Optimal Nonlinear Predictive Visual Servoing of a Wheeled Mobile Robot

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Abstract: - This paper develops methodologies and techniques for control architecture design, path tracking laws and posture estimation of a vision-based wheeled mobile robot (WMR). To solve the problem of position/orientation tracking control of the WMR, two kinematical optimal nonlinear predictive control laws are developed to manipulate the vehicle to asymptotically follow the desired trajectories. A Kalman filtering scheme is used to reduce the bad effect of the imaging noise, thereby improving the accuracy of pose estimation. The experimental system is composed of a wireless RS232 modem, a DSP-based controller for the wheeled mobile robot and a vision system with a host computer. A computation-effective and high-performance DSP-based controller is constructed for executing the developed sophisticated path tracking laws. Simulation and experimental results are included to illustrate the feasibility and effectiveness of the proposed control laws.

Key-Words: - Predictive control, Mobile robot, Path tracking, Nonlinear characteristics, Kalman filtering

1 Introduction
This study has been mainly excited by a wide variety of practical mobile robots applications due to their ability to work in various domains. WMRs have already gained widespread applications, such as planetary exploration, materials transportation, military tasks, manufacturing servicing, hazardous environment and mine excavation. To achieve the aforementioned tasks, the WMR requires sensing of the environments, intelligent trajectory planning, navigation and path tracking control. This desired autonomous or intelligent behavior has motivated an intensive research over the past decades.

To achieve path-tracking control for the WMRs, many sophisticated control approaches have been investigated by several researchers. The existing tracking control methods for the WMR can be classified into five categories: (i.) sliding mode [1]; (ii.) nonlinear control [2-3]; (iii.) fuzzy control [4]; (iv.) neural network control [5]; and (v.) adaptive backstepping control [6]. In 2007, Qiuling et al. used sliding mode control for tracking control, which is complicated and computationally expensive. The generated velocity command with respect to time is not a smooth curve in [1]. Lei et al. introduced the fuzzy tracking control approaches [4] in 2006. But it is very difficult to formulate the fuzzy rules, which are usually obtained from the trial-and-error procedure. In 2006, the computational expensive neural network was adopted by Heinen et al. [5]. The algorithm requires on-line learning in order to make the robot perform properly.

In this paper, we introduced two optimal nonlinear predictive control approaches [7] for manipulators. The control laws minimized a quadratic performance index of the state predicted tracking error. The algorithms were shown to improve tracking accuracy of the manipulators.

In addition, Kalman filter police has been proposed in [8]. In this paper, we extend the Kalman filter method to deal with the pose estimation problem of the WMR with corrupted imaging noise.

This paper is organized as follows. Camera-Space Tracking Control is presented in Section 2, and Section 3 aims at developing two optimal nonlinear predictive control approaches. Section 4, an extended Kalman filtering scheme is adopted to filter out the corrupted noise in the images. In Section 5 simulation and experimental results are presented. Section 6 concludes this paper.

2 Camera-Space Tracking Control
2.1 Vision-based Control System
The control objective for the vision-based tracking problem is to manipulate the WMR to follow the desired trajectory. This system is composed of a ceil-mounted fixed camera whose outputs are connected to a host computer, and a
WMR with two atop different color, round marks (in Fig.1).

The host computer is used to periodically provide the position and orientation information of the WMR for tracking the reference trajectory, via the wireless RS232 modem. The operation of such a control system can be easily understood from the system block diagram shown in Fig.2.

2.2 Preliminary Background

2.2.1 Kinematic Model

Consider a nonholonomic WMR under the assumption of pure rolling and non-slipping. Its kinematic model is given by

\[ \dot{q} = S(q) \cdot u \]  (1)

Where \( q(t), \dot{q}(t) \in \mathbb{R}^3 \) are defined by

\[ q = \begin{bmatrix} x_c & y_c & \theta \end{bmatrix} \quad \dot{q} = \begin{bmatrix} \dot{x}_c & \dot{y}_c & \dot{\theta} \end{bmatrix}^T \]  (2)

\( x_c(t), y_c(t) \) and \( \theta(t) \) represent the position/orientation of the WMR in Fig.3 respectively, the matrix \( S(\cdot) \in \mathbb{R}^{3 \times 2} \) is defined as follows:

\[ S(q) = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{bmatrix} \]  (3)

And the velocity vector \( u(t) \in \mathbb{R}^2 \) is denoted by

\[ u = \begin{bmatrix} v_1 \\ v_2 \\ \theta \end{bmatrix}^T = \begin{bmatrix} v_1 \\ \theta \end{bmatrix}^T \]  (4)

We defined one another Cartesian position/orientation in the camera space by \( \bar{q} = \begin{bmatrix} \bar{x}_c \\ \bar{y}_c \\ \bar{\theta} \end{bmatrix} \), and thus, the kinematic model in the camera-space takes the same form

\[ \dot{\bar{q}} = S(\bar{q}) \cdot \bar{u} \]  (5)

Where the velocity vector \( \bar{u}(t) = \begin{bmatrix} \bar{v}_1 \\ \bar{v}_2 \end{bmatrix} \in \mathbb{R}^2 \) represent the linear and angular velocities in the camera-space.

In order to control the WMR in the camera-space to track a camera-space desired trajectory (i.e., \( x_{rc}, y_{rc} \) and \( \theta_{rc} \)), task-camera space transformations is required such that the proposed velocity vector in the task-space is able to effectively and correctly drive WMR in the camera-space.

Instead of generating the reference trajectory in the task-space using the vision system, we specify the desired velocity vector \( \bar{u}_{rc} \) in the camera-space such that the reference trajectory can be produced in the camera-space via the following expression

\[ \dot{\bar{q}} = S(\bar{q}) \cdot \bar{u}_{rc} \]  (6)

2.2.2 Kinematic Tracking Control Design

To develop the control law, we have to find out the open loop error system in terms of \( \bar{x}(t), \bar{y}(t), \) and \( \tilde{\theta}(t) \in \mathbb{R}^1 \) which denote the differences between the camera-space Cartesian posture and the desired posture by...
\[ \dot{x} = \ddot{y}_{re} - \dot{y}_{re} \quad \ddot{y} = \ddot{y}_{re} - \dot{y}_{re} \quad \ddot{\theta} = \dot{\theta} \quad \Theta = \Theta \quad (7) \]

To facilitate the control design process, we employ a well-known global invertible transformation between \( \dot{x}(t) \), \( \ddot{y}(t) \) and \( \Theta(t) \) the auxiliary error signal.

\[
e(t) = \begin{bmatrix} \epsilon_1(t) & \epsilon_2(t) & \epsilon_3(t) \end{bmatrix}^T \in \mathbb{R}^3
\]

\[
\begin{bmatrix}
\epsilon_1 \\
\epsilon_2 \\
\epsilon_3
\end{bmatrix} =
\begin{bmatrix}
\cos \ddot{\theta} & \sin \ddot{\theta} & 0 \\
-\sin \ddot{\theta} & \cos \ddot{\theta} & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta}
\end{bmatrix}
\quad (8)
\]

We observe that \( \epsilon_1(t), \epsilon_2(t), \epsilon_3(t) \) denotes the errors in the tangent direction, normal direction and orientation, respectively. With the invariability of transformation in Eq. (8), it is easy to prove that

\[
\lim_{x \to \infty} \epsilon_1(t), \epsilon_2(t), \epsilon_3(t) = 0 \iff \lim_{x \to \infty} \ddot{x}(t), \ddot{y}(t), \dot{\theta}(t) = 0
\]

by considering

\[
\begin{bmatrix}
\dot{\epsilon}_1 \\
\dot{\epsilon}_2 \\
\dot{\epsilon}_3
\end{bmatrix} =
\begin{bmatrix}
-1 & \epsilon_2 \\
0 & -\epsilon_1 \\
0 & -1
\end{bmatrix}
\begin{bmatrix}
\ddot{v}_1 \\
\ddot{v}_2 \\
\ddot{v}_3
\end{bmatrix}
+ \begin{bmatrix}
\ddot{v}_1 \cos \epsilon_3 \\
\ddot{v}_1 \sin \epsilon_3 \\
\ddot{v}_2 \sin \epsilon_3
\end{bmatrix}
\quad (9)
\]

Let

\[
P = \begin{bmatrix}
-1 & \epsilon_2 \\
0 & -\epsilon_1 \\
0 & -1
\end{bmatrix}, u = \begin{bmatrix}
\ddot{v}_1 \cos \epsilon_3 \\
\ddot{v}_1 \sin \epsilon_3 \\
\ddot{v}_2 \sin \epsilon_3
\end{bmatrix}, \Pi = \begin{bmatrix}
\ddot{v}_1 \\
\ddot{v}_2 \\
\ddot{v}_3
\end{bmatrix}
\]

\[
J_1 = \frac{1}{2} e^T(t + h) \cdot Q \cdot e(t + h) + \frac{1}{2} \bar{u}^T \cdot R \cdot \bar{u}
\quad (13)
\]

\[
\bar{u} = \begin{bmatrix}
\ddot{v}_1 \\
\ddot{v}_2 \\
\ddot{v}_3
\end{bmatrix}, Q = Q^T \geq 0, R = R^T > 0
\quad (14)
\]

where \( Q = Q^T \geq 0 \in \mathbb{R}^{3 \times 3} \) is a semi positive-definite matrix and \( R = R^T > 0 \in \mathbb{R}^{2 \times 2} \) is a positive definite matrix. The minimization of \( J_1 \) with respect to \( u(t) \) yields:

\[
\frac{\partial J_1}{\partial \bar{u}} = (\ddot{e}^T \cdot Q \cdot e(t + h) + R \cdot \bar{u}^T) = 0
\quad (15)
\]

Therefore, we have the optimal control vector as

\[
\bar{u}^*(t) = -(P^T \cdot Q \cdot P)^{-1} \cdot P^T \cdot Q \cdot (h^{-1} \cdot e(t) + \Pi) = \begin{bmatrix}
\ddot{v}_1 \\
\ddot{v}_2
\end{bmatrix}
\quad (16)
\]

### 3.2 A Finite Horizon Nonlinear Predictive Control

In this section we present kinematically tracking controllers based on minimization of the predictive state tracking errors. The two nonlinear predictive control schemes are developed to allow the vehicles position and orientation tracking of a desired reference trajectory.

**3.1 A fixed End Point Predictive Controller**

We improve tracking accuracy at next instant \( (t + h) \), where \( h \) is a small time increment. That is, the tracking error is defined as:

\[
e(t + h) \simeq e(t) + h \cdot \dot{e}(t)
\quad (11)
\]

\[
\simeq e(t) + h \cdot (P \cdot \bar{u} + \Pi)
\quad (12)
\]

In order to find the control vector \( u(t) \) that improves tracking error at next instant, we consider a dynamic performance index that penalizes the predictive tracking error and control efforts.

\[
J_2 = \frac{1}{2} \int_0^h e(t + T)^T \cdot Q \cdot e(t + T) \cdot dT + \frac{1}{2} \int_0^h \bar{u}^T(t + T) \cdot R \cdot \bar{u} \cdot dT
\quad (18)
\]

where \( h \) is the predicted control signal horizon, \( Q = Q^T \geq 0 \in \mathbb{R}^{3 \times 3} \) is a positive semi definite matrix and \( R = R^T > 0 \in \mathbb{R}^{2 \times 2} \) is a positive definite matrix. The minimization of \( J_2 \) with respect to \( \bar{u}(t) \) yields:

\[
\frac{\partial J_2}{\partial \bar{u}} = 0
\quad (19)
\]

\[
\bar{u}^* = -(\frac{1}{2} h^2 P^T Q P + R)^{-1} \cdot \left(\frac{1}{2} h^2 P^T Q \Pi - P^T Q e(t)\right)
\quad (20)
\]

### 4 Pose Estimation Using Kalman Filtering

When the noise in the real mobile robot occurs, Kalman filter police has been proposed in [8]. In this paper, we extend the Kalman filter method to deal with the pose estimation problem of the WMR with corrupted imaging noise.
5 Simulations and Experimental Results

5.1 Simulation Results

Before preceding the following experiment, numerical simulations using Matlab/Simulink were used to illustrate the feasibility of the proposed controllers and to verify the effectiveness of the proposed methods.

Line trajectory tracking responses. Fig. 4 show the A fixed-end point predictive controller: (a) tracking response. For the finite-horizon nonlinear predictive controller: (b) tracking response.

Fig. 4 Line trajectory tracking responses

Circle trajectory tracking responses. Fig. 5 show the A fixed-end point predictive controller: (a) tracking response. For the finite-horizon nonlinear predictive controller: (b) tracking response.

Fig. 5 Circle trajectory tracking responses

Fig. 6(a) depicts the line-tracking path that is corrupted with the noise. Fig. 6(b) shows that the corrupted noise is removed. Fig. 6(c) depicts the circle-tracking path that is corrupted with the noise. Fig. 6(d) shows the filtered circle trajectory. Fig. 6 reveals that the noise effect on posture estimation has been significantly reduced.
5.2 Experimental Results

A recent picture of the wheeled mobile shows in Fig. 7. Fig. 8 depicts the flowchart of the control codes programming, which is helpful for the coding of the control laws for the path tracking and velocity PID control.

Fig. 9(a) shows the resulting tracking responses for the camera-space desired trajectory line. Furthermore, Fig. 9(b) displays the resulting tracking responses for the camera-space desired circle trajectory.

The computer simulation results and experimental results have verified that two the predictive path tracking controllers really achieve the mission of path tracking control. From the experimental results illustrated in Fig. 6 there exist fluctuations caused by the image noise. Although sometimes the WMR seems to detour the desire trajectory in the camera space, the image-recognition process maybe cause the errors. This situation can be avoided if the experimental setup system is using Kalman filter.

Fig. 7 A recent picture of the wheeled mobile

Fig. 6 The corrupted noise and Kalman filter trajectory tracking responses

Fig. 8 Flowchart of the control codes programming
Fig.10(a) (b) shows that the Kalman filter tracking responses for the desired line trajectory experimental result. The Kalman filter indeed removes out measurement noise from measurements.

6. Concludes
Two kinematical, predictive path tracking control laws have been proposed in order to achieve asymptotical path tracking. The designed system together with proposed control laws have been successfully used to manipulate the WMR follow the desired reference trajectories. There are noise reduction for pose estimation of the WMR by applying the Kalman filter to remove the noise corrupted the captured images, so the Kalman filter can be applied in order to reduce the noise. Computer simulations and experimental results have also shown the feasibility and effectiveness of the proposed Kalman filter associated with imaging processing.

References