
<td>400</td>
</tr>
<tr>
<td>σᵣ, %</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>tᵢₚ, min</td>
<td>500</td>
<td>300</td>
<td>380</td>
<td>380</td>
<td>550</td>
<td>200</td>
</tr>
<tr>
<td>σᵣ, %</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Δy</td>
<td>ₗₘₜₐₓ, °C</td>
<td>2.5</td>
<td>1.8</td>
<td>1.9</td>
<td>1.7</td>
</tr>
</tbody>
</table>

are in dashed lines. For more realistic simulation investigations the nominal and perturbed plant in Simulink are modelled by a higher order time lag.

The comparison among the different FLC systems shows little difference. The best with respect to robust properties is FLC₂ with evenly allocated MFs for the input e. It has the smallest |Δy|ₗₘₜₐₓ, which is a measure for robustness and no overshoot for...
nominal and perturbed plant. The performance of the FLC₃ system is the worst - with longest settling time, high and robustness and overshoot for perturbed plant. The FLC system is less sensitive to the allocation of the MFs of the input e and the output than the SI FC system.

5 Conclusions and Future Work
The main contributions of the present paper confine to the following.

1. A simple and general design method for incremental PI-like process FLC, based on modification of the SI FC stability and robustness approach in [15], is developed for plants with time delay. It uses the distinguished similarity between the SI FC control curve and the $e$-$\Delta u$ projection of the FLC control surface – both assumed sector bounded, and only basic approximate expert information about the plant.

2. The FLC design method is embedded in a simple algorithm for tuning of the FLC parameters from the requirement for robust stability and robust performance of the closed loop system.

3. The tuning algorithm is applied to design three FLCs with differently allocated MFs in the fuzzy unit for the stabilisation of the air temperature in an electrical furnace. The step responses of the closed loop FLC and the corresponding SI FC systems with nominal and perturbed plant are simulated in Simulink and the performances compared.

The FLC systems preserves stability and performance for high range of plant model uncertainties, retaining at the same time good nominal behaviour, which can hardly be ensured by other design techniques. The tuning procedure can easily be embedded in industrial programmable FLCs.

Future work is foreseen in experimentation of the developed method for the design of FLCs for laboratory pilot plants using MATLAB Real Time facilities.

References: