Comparison of Global Histogram Methods for 2D and 3D Entropy Based Image Segmentation

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Abstract: - In this paper we explain two functional models for entropy based image segmentation: using global 2D and 3D image histograms of grayscale images. We compare results obtained by both methods for different type of images such as: biomedical and micro objects, digital video records and natural photographs. Short introduction of multistage gradient 3D entropy segmentation for texture analysis is also introduced. 2D and 3D entropy segmented images can be combined to achieve better results in applications such as digital video databases and multimedia information retrieval frameworks, micro object classification for material analysis and biomedical applications.

Key-Words: - 2D, 3D global entropy image segmentation

1 Introduction
Image acquisition and processing is becoming more and more popular in many consumer and specialized electronic systems today. Since space, medical and military applications are de-facto standard technological fields where image processing is taking place, many high speed systems were developed to analyze and store all these data. Today we have new challenges; there are many new commercially available multimedia rich applications; such as internet image databases, digital video and video on demand etc. Many new semantic based models for video content were developed. These systems need to analyze visual content [1] in multidimensional and dynamic environment. Here image and motion segmentation take an important role in development of new object based video compression and user interaction models.

In past five years our research was focused on development of new statistically based models for motion image sequences segmentation. Our model is well explained in [2]. In this article we describe two types of image segmentation using global image histogram multistage entropy function. We describe how 2D (grey level pixel probability distribution function) and 3D (joint pixel probability distribution function) can be used for automatic and optimal image segmentation in different applications such as: natural photography, micro objects inspection systems and digital video records for security applications. All of these applications need a fast and reliable model for automatic image and motion segmentation. Many non specialized applications such as consumer digital photography data bases need a more complex approach to achieve reliable feature and object segmentation. These applications use color images with higher spatial resolution. Color based model for image segmentation use both 2D and 3D multistage entropy functions and color image histograms.

2 2D Entropy based image segmentation
Entropy function of 2D grey scale image histograms can be used for automatic image segmentation [3]. Since 2D image histograms represent only the probability distribution of single pixels inside an image, segmented images lose some important spatial data (object edges and surrounding brightness transition zones between objects and background). 2D entropy function can be used for fast segmentation, but all images need to be processed with high pass or background removal filter first. This is important especially for microscopic images. In some cases of object retrieval these techniques can be used for general purpose applications like; multimedia data segmentation and motion detection.

2.1 Entropy calculations
All statistical operations are performed on normalized image histograms (or PDF – probability distribution functions). For 8 bit grey level images we have corresponding 2D normalized histogram:

\[ P_{(2D)} = \{ p_0, p_2, ..., p_8, ..., p_{255} \} \]  

(1)

where \( P_{(2D)} \) is normalized image histogram, \( p \) - is probability to occur corresponding pixel having brightness \( b \), where \( b = 0, 1, ..., 255 \).
Now we calculate image entropy $H(P_{(2D)})$ using discrete histogram $P_{(2D)}$.

$$H(A) = -\sum_{i=0}^{b} p_i \log p_i \quad (2)$$

$$H(B) = -\sum_{i=b}^{255} p_i \log p_i \quad (3)$$

$$H_b = -\log P(A) - \log P(B) - \frac{H(A)}{P(A)} - \frac{H(B)}{P(B)} \quad (4)$$

Where $b = \text{var from 0 to 255}$

$A = \text{var from 0 to b}$

$B = \text{var from 255 to b}$

$$H(P_{(2D)}) = \{H_b, H_1, ..., H_{255}\} \quad (5)$$

After this we can find entropy maximum,

$$\max H(P_{(2D)}) \quad (6)$$

Maximum of entropy function defines the brightness threshold value – $b^*$ for the image (Fig. 1.a and Fig. 1.b).

![Fig.1.a, Grey scale image (right) and its 2D PDF (left)](image)

![Fig.1.b, BW image (right) calculated using threshold level defined by 2D entropy function (left)](image)

### 2.2 Multistage 2D entropy segmentation

Single stage 2D entropy function is fast and automatic approach for threshold different types of images. But in many cases especially when we work with outdoor and complex images this is not sufficient for optimal object segmentation. Thus the multistage 2D entropy function can be used. Multistage entropy function [4] defers from single stage only by number of iterations and histogram divisions by part A and B. Simple 4 stage entropy function can provide us the possibilities to automatically threshold input image on 4 layers. Algorithm is as follow:

1. Calculate $b^*$ from $H(P_{(2D)})$, $b = \text{var from 0 to 255}$
2. Divide histogram in two equal histograms $P_1_{(2D)}$ and $P_2_{(2D)}$

$$p_{1,b} = \begin{cases} p_b & \text{if } b^* \leq \text{max} \\ 0 & \text{if } b^* > \text{max} \end{cases} \quad (7)$$

$$p_{2,b} = \begin{cases} p_b & \text{if } b^* \geq \text{max} \\ 0 & \text{if } b^* < \text{max} \end{cases} \quad (8)$$

Where $b = \text{var from 0 to 255}$

3. Calculate $H(P_1_{(2D)})$ and $H(P_2_{(2D)})$
4. Calculate $b_1^*$ for $P_1_{(2D)}$
5. Calculate $b_2^*$ for $P_2_{(2D)}$
6. Now we have $b_1^*$, $b^*$, $b_2^*$ that are maximum entropy functions calculated for different $P_{(2D)}$ intervals. We use these values to threshold input image on 4 layers (Fig. 2.a and Fig. 2.b).

![Fig.2.a, Multi threshold – 4 layers, using fixed values](image)

![Fig.2.b, Multi threshold – 4 layer image, using multistage 2D entropy (right) and its entropy (left)](image)

This function can be used to create image object extraction mask for multilayer object segmentation. We can continue this local entropy segmentation using the same procedure for any new entropy region inside image histogram. Practical limits for multi threshold to achieve better image segmentation are 4, 8 and 16 layers. In case we have motion images, we can use this procedure for calculating resulting background image.
3D Entropy based image segmentation

2D entropy function is commonly used for many image segmentation applications. 2D entropy lacks his robustness when multiple images with large variety of gradient brightness between each other and background need to be segmented. Here 3D entropy function comes in sense.

Before this we need to define and calculate 3D image histogram $P_{(3D)}$, this type of histogram represents the probability distribution of joint pixels (Fig. 3.a). Using this histogram we can calculate 3D entropy function (Fig. 3.b). Finding its maximum we can threshold image much better than using 2D entropy. This type of thresholding will provide us information regarding two types of joint pixels: homogeneity zones (joint pixels having the same brightness) and non homogeneity (object surrounding areas and shades).

3D entropy function is calculated as a sum of all entropy functions calculated for 3D histogram columns and rows. The maximum of this function can be found dividing 3D histogram in to 4 separate regions; A, B, C, D (Fig. 4). Zones A and B corresponds to object and background, and zones C and D corresponds to object shades and surrounding areas brightness gradient. In most of cases 3D entropy maximum lays on the main diagonal passing through A and B quadrants.

We calculate image 3D entropy function $H3D(P_{(3D)})$ using discrete histogram $P_{(3D)}$. 3D entropy is calculated for each column and row inside $P_{(3D)}$ using method described in section (2.1). After this we create two 3D entropy functions, first is called gradient entropy function and is presented by:

$$H3Dc_{(3D)} = \begin{pmatrix}
P_{0,0} & P_{0,1} & \ldots & P_{0,j} & \ldots & P_{0,255} \\
P_{1,0} & P_{1,1} & \ldots & P_{1,j} & \ldots & P_{1,255} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
P_{255,0} & P_{255,1} & \ldots & P_{255,j} & \ldots & P_{255,255}
\end{pmatrix}$$

And second for rows:

$$H3Dr_{(3D)} = \begin{pmatrix}
Hr_{1,0} & Hr_{1,1} & \ldots & Hr_{1,j} & \ldots & Hr_{1,255} \\
Hr_{2,0} & Hr_{2,1} & \ldots & Hr_{2,j} & \ldots & Hr_{2,255} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
Hr_{255,0} & Hr_{255,1} & \ldots & Hr_{255,j} & \ldots & Hr_{255,255}
\end{pmatrix}$$

After this we need to find entropy maximum:

$$\max (H3Dc(P_{(3D)}) + H3Dr(P_{(3D)})) / 2$$

Some implementations may consider 3D entropy function as a simultaneously calculated for each one point inside 3D histogram. We prefer to create 3D entropy function as a fraction of 2D columns and rows entropy. This kind of representation gives us the right to make image segmentation based on entire 3D entropy (Fig. 5) and on 3D gradient entropy, without having the need to recalculate 3D entropy each time we need to make segmentation for homogeneity and gradient zones.
3.1 Image segmentation using 3D gradient entropy

Because 3D entropy is based on 2D entropy calculation of single columns and rows we introduce a new 3D gradient entropy function. This function examines only the columns of 3D histogram, interpreting values on main homogeneity diagonal as zeros. This approach gives us a powerful method to calculate entropy only of brightness transition zones. This technique gives us the power of local adaptive 2D entropy segmentation method and histogram equalization using global 3D image histogram (Fig. 6).

3D gradient entropy can help us to find target object even if we work with noisy images having bad spatial resolution and contrast (Fig. 7).

3.2 Multistage 3D and gradient entropy

In section 2.2 we explained how a multistage segmentation mask can be calculated using $P_{(2D)}$ entropy based histogram division and local entropy definition. No considers we have 3D entropy function calculated for $P_{(3D)}$. In section 3 we define how simple BW threshold value can be calculated more precisely taking in account joint pixel probability distribution. Nothing easier if we treat 3D histogram quadrant A and B, C, D as we treated 2D histograms $P_{1(2D)}$ and $P_{2(2D)}$. In this case we will define $P_{1(3D)}$ and $P_{2(3D)}$ histograms (Fig. 7). Remember to fill all non target histogram quadrants as filled with zeros. Thus the multistage 3D entropy function is calculated.

3D gradient multistage entropy is calculated by calculating entropy functions for all columns of 3D histogram, treating main diagonal as filled with zeros. Difference between 3D multistage entropy and 3D gradient multistage entropy is shown on Fig. 8. Because gradient entropy analyze and group only image shading zones it can be used as a stage before image texture analysis.
Entire 3D entropy function trends to separate 3D histogram mainly inside main homogeneity diagonal, making easier to better extract image homogeneity zones and surrounding areas (Fig. 8,b). In opposite to this 3D gradient entropy separate 3D histogram in to different pikes around homogeneity diagonal (Fig. 8,e).

4 Complex method for multidimensional entropy based image segmentation

Above we have explained two global histogram methods for 2D and 3D entropy based multistage image segmentation. These methods can be combined in to multidimensional entropy image. Results obtained from 2D and 3D multistage entropy give us a good understanding about image object laying in large spatial areas and their edges and surrounding areas and shades. 3D gradient entropy multistage segmentation give us the information regarding image global shading, thus giving a god background for future texture analysis and processing using global histogram methods. Combining results achieved by 2D multistage entropy and 3D gradient entropy provide valuable information regarding large and small object statistical representation. Multistage entropy image 3D histograms have less data to be compared, making image classification faster, than using entire 3D histograms. Combined images are more similar to original one (Fig. 9).

Fig. 8, Multistage entropy image (a), and 3D entropy function (b) and histogram (c), 3D gradient entropy image (d), and its 3D entropy function (e), and histogram (f).

The most important in new model for complex multidimensional entropy based image segmentation is that we have more information for spatial image regions – objects and background, including information about the brightness gradient inside them. Because 3D image histograms can be interpreted in several ways; main diagonal homogeneity entropy, entire entropy and 3D histogram columns entropy, we can extract corresponding data for each one selected area inside 3D histogram space and finding corresponding objects inside grey scale image.

5. Multidimensional motion image segmentation using 3D multistage entropy function

In many cases image information is non static so or we need to segment images obtained in non fixed conditions. A concrete case described here is how 3D multidimensional image entropy can help us to achieve better and faster digital video segmentation. Such segmentation is important for many content analysis post processing techniques used in modern video and image data base indexing and summarization systems. All these methods work on vector space dealing with objects instead of images. For reliable motion based image segmentation we use 3 statistical criterions for digital video key frame detection: Kolmogorov, Pirson and Complex, our model is well explained in [2]. Using 2D and 3D multistage entropy images we can faster the process of motion detection simultaneously achieving object segmentation and statistical classification, see section 7.
7. Experimental results

3D image entropy is a global method to statistically process digital images producing the similar results as statistical mode filtration, histogram stretching and local 2D entropy definition. Improvements can be significant in images that have large amount of low pass noise, such as scanned documents (Fig. 10).

![Image](image1.png)

Fig. 10, Image segmented using 2D entropy function (a) and 3D gradient entropy (b).

Multistage entropy improves video [6] motion segmentation (Fig. 11).

![Image](image2.png)

Fig. 11, Improving motion segmentation using multistage entropy. Original sequence (a) and its corresponding 8, 4 and 2 layer entropy frames are shown. Shows statistical criterion calculated for original 8 bit image (b), for 4 bit entropy image (c), for 3 bit entropy (d) image and for 1 bit entropy image (e).

4 Conclusion

After this short overview on different types of image entropy we can conclude that entropy segmentation plays an important role in mass image analysis systems. In some simple cases when working with good and filtered images we can use single stage 2D entropy function. For better region segmentation in more shaded images we can use multistage 2D entropy function. For better spatial object segmentation we definitely recommend the use of global 3D entropy function and its multistage variant. In applications that we use large images having different content (photographs) we recommend to use multidimensional 2D and 3D multistage and gradient entropy function, providing much easier and faster way for region segmentation. Entropy segmented images can help us to achieve better and faster motion segmentation, when we need to process long time video records and multimedia data. Entropy segmentation can be used in color images, but in this case we need to convert RGB data format to HSL color space and threading this 3D space with multidimensional entropy function. Several improvements need to be done for proposed here 3D gradient entropy. The main problem is how to achieve better strategy for statistical image artifacts removal simultaneously with 3D entropy calculation.

References:


