The Effects of Fuzzy Forecasting Models on Supply Chain Performance

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Abstract: - One of the important activities which directly affect the supply chain performance is the demand forecasting. But selection of the appropriate forecasting model that best fits the demand pattern is not an easy decision. This paper analyze the effects of fuzzy linear regression, fuzzy time series and fuzzy grey GM (1,1) forecasting models on supply chain performance quantifying the demand variability through the stages.

Key-Words: - Forecasting, Supply chain, Fuzzy regression, Fuzzy time series, Fuzzy grey GM (1,1).

1 Introduction

In today’s competitive business environment the performance of companies and even countries are mainly based on customer relations; understanding the needs of customer and ability of immediate response to those needs. Supply chain (Sc) systems clearly includes all activities of this complex system required to satisfy the customer demand and can be defined as “the network of organizations that are involved, through upstream and downstream linkages, in the different process and activities that produce value in the form of products and services in the hand of ultimate customer”[1]. Rethinking the definition of Sc exposes that the performance of a successful Sc system concerns directly with appropriate demand information flow [2]. A well-known phenomenon of the Sc systems is the variability of the demand information between the stages of the supply chain (i.e. the whiplash or bullwhip effect (WE).)

This variability increases as the demand data moves upstream from the customer to the other stages of the SC system engendering undesirable excess inventory levels, defective labor force, cost increases, overload errors in production activities and etc.

Forrester [3, 4] with a simple Sc simulation consisted of retailer, wholesaler, distributor and factory levels, discovered the existence of WE as ‘demand amplification’ in Sc model. He argued about the possible causes and suggested same ideas to control the WE. He emphasized on the decision making process in each phase of SC and betrayed that this process could be the main reason the demand amplification through the chain from the retailer to the factory level [2]. Later Sterman; in 1989, studied this phenomenon using his well-known simulation model; the beer game, and apprised that inaccurate judgments made by the participants and the adoption of this judgments to the Sc system is the main reason of this phenomenon. Lee et al. [5, 6, 7, 8] declared causes of WE as: price fluctuations, rationing game, order batching, lead times and demand forecast updating; which is also the main scope of this paper. Baganha et al. [9], Graves[10], Drezner et al [11], Chen et al.[12] and Li et al. [13] also studied WE from the perspective of information sharing and demand forecasting / updating.

Accurate demand forecasting is one of the major minimization tools for WE but, finding the adequate model for the demand pattern is snarl [2]. Though past the studies about forecasting bring to light that, under relatively few data information and uncertainties; just like the situation in many Sc systems, the fuzzy forecasting models such as fuzzy time series (FTs) [14, 15, 16, 17] and fuzzy linear regression (FR) [18, 19, 20] and fuzzy grey GM (1,1) (FGG) forecasting models [17, 21] performed successfully, not much attention is paid on these systematic. This paper focuses on the effects of selected fuzzy forecasting models on the Sc performance via demand variability in the system.

The rest of this paper is organized as follow. In section 2, FR, FTs and FGG forecasting models are introduced. Section 3 deals with the quantification of WE for evaluation of Sc performance. In Section 4 and 5 proposed Sc simulation model is discussed and the effects of selected forecasting models on Sc performance is examined. Finally in section 6 research findings and conclusions are presented.

2 FR, FTs, FGG Forecasting Models

2.1 FR Forecasting Model
Linear regression: which shows the relation between response or dependent variable \( y \) and independent or explanatory variable \( x \), can be formulated considering the relation of \( y \) to \( x \) as a linear function of parameters with:

\[
Y = f(x) = \theta^T X
\]

where \( \theta \) is the vector of coefficients and \( X \) is the matrix of independent variable [2]. The application of linear regression model is suitable for the systems in which the data sets observed are distributed according to a statistical model (i.e., unobserved error term is mutually independent and identically distributed). But generally, fitting the demand pattern of a real Sc to a specific statistical distribution is not possible. The FR model introduced by Tanaka et al. [18, 19] in which “deviations reflect the vagueness of the system structure expressed by the fuzzy parameters of the regression model” (i.e. probabilistic) is suitable for the declared demand patterns and basically can be formulated as:

\[
\bar{y} = (c_0, s_0) + (c_1, s_1)x_1 + (c_2, s_2)x_2 + \ldots + (c_n, s_n)x_n
\]

where \( c_k \) is the central value and \( s_k \) is the spread value, of the kth fuzzy coefficient; \( \bar{X}_k = (c_k, s_k) \), usually presented as a triangular fuzzy number (TFN). And this representation is fact that relaxes the crisp linear regression model. Using fuzzy triangular membership function for \( \bar{X}_k \), the minimum fuzziness for \( \bar{X}_k \) can be maintained with a linear programming (LP) model as follow [2, 19, 22];

\[
Z = \text{Min}\left\{ m x_0 - (1-h) \sum_{i=1}^{n} \sum_{k=0}^{m} s_k x_{ik} \right\}
\]

\[
st \sum_{k=0}^{m} c_k x_{ki} - (1-h) \sum_{k=0}^{m} s_k x_{ki} \leq y_i
\]

\[
\sum_{k=0}^{m} c_k x_{ki} - (1-h) \sum_{k=0}^{m} s_k x_{ki} \geq y_i
\]

\[
\forall_k = 1,2,3,\ldots, n \quad \forall_i = 1,2,3,\ldots, n
\]

2.2 FGG Forecasting Model

The grey system theory introduced by Deng [24, 25] can simply be summarized as a methodology that concerns with the systems comprising uncertainties and lack of sufficient amount of information; in which, the term 'grey' indicates the system information that lies between the clearly and certainly known ones and the unknown part of the system [2]. The discrete time sequence data is used to expose a regular differential equation; GM (n,m), with the accumulated generating operation (AGO) and inverse accumulated generic operation (IAGO). In GM (n,m), terms n and m represents the order of ordinary differential equation and the number of grey variable respectively, defining the order of AGO and IAGO. In addition to the facts defined above; if the data sets collected from the system are linguistic, FGG forecasting models may perform successfully.

The FGG forecasting model introduced by Tsaur [21]; which assumes data series collected from the system are symmetrical triangular fuzzy numbers (TFN), is very similar to the crisp one and can be explained with the following equations. The original data series collected from the model is:

\[
\hat{D}^y = \left( \hat{D}_1^y + \hat{D}_2^y + \hat{D}_3^y + \ldots + \hat{D}_n^y \right)
\]

where \( n \) is the number data collected from the system. And \( \hat{D}^y \); the new fuzzy data sequence generated with AGO, can be shown with the following equation as:

\[
\hat{D}^x = \left( \hat{D}_1^x + \hat{D}_2^x + \hat{D}_3^x + \ldots + \hat{D}_n^x \right)
\]
where \( \hat{D}_k \); \( \forall k = 1,2,\ldots, n \), is a symmetrical TFN with central and spread values \( \sum_{i=1}^{k} D_i^0, \sum_{i=1}^{k} s_i^0 \); \( \forall i = 1,2,\ldots, k \) respectively. And the fuzzy GM(1,1) model is denoted as;

\[
d\hat{D}^V / dk + a\hat{D}^V = \hat{b}
\]

where \( a \) is the developing coefficient and STFN \( \hat{b} \) denotes the fuzzy grey input with the central value \( \hat{b} \) and the spread value \( b_t \) and the membership function for \( b \) is constructed as follow.

\[
\mu_b(\alpha) = \begin{cases} 
\frac{1 - |\alpha - \hat{b}|}{\hat{b}}, & b - b_1 \leq \alpha < b + b_t \\
0, & \text{otherwise}
\end{cases}
\]

By setting the sampling interval one unit as in crisp model \( d\hat{D}^V / dk \) can be rewritten as

\[
d\hat{D}^V / dk = \hat{D}_k^V + \hat{D}_{k-1}^V = \hat{D}_k^V, \quad \forall k = 2,3,\ldots,n
\]

or

\[
d\hat{D}^V / dk = \hat{D}_k^V + \hat{D}_{k+1}^V = \hat{D}_k^V, \quad \forall k = 1,2,\ldots,n
\]

where \( \hat{D}_k^V \) is a fuzzy number with central and spread values; \( \hat{D}_0^k \) and \( \hat{D}_1^k \), \( \forall k = 2,3,\ldots,n \). Let the average of \( \hat{D}_k^0 \) and \( \hat{D}_{k+1}^0 \) (which is a STFN with the central value \( Q_{k+1} \) and the spread value \( p_{k+1} \)) be the second part of equation (6) as;

\[
\hat{D}_k = -a\hat{O}_{k+1} + \hat{b}, \quad \text{where } \hat{O}_{k+1} = 1/2(\hat{D}_{k+1}^V + \hat{D}_{k+1}^V)
\]

and the spread value

\[
p_{k+1} = 1/2(\sum_{i=1}^{k} D_i^0 + \sum_{i=1}^{k+1} s_i^0)
\]

As the spread determines fuzziness; values of unknown variables \( a, b \) and \( b_t \) can be obtained from the solution of the following LP model with the objective function that minimizes the spread value of STFN \( b \).

After solving the LP problem; similar to the crisp grey GM(1,1) model, Tsaur suggested that estimated fuzzy number \( \hat{D}_k^0 \) with lower bound \( \hat{D}_k^{low} \) and upper bound \( \hat{D}_k^{upr} \); \( \hat{D}_k = \hat{D}_k^{low}, \hat{D}_k^{upr} \), could be obtained. Finally the fuzzy forecast value for period \( k+1 \); \( \hat{D}_k^0 = \hat{D}_k^{low}, \hat{D}_k^{upr} \), could be determined as follow

\[
\hat{D}_k^0 = \hat{D}_k^{upr} - \hat{D}_{k-1}^{upr}, \text{for } k \geq 2 \text{ and } g = low, upr.
\]

### 3 The Measuring Sc Performance with Quantified WE

This study evaluates the Sc performance by quantifying the demand variability (i.e. WE) in various stages of the proposed Sc simulation using Chen et al.’s model [9,10] that defines WE as the ratio of demand variances of two consequent stages. The smaller the WE the better the Sc performance will be.

Chen et al.; assuming the customer demand in period \( t \) to the retailer (\( D_t \)) as random variables; defined \( D_t \) with the following equation.

\[
D_t = \mu + D_{t-1} \rho + \varepsilon_t
\]

where \( \mu \) and \( \rho \) denotes a non negativity constant and correlation parameters \( \rho < 1 \) respectively (as \( \rho \) indicates the relationship between demands \( \rho = 0 \) betrays the independent identically distributed (i.i.d.) demand). The variance of \( D_t \) is emerged as

\[
Var(D_t) = \frac{\sigma^2}{1 - \rho^2}
\]

Assuming the inventory system in the retailer is order-up-to policy with fixed lead time and the forecasting technique used is simple moving average, the orders placed by the retailer at period \( t \); \( q_t \) (which is an important variable owing to the fact that WE is quantified as \( Var(q) \) relative to \( Var(D) \)) is denoted as:

\[
q_t = y_t - y_{t-1} + D_{t-1}
\]

and \( Var(q)/Var(D) \) have the following relation:
\[
\frac{\text{Var}(q)}{\text{Var}(D)} \geq 1 + \left( \frac{2L}{p} + \frac{2L^2}{p^2} \right)(1 - \rho^p)
\] (18)

where \(L\) is the period and \(p\) is the number of observations.

### 4 The Simulation Model

In this study a near beer distribution game model is used; which is extended from the Paik’s [26] model with predetermined cost items (holding, setup / production), inventory restrictions, production restrictions and delay functions. The beer game model; which is an effective tool for investigating the behaviors and factors effecting Sc performance as it successfully can reflect the attitude of the real life models, was introduced by Sterman [27].

The model proposed here, is simply a two staged Sc system consists of a retailer and a factory. Due to its common use and successful primed fuzzy functions; Matlab is the adjudicated simulation tool. Demand information in each period can be either crisp or fuzzy depending on the forecasting model that will be analyzed; similar to our previous model [2]. The model evaluates the Sc performance by computing the ratio of demand variances of consequent stages; i.e.

\[
\frac{\text{Var}D_{S_i}}{\text{Var}D_{S_{i+1}}}, \text{where } D_{S_i} \text{ denotes the demand from stage } S \text{ to upstream stage } S_{i+1}\text{ (i.e } i = c, r, f \text{ where } c, r, f \text{ represent customer, retailer, factory respectively. The cost, delay and factory production capacity parameters are variable and their values depend on the analyzer. The generic decision rule in each time period } t \text{ can be summarized with the following equation [2, 27].}
\]

Upstream Order Quantity = [Forecast Value + Correction of Inventory + Correction for Supply Line] \hspace{1cm} (19)

Simple exponential smoothing (EXS) model is used as a crisp forecasting technique for comparison. The customer order received from retailer in period \(t\) is taken as a base for forecasting the forthcoming demand, and each time the order received, forecasting function update its structure according to the new demand information. After estimating the forthcoming demand; simulation model, using the decision rule in (19) and other parameters (i.e. cost, lead time, availability, demand pattern etc.), makes an ordering decision to upper echelon of Sc. And the ratio of variability between the customer orders, retailer orders and manufacturing decision of factory shows the performance of Sc system based on the selected forecasting model.

### 5 Experiment

The following figures illustrate randomly generated \(D_c\) (the same for all simulation runs), and calculated \(D_c\) and \(D_f\) values derived from the simulations using selected forecasting methods for a time horizon of 30 periods. And to reflect the response of Sc performance to the selected forecasting model, the calculated standard deviation values are given Table-1. In the simulations, the production capacity of factory is taken as 100 units per period and on hand inventory in each echelon at time zero is set to again 100 units. The set of \(D_c\) values for 30 periods; \(D_{set}\) are given below. The results gathered from FTs forecasting model are not illustrated as they are similar to FR results.

\[D_{set} = \{66.2297; 62.7269; 76.2016; 56.5420; 36.5401; 47.0135; 1.0196; 59.4657; 52.3389; 38.1779; 36.9058; 28.3868; 49.0454; 57.5869; 43.3928; 40.0020; 49.2804; 46.5048; 30.8547; 75.8510; 58.8182; 75.6188; 40.0454; 27.6257; 66.1530; 50.8240; 34.8758; 48.2174; 9.8230; 71.6784\}.\]

#### Table-1. Standard deviation values

<table>
<thead>
<tr>
<th>Forecasting Model</th>
<th>EXS</th>
<th>FR</th>
<th>FGG</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Sd_c)</td>
<td>18.24</td>
<td>18.24</td>
<td>18.24</td>
</tr>
<tr>
<td>(Sd_r)</td>
<td>65.39</td>
<td>28.86</td>
<td>81.13</td>
</tr>
<tr>
<td>(Sd_f)</td>
<td>33.28</td>
<td>29.42</td>
<td>44</td>
</tr>
</tbody>
</table>

Fig.1. The demand response to EXS forecasting model (i.e. W.E.)
6 Research Findings and Conclusion

In this study the effects of selected fuzzy forecasting models (FR, FGG and FTs) to Sc performance is examined by using measured demand variability (i.e. WE). As a crisp base technique, EXS forecasting model is chosen for comparison. The results exposed that the fuzzy forecasting model used in the study; except FGG, considerably increased the performance of proposed Sc system decreasing demand variability through the chain. Further researches can be made using fuzzy lead times as to more adapt the model to complex real-world Sc systems.

References:
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