Implementation structure of 3-D FIR digital filters based on the transformation method

GUERGANA MOLLOVA
Dept. of Computer Aided Engineering
University of Architecture, Civil Engineering and Geodesy
1 Hr.Smirnenski Blvd., 1046 Sofia
BULGARIA

Abstract: - In this paper, we show the application of Chebyshev structure proposed by McClellan and Chan for design of three-dimensional (3-D) filters based on the McClellan transformation. We consider the implementation of 3-D FIR cone-shaped filters designed recently by Mollova and Mecklenbräuker. The Chebyshev structure is originally developed for 2-D digital filters designed by transformation method, but it is also applicable for 3-D filters design. By using analytical expressions for transformation function and 3-D direct convolution, an effective realization of the transformation subfilter is proposed.

Key-Words: - Digital filter implementation, FIR digital filters, Three-dimensional (3-D) filters, McClellan transformation, Chebyshev structure, 3-D cone FIR filters

1 Introduction. Problem review
Over the last decade a number of different approaches for design of multidimensional (M-D) digital filters have been developed. These filters have many applications in fields such as radar, seismic, biological, and image processing. An efficient and powerful tool in designing M-D filters is McClellan transformation. A special attention is paid to constructing two-dimensional (2-D) filters, but higher-dimensional filters have been also considered. The technique was first developed by McClellan [1] and then subsequently applied by different authors. This transform is based on the design of M-D transformation function necessary to map 1-D filter prototype into resulting M-D filter. Several techniques [2-5] have been presented to enhance the design capability of the McClellan transform (including different examples – e.g. fan filters, diamond-shaped filters, quadrantly elliptically/circularly symmetric filters, directional filters, etc.). It is also shown that this transform is easy to perform and gives efficient 2-D realization structure. Mecklenbräuker and Mersereau proposed a family of 2-D structures [6] (direct transformed, transpose direct transformed, and cascade structures) for implementing 2-D FIR digital filters designed by means of a transformation. They prove that these implementations are more efficient than the direct implementations and FFT implementation for filters up to degree 50x50. Another new implementation structure which is currently widely applied is created by McClellan and Chan [7]. They suggested the use of the Chebyshev recursion for implementing the transformation method. It is demonstrated that this structure requires a minimum number of multiplications and uses directly as coefficients the impulse response samples of the 1-D prototype filter. In addition, it has a good round-off noise performance compared with the other methods for implementation. Charalambous in his work [8] shows that the Chebyshev structure can be used for implementing the resulting M-D multiplierless spherically symmetric FIR filters. The systolic multi-layer architecture for 2-D adaptive filtering is proposed [9], based on the Chebyshev recursion and the McClellan transform.

A new reversed Chebyshev structure is proposed by Lien and Tang [10] possessing a better round-off noise error performance. It is discussed that this reversed structure is suitable to be implemented using the 2’s complement truncation operation. The round-off noise upper bounds which are independent of the transformation filters have been also established [10].

All these investigations show that the Chebyshev and the reversed Chebyshev structures have the best round-off error performance, but they have the drawback of more storage. The direct transformed structure is not suitable for fixed-point implementation because of the large values of the round-off noise. The cascade structure can be modified in view of some reduction of the noise and the number of operations required [11].

In this paper, we discuss the application of the Chebyshev structure proposed by McClellan and Chan [7] for design of 3-D FIR cone-shaped filters based on the McClellan transformation. A new method for design of 3-D cone-shaped filters is recently reported by Mollova and Mecklenbräuker [12, 13]. A minimization of the ISE (Integral Squared Error) function is presented in another work [14] by using double integration in the frequency domain of designed 3-D cone filters. These new techniques give closed-form analytical solutions for
the transform coefficients (which take part in the
definition of the transform function). We are going now
to extend our previous investigations in direction to
implementation of designed 3-D cone FIR filters.

2 Design of 3-D cone filters based on the transformation

2.1 The McClellan transformation
We introduce shortly below the theoretical background
of the McClellan transformation method and its
application for design of 3-D FIR cone digital filters.
The derivation of the Chebyshev structure will be
explained in the next section.

Consider a 1-D zero-phase FIR filter of odd-length
2N_{1D}+1 with frequency response given by:
\[ H(\omega) = \sum_{n=0}^{N_{1D}} a(n) \cos(n\omega), \]  \hspace{1cm} (1)
where \( T_n[x] \) is the \( n \)-th order Chebyshev polynomial
[15]. In above expression the coefficients \( a(n) \) are
defined by:
\[ a(n) = \begin{cases} h(0), & n = 0 \\ 2h(n), & n \neq 0 \end{cases}, \]
where \( h(n) \) is the impulse response of the 1-D filter.

The 1-D filter prototype can be mapped into 3-D
filter by applying the substitution \( \cos(\omega) = F_3(\omega_1, \omega_2, \omega_3) \),
where \( F_3(\omega_1, \omega_2, \omega_3) \) is the transformation function. The
following condition:
\[ |F_3(\omega_1, \omega_2, \omega_3)| \leq 1, \quad \text{for } \omega_1, \omega_2, \omega_3 \in [-\pi, \pi] \]
has to be satisfied for scaled or “well-behaved”
transform function. Thus the frequency response of the
resulting 3-D filter can be written as:
\[ H(\omega_1, \omega_2, \omega_3) = h(0) + \sum_{n=1}^{N_{1D}} 2h(n) T_n[F_3(\omega_1, \omega_2, \omega_3)], \]  \hspace{1cm} (2)
where \( \omega \) is the 1-D frequency variable and \( (\omega_1, \omega_2, \omega_3) \)
is a point in the 3-D space. The 3-D surfaces created by
McClellan transformation are called isopotential
surfaces.

Therefore, the 3-D design procedure is decomposed
to the design of a 1-D prototype filter and the application
of a correctly determined transformation function. The
choice of this function is very important in the overall
design (because it determines the shapes of the surfaces
of designed 3-D filters). The values along these surfaces
are fixed by means of the 1-D prototype frequency
response.

2.2 Basics of the 3-D cone filters design
In our previous work [12, 13], we considered an ideal
double cone filter oriented in \( \omega_3 \)-direction. At first, we
defined our first-order transformation function
\( F_3(\omega_1, \omega_2, \omega_3) \) in three frequency variables. Then by
applying a number of initial constraints [12, 13] on the
transform function (in order to have a true mapping from
1-D lowpass prototype to 3-D lowpass cone filter), we
derived the following scaled transformation function:
\[ F_3(\omega_1, \omega_2, \omega_3) = \\
\quad = t_{11}(1 + \cos(\omega_1) \cos(\omega_2) \cos(\omega_3)) + t_{10}(\cos(\omega_1) + \cos(\omega_2) \cos(\omega_3)) + \\
\quad + t_{01}(\cos(\omega_2) + \cos(\omega_1) \cos(\omega_3)) + t_{00}(\cos(\omega_3) + \cos(\omega_1) \cos(\omega_2)) + \\
\frac{1}{3\cos(2\theta)}(\cos(\omega_1) - \cos(\omega_2) \cos(\omega_3) \cos(\omega_3) - \cos(2\theta)), \]
where \( \theta \) is the angle of inclination of the cone, i.e. the
angle between the cone surface and the \( (\omega_1, \omega_2) \)-plane.
The following very simple analytical expressions for
transform coefficients are developed:
\[ \begin{align*}
  t_{11} &= -t_{10} = -t_{01} = \frac{\cos(2\theta)}{2(3-\cos(2\theta))} \\
  t_{00} &= 2 - \frac{\cos(2\theta)}{2(3-\cos(2\theta))}.
\end{align*} \]  \hspace{1cm} (3)

Below we prove that this concise and closed form of
relations (3) leads to an effective implementation of
designed 3-D filters using the Chebyshev structure.

3 Implementation of the resulting 3-D
cone filters using the Chebyshev structure

3.1 Derivation of the Chebyshev structure
The frequency response of a 3-D filter obtained with the
McClellan transform is given by (2), where the \( n \)-th
order Chebyshev polynomial \( T_n[x] \) is defined by the
following recursive relations:
\[ T_0[x] = 1 \]
\[ T_1[x] = x \\
T_n[x] = 2xT_{n-1}[x] - T_{n-2}[x], \quad n \geq 2, \quad -1 \leq x \leq 1. \]  \hspace{1cm} (4)
If we replace the variable \( x \) by transformation function
\( F_3(\omega_1, \omega_2, \omega_3) \) in (4), we obtain the following relation:
\[ T_n[F_3(\omega_1, \omega_2, \omega_3)] = 2F_3(\omega_1, \omega_2, \omega_3)T_{n-1}[F_3(\omega_1, \omega_2, \omega_3)] - \\
- T_{n-2}[F_3(\omega_1, \omega_2, \omega_3)]. \]  \hspace{1cm} (5)

The network realization of the Chebyshev recursion
(5) is given in Fig.1. Using this recursion and expression
(2), we can obtain the final modular realization of
designed 3-D filter \( H(\omega_1, \omega_2, \omega_3) \) as demonstrated in
Fig. 2.

We also replaced \( F_3(\omega_1, \omega_2, \omega_3) \) by \( H_s \) (for subfilter)
in the common structure from Fig. 2 for simplicity. As can
be seen, this structure uses directly the impulse response
samples of the 1-D filter prototype \( h(0), h(1), \ldots, h(N_{1D}) \).
The main feature is the modularity: several
3.2 Realization of the subfilter \( H_s \)

The subfilter (or subnetwork) with frequency response they are expressed as a function of the angle derived [12], see relations (6) below. As can be seen, the graphical representation of \( h_3(k_1,k_2,k_3) \) is given in [12]. One possible solution is to realize this subfilter using 3-D direct convolution. We use the following statement for the transformation subfilter:

\[
\begin{align*}
T_{n-3}[F_3(\omega_1,\omega_2,\omega_3)] &= h(0) + 2h(1) + 2h(2) + 2h(3) + 2h(N_{ID}-2) + 2h(N_{ID}-1) + 2h(N_{ID}) \\
T_{n-2}[F_3(\omega_1,\omega_2,\omega_3)] &= y(n_1,n_2,n_3) = \\
&= \sum_{[k_1,k_2,k_3] \in R_t} \sum_{[k_1,k_2,k_3] \in R_t} h_3(k_1,k_2,k_3) x(n_1-k_1, n_2-k_2, n_3-k_3),
\end{align*}
\]

where \( R_t \) is a 3x3x3 region of support of the sequence \( h_3(k_1,k_2,k_3) \) and \( y(n_1,n_2,n_3) \) is the output sequence of the subfilter. By applying relations (6) and some mathematical calculations, we obtain the final result for the output of the subfilter, as given by equation (7).

Therefore, we obtain a realization with 5 multiplications and 26 additions per output sample for the subfilter \( H_s \) (see eq.(7)). The total number of multiplications and additions for the whole network can be also determined (taking into account that we have \( N_{ID} \) subfilters in the network, where the length of 1-D filter prototype is \( 2N_{ID}+1 \)). This realization is very effective (for comparison – 2-D transformation subfilter [15] requires 5 multiplications and 8 additions). In other word, our 3-D subfilter requires the same number of multipliers in hardware realization as the 2-D transformation filters.

As proved by [15], the storage of Chebyshev structure is approximately double that of direct convolution, but the number of multipliers per output grows only linearly with \( N_{ID} \).

\[
\begin{align*}
A &= -\frac{\cos(2\theta)}{2(3-\cos(2\theta))}, \\
B/2 &= \frac{2}{4(3-\cos(2\theta))}, \\
D/2 &= \frac{2+\cos(2\theta)}{4(3-\cos(2\theta))}, \\
E/2 &= -\frac{\cos(2\theta)}{8(3-\cos(2\theta))}, \\
K/2 &= \frac{16(3-\cos(2\theta))}{16(3-\cos(2\theta))},
\end{align*}
\]

\[
\begin{align*}
&h_3(k_1,k_2,k_3) = \\
&= \begin{cases} 
(k_1 = \pm 1, k_2 = k_3 = 0) & \text{or} \ (k_1 = 0, k_2 = \pm 1) \\
(k_1 = 0, k_2 = \pm 1, k_3 = k_2) & \text{or} \ (k_1 = 0, k_2 = \pm 1, k_3 = k_2) \\
(k_1 = 0, k_2 = \pm 1, k_3 = -k_1) & \text{or} \ (k_1 = 0, k_2 = 0, k_3 = k_2) \\
(k_1 = 0, k_2 = \pm 1, k_3 = -k_1) & \text{or} \ (k_1 = 0, k_2 = 0, k_3 = -k_1) \\
(k_1 = \pm 1, k_2 = k_3 = k_1) & \text{or} \ (k_1 = \pm 1, k_2 = k_3, k_2 = -k_1) \\
(k_1 = \pm 1, k_2 = k_3 = k_1) & \text{or} \ (k_1 = \pm 1, k_2 = k_3, k_2 = k_1)
\end{cases}
\]

Therefore, we obtain a realization with 5 multiplications and 26 additions per output sample for the subfilter \( H_s \) (see eq.(7)). The total number of multiplications and additions for the whole network can be also determined (taking into account that we have \( N_{ID} \) subfilters in the network, where the length of 1-D filter prototype is \( 2N_{ID}+1 \)). This realization is very effective (for comparison – 2-D transformation subfilter [15] requires 5 multiplications and 8 additions). In other word, our 3-D subfilter requires the same number of multipliers in hardware realization as the 2-D transformation filters.

As proved by [15], the storage of Chebyshev structure is approximately double that of direct convolution, but the number of multipliers per output grows only linearly with \( N_{ID} \).
\[y(n_1, n_2, n_3) = Ax(n_1, n_2, n_3) +
+ B/2 \left[ x(n_1, n_2 + 1, n_3) + x(n_1, n_2 - 1, n_3) + x(n_1 - 1, n_2, n_3) + x(n_1 + 1, n_2, n_3) \right] +
+ D/2 \left[ x(n_1, n_2, n_3 + 1) + x(n_1, n_2, n_3 - 1) \right] +
+ K/2 \left[ x(n_1 + 1, n_2 + 1, n_3 + 1) + x(n_1 - 1, n_2 - 1, n_3 - 1) + x(n_1, n_2 - 1, n_3 - 1) + x(n_1, n_2, n_3 - 1) + x(n_1 + 1, n_2, n_3 + 1) + x(n_1 + 1, n_2, n_3 - 1) +
+ x(n_1, n_2, n_3 + 1) + x(n_1, n_2, n_3 - 1) +
+ E/2 \left[ x(n_1, n_2, n_3 + 1) + x(n_1, n_2, n_3 - 1) \right] + \quad (7) \]

4 Conclusion

In this paper, we consider the application of Chebyshev structure for implementation of 3-D cone filters based on the McClellan transformation. By using closed-form relations for \(h_3(k_1, k_2, k_3)\) and 3-D direct convolution, we propose an efficient realization of the transformation subfilter. We can assume that our implementation structure will be very attractive for real-time adaptive filtering.

References: