Abstract: Dynamic modeling of osteoarticular system is necessary because the exertion of various actions and natural physiological movements are essentially dynamic. The wide mobility of human body leads to the necessity of modeling the osteoarticular system as a mechanism with a large number of degrees of freedom. The differential equations system obtained are very complex and assume numerical integration. Very often we use a simplified model because the phenomena produced are so complex that accurate mathematical reproduction is practically impossible. A dynamic model must provide a good estimate of total weight and mass distribution as well as transmissibility and amortization properties for bones, muscles, joints, blood and skin.

The paper presents a dynamic functional model considering the human upper limb as a mechanic system with 5 degrees of freedom in the case the segments are moved by their own weight forces. The bones and the muscles were modeled in Solid Works, the model of the upper limb obtained being very close as form to the real one. The differential equations of motion were solved using Lagrange formalism.

Key-Words: modeling, human upper limb, dynamic model, osteoarticular system

1 Introduction

The research regarding the mechanical aspect of the human body’s osteoarticular system can be successfully made through classical and modern calculus and experimental engineering methods. Thus, the osteoarticular system can be seen by the engineer as being a deformable special structure, with a considerable complexity regarding its geometry, elastic properties and its functions.

Although the biomechanical modeling of the osteoarticular system is subject to the same general laws and principles which are used in engineering, we still have to consider the fact that there are some differences which limit the possibilities of this research method. As a consequence a model must be conceived and investigated as to determine within certain very accurate limits, the behavior of the original system. The inbred variability of geometry and of the mechanical properties of the osteoarticular system, from person to person, is one of the aspects that generate considerable supplementary difficulties regarding the accomplishment of biomechanical researches and which always must be taken into consideration.

The dynamic modeling of the osteoarticular system is necessary because the performance of some actions and common physiological motions is in its essence dynamic.

The high mobility of the human body leads to the necessity of the modeling of the osteoarticular system as a mechanism with a high degree of flexibility. The differential equation system which is obtained is complex and it implies numerical integration. In most cases, we use a simplified model because the occurring phenomena are so complex that a precise mathematical replica is virtually impossible.

The 3D model determination of some fundamental sizes in the study of dynamics is necessary for: improving the prosthesis and the metallic implants of the upper limb and their making in accordance with the particularities of each case, the assessment in which bone fractures are made (in most of the cases dynamic: falls, slips, impacts), with implications in forensic medicine; knowing the circumstances in which the fractures occur through shock etc.

In order to carry out a functional dynamic model of the upper limb as closely as possible to the real one, it is necessary to study the properties of the biological materials that make up the human body, so that the measures involved in the differential equations are as correct as possible [2], [5], [10].
2 The Dynamic Model of the Human Upper Limb

It is consider the upper limb’s structure as a mechanical system with 5 degrees of freedom, subject only to its own friction forces, so that the differential equations of motion will be determined by using Lagrange’s equations [1], [3].

Taking into consideration the simplified geometrical model and the spatial structure which physically models (molds) the upper human limb, the total kinetic energy is obtained:

\[
E_k = \frac{1}{2} \cdot J^{(ms)}_{Z0} \cdot \dot{q}_1^2 + \frac{1}{2} \cdot J^{(ms)}_{Z1} \cdot \dot{q}_2^2 + \frac{1}{2} \cdot J^{(ms)}_{Z2} \cdot \dot{q}_3^2 \\
+ \frac{1}{2} \cdot J^{(r+p+d)}_{Z3} \cdot \dot{q}_4^2 + \frac{1}{2} \cdot J^{(r+p+d)}_{Z4} \cdot \dot{q}_5^2
\]

(1)

where \( z_0, z_1, z_2 \) represent the rotation axis corresponding to the three motions at level of the shoulder (flexion-extension, abduction-adduction, rotation), \( z_3 \) the axis corresponding to the flexion-extension motion at the elbow level, \( z_4 \) the rotation axis corresponding to the flexion-extension motion at the level of the wrist, and \( J_{Z0}, J_{Z1}, J_{Z2}, J_{Z3}, J_{Z4} \), the inertia moments in connection to these axes.

The following notes have been made: \( ms \) – upper limb, \( r \) – radius, \( p \) – palm, \( d \) – fingers, also standing for the lengths of the afferent parts.

The inertia moments in connection with the axis \( z_0, z_1, z_2, z_3, z_4 \) are calculated by applying Steiner’s formulas [3]:

\[
J^{(p+d)}_{Z4} = J^{p+d}_{IIZ4, c(p+d)} + \left( \frac{p + d}{2} \right)^2 m_{(p+d)} =
\]

(2)

\[
J^{(r+p+d)}_{IIZ4, c(p+d)} + \frac{(p + d)^2}{4} m_{(p+d)}
\]

\[
J^{(r+p+d)}_{IIZ3, c(p+d)} = J^{p+d}_{IIZ3, c(p+d)} + \left( \frac{r + p + d}{2} \right)^2 m_{(p+d)} + J^{(r)}_{Z3}
\]

(3)

\[
J^{(ms)}_{Z2} = J^{p+d}_{IIZ2, c(p+d)} + \left( \frac{p + d}{2} \sin q_5 \right)^2 m_{(p+d)} +
\]

\[
J^{(r)}_{IIZ2, c(r)} + \left( \frac{r}{2} \sin q_4 \right)^2 m_{(r)} + J^{(h)}_{IIZ2, c(h)} + \frac{h^2}{4} m_{(h)}
\]

(4)

\[
J^{(ms)}_{Z1} = J^{p+d}_{IIZ1, c(p+d)} + \left( \frac{p + d}{2} \cos q_5 \right)^2 m_{(p+d)} +
\]

\[
J^{(r)}_{IIZ1, c(r)} + \left( \frac{r}{2} \cos q_4 \right)^2 m_{(r)} + J^{(h)}_{IIZ1, c(h)} + \frac{h^2}{4} m_{(h)}
\]

(5)

\[
J^{(ms)}_{Z0} = J^{p+d}_{IIZ0, c(p+d)} + \frac{(p + d)^2}{4} m_{(p+d)} + J^{(r)}_{IIZ0, c(r)} +
\]

\[
r^2 \frac{m_{(r)}}{4} + J^{(h)}_{IIZ0, c(h)} + \frac{h^2}{4} m_{(h)}
\]

(6)

Although the bones do not show any homogeneous structure, the value of the weight is that calculated based on the average density: \( \rho = 1,3 \) g/cm\(^3\). Approximation leads to results compatible with those from literature [4], [6], [12]. For the arm with muscles the muscular density \( \rho = 1,13 \) g/cm\(^3\) has been taken into consideration.

With the help of the Solid Works program [14], knowing the bone dimensions and the muscle and bone densities, these temporary figures can be directly calculated by putting the reference systems directly into couples with the \( z \) axis orientated along the axis of those respective couples. Figure 1 shows the human upper limb model for which the inertia moment calculus is made, the axis in connection to which this calculus is made, as well as the results obtained [7], [8].

Fig.1. The calculus for the inertia moments for the whole arm (upper limb)

The kinetic energy derivates compared to generalized speeds and their derivates compared to time, as well as the derivates of the kinetic energy compared to the generalized coordinates become:
\[ \dot{q}_2 = \frac{p + d}{2} \dot{q}_2 m_{(p+d)} \sin 2q_5 - \frac{r}{2} \dot{q}_4 m_{(r)} \sin 2q_4 \]

\[ + J_{Z1}^{(m,s)} \dot{q}_2 = -m_4 g \frac{h}{2} \cos q_1 \cos q_2 - m_4 g (h \cos q_1 \cos q_2 + \frac{r}{2} \cos q_4 \cos q_2) - m_{(p+d)} g (h \cos q_1 \cos q_2 + r \cos q_4 \cos q_2) - \frac{p + d}{2} \cos q_4 \cos q_2 \]

\[ \dot{q}_3 = \frac{p + d}{2} \dot{q}_2 m_{(p+d)} \sin 2q_3 + \frac{r}{2} \dot{q}_4 m_{(r)} \sin 2q_4 \]

\[ + J_{Z2}^{(m,s)} \dot{q}_3 = 0 \]

\[ J_{Z3}^{(p+d)} \ddot{q}_4 - \frac{1}{2} m_{(r)} \frac{r}{2} \sin 2q_4 (\dot{q}_4^2 - \dot{q}_2^2) = -m_{r} \frac{r}{2} \cos q_4 - m_{(p+d)} g (r \cos q_4 + \frac{p + d}{2} \cos q_3) \]

\[ J_{Z4}^{(p+d)} \ddot{q}_5 - \frac{1}{2} m_{(p+d)} \frac{p + d}{2} \sin 2q_5 (\dot{q}_3^2 - \dot{q}_2^2) = -m_{(p+d)} g (\frac{p + d}{2} \cos q_5) \]

By replacing in the equations (7)-(10) the figures for the inertia moments we have:

\[
\begin{align*}
I_{Z1}^{(m,s)} & = I_{x1} = 0.179203192 \text{ kg}\cdot\text{m}^2 \\
I_{Z2}^{(m,s)} & = I_{y2} = 0.180021932 \text{ kg}\cdot\text{m}^2 \\
I_{Z2}^{(p+d)} & = I_{z2} = 0.001317225 \text{ kg}\cdot\text{m}^2 \\
I_{x3}^{(p+d)} & = I_{xx} = 0.023420715 \text{ kg}\cdot\text{m}^2 \\
I_{y3}^{(p+d)} & = I_{yy} = 0.000662244 \text{ kg}\cdot\text{m}^2 \\
\end{align*}
\]

weights: \( m_{(p+d)} = 0.14366 \text{ kg (hand)}, m_{(r)} = 0.83461 \text{ kg (forearm)}, m_{ho} = 0.103785 \text{ kg (upper part of the human upper limb)}, m_{aul} = 2.01612 \text{ kg (upper limb)} \)

and the bone sizes \( h = 0.29 \text{ m}, r = 0.235 \text{ m}, p + d = 0.16 \text{ m} \), the following differential equations system is obtained:

\[ \ddot{q}_1 = -23.78 \cos q_1 - 7.22 \cos q_4 - 0.62 \cos q_5 \]

\[ \ddot{q}_2 = 0.06 \dot{q}_2 \dot{q}_5 \sin 2q_5 + 0.54 \dot{q}_2 \dot{q}_4 \sin 2q_4 - 21.60 \cos q_1 \cos q_2 - 71.83 \cos q_2 \cos q_4 - 0.62 \cos q_2 \cos q_5 \]

\[ \ddot{q}_3 = -8.46 \dot{q}_3 \dot{q}_5 \sin 2q_5 - 75.38 \dot{q}_3 \dot{q}_4 \sin 2q_4 \]

\[ \ddot{q}_4 = 2.09 \dot{q}_3^2 \sin 2q_4 - 2.09 \dot{q}_2^2 \sin 2q_4 - 55.25 \cos q_4 - 4.78 \cos q_5 \]

\[ \ddot{q}_5 = 8.33 \dot{q}_3^2 \sin 2q_5 - 8.33 \dot{q}_2^2 \sin 2q_5 - 186.66 \cos q_5 \]

3 Solving the Differential Equation System for the Simplified Model of the Human Upper Limb

The integration of the differential equations system that represents the reduced model of the upper limb has been made in MatLab 7.01, using the following notations [6], [9], [11], [13]:

\[ \dot{q}_1 = y(1), \dot{q}_2 = y(2), \dot{q}_3 = y(3), \dot{q}_4 = y(4), \dot{q}_5 = y(5) \]

\[ \dot{q}_1 = y(6), \dot{q}_2 = y(7), \dot{q}_3 = y(8), \dot{q}_4 = y(9), \dot{q}_5 = y(10) \]

On the basis of the numerical values and the articulator variables and the articulator speeds obtained as a result of integrating for \( t \in [0,0.4] \) and the initial conditions different from zero, the graphic representations of the motion laws and speed corresponding to these results have been obtained (figure 2).

As solving of the differential equation system is numerical, the statistical processing of the data symbolized in figure 2 is particularly important for the variation of the articulator variables \( q_i \) written down as \( y_{i1} \) in the MatLab program. The main statistical estimates are represented in figure 3 from which results: the mean values during the specified time, the median values and the global variation interval indicated as such through the respective extreme values.
Fig. 2. Graphic diagram of the motion laws and speeds, $t \in [0,0.4]$

Fig. 3. Statistic estimator for variation or articular variables $q_i$, $t \in [0,0.4]$

4 Conclusion
In order for the results of the dynamic modeling and to process a prosthetics for the upper limb based on outcome obtained by integrating the differential equations, the approximation of the functions $y_i = f(t)$ can be made through the orthogonal polynomial method. After obtaining them, the opposed method can be applied in the differential equations of motion (12)-(16) to calculate the generalized forces that can be put in effect with the help of couple engines which will develop active moments in accordance with the results from the mentioned equations.

The variant studied in the paper can be extended to the case in which the initial conditions are null and the time period is longer than 0.4 seconds.

Both the obtained representations as well as the possible due to the changing of the initial conditions or the period of time, allow multiple subsequent uses.

References: