Mixed Convection in Vertical Ducts

NICOLAS GALANIS[1] and AMIN BEHZADMEHR[2]
[1]Mechanical Engineering, Université de Sherbrooke, Sherbrooke Qc, CANADA
nicolas.galanis@usherbrooke.ca
[2]Mechanical Engineering, University of Sistan & Baluchestan, Zahedan, IRAN
amin.behzadmehr@USherbrooke.ca

Abstract: This article is a review of research on mixed convection in vertical ducts published in the last 20 years. It includes experimental, analytical and numerical results and illustrates the complexity of these flows and their differences from forced convection flows. These differences include the modification of the developing velocity and temperature profiles, the initiation of flow reversal at different radial and axial positions and the significant effects of buoyancy intensity and direction on the flow regime, the Nusselt number and the friction coefficient.

Key-Words: Buoyancy, Nusselt number, experiments, numerical simulations, analytical solutions.

1 Introduction

Fluid flow with heat transfer occurs in many industrial installations such as pressurized water reactors, supercritical boilers, solar collectors and heat exchangers. When the fluid motion is induced by some external means (pump, blower, wind, etc.) the process is called forced convection. If it arises from an external force field (e.g. gravity) acting on density gradients induced by the transport process, it is called free (or natural) convection. At the interface between forced and natural convection the flow is driven simultaneously by external means and by a force field acting on density gradients; this is called mixed (or combined) convection. This third mode has received very little attention in heat transfer text books. For instance, early heat transfer treatises devote separate chapters to forced and free convection (Jakob 1949) or are entirely dedicated to forced convection (Kays & Crawford 1993, Shah & London 1978). Even very recent text books (Incropera et al 2007, Holman 2002) do not devote more than one or two pages to the subject of mixed convection. This is due to the complexity of mixed convection flows and to the sometimes contradicting results reported in the literature. A review of early publications on mixed convection in vertical tubes was written by Jackson et al (1989). The present paper provides a summary of more recent experimental, analytical and numerical studies on this subject with emphasis on the latter. It is restricted to flow of Newtonian fluids with negligible radiation effects, body forces due only to gravity and straight vertical ducts of constant geometry.

2 Experimental Results

Metais & Eckert (1964) analysed most of the available literature and produced charts (Fig. 1) which establish the boundaries between forced, mixed and natural convection as well as those between laminar and turbulent flow regimes in circular constant diameter ducts. This compilation illustrates the fact that the transition from laminar to turbulent conditions occurs at Reynolds number lower than 2000 as a result of heating.

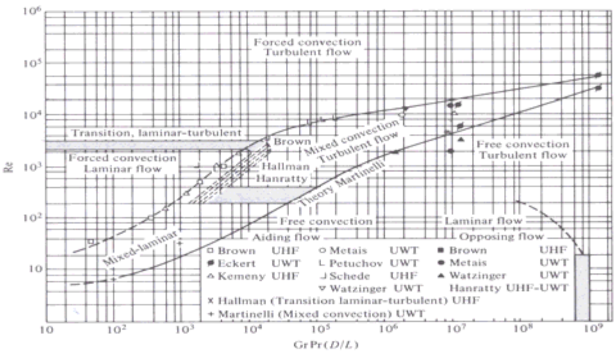

Figure 1. Regimes of free, forced and mixed connvection in vertical tubes (Metais & Eckert. 1964)

2.1 Laminar flow

Zeldin & Schmidt (1972) investigated the effects of heating on developing laminar upward flow of air in an isothermal vertical tube. They used a Pitot tube and thermocouples to measure the axial velocity and fluid temperature for Re = 380 and Gr = 12630. They thus showed that the velocity profile is quite different from the forced convection one and that the maximum velocity may not occur at the centreline (Fig. 2). Morton et al (1989) demonstrated the occurrence of flow reversal near the centreline for aiding flow in an isothermal tube with Re = 25 and Gr = 5000. Recently, Maré et al (2008) determined the velocity vectors in a vertical coaxial double-duct heat exchanger for parallel ascending flow of water using the PIV technique. For flow rates of the same order of magnitude in the inner tube and the annulus, and temperature differences of about 20 °C, they observed that flow reversal occurs in

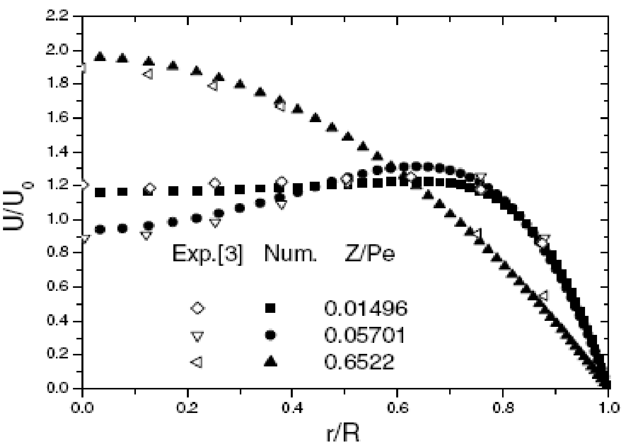

Figure 2. Measured (Zeldin & Schmidt 1972) and calculated (Behzadmehr et al 2003) velocity profiles

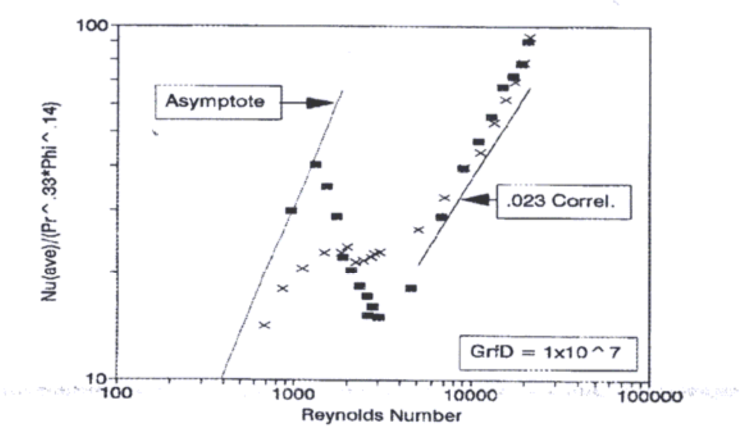

Figure 3. Upflow (■) and downflow (x) for same Grashof number (Joye 1996)

both streams over large axial distances for both heating and cooling of the flow in the inner tube.

The Nusselt number for laminar mixed convection (Nu_m) can be evaluated by first calculating the corresponding Nusselt numbers for forced (Nu_f) and natural (Nu_n) convection and then combining them as follows:

$$(Nu_m)^3 = |(Nu_f)^3 \pm (Nu_n)^3| \qquad (1)$$

Here the plus and minus signs apply respectively to flows with aiding and opposing buoyancy. Therefore, for laminar mixed convection with aiding buoyancy $Nu_m > Nu_f$ while the opposite is true for opposing buoyancy.

2.2 Transition and turbulent flow

As mentioned before, due to heating the transition from laminar to turbulent flow occurs at Reynolds numbers significantly lower than 2000. Another interesting effect of heating on the flow regime was illustrated by Steiner (1971) who measured air velocities and temperatures in a vertical pipe with UHF. These results show that the flow was laminar for Reynolds numbers considerably higher than the transition limit for isothermal flows. Specifically, he obtained laminar velocity and temperature profiles for Re=14900 with Gr/Re= 14550, for Re=9800 with Gr/Re= 12400 and for Re=7100 with Gr/Re= 16490. He thus demonstrated the phenomenon of reverse transition (or laminarization) which is due to the buoyancy induced acceleration and is similar to the observed laminarization of accelerated isothermal flows.

Petukhov (1976) presented data for vertical pipes with $2 \leq Pr \leq 6$ and developed a correlation for the Nusselt number. Later, Bernier & Baliga (1992) used a thin semitransparent film heater, glued inside a Plexiglas pipe, and dye injection to visualize upward water flows subjected to a uniform wall heat flux. This technique allowed the direct visualization of fluid-flow phenomena such as recirculation cells and laminar-turbulent transition. Vilemas et al (1992) investigated the local heat transfer in a vertical gas-cooled tube with aiding flow mixed convection in turbulent flow regime. They developed a correlation for calculating local heat transfer along the tube length in the case of weak and strong buoyancy forces. Joye (1996) studied mixed convection in an isothermal vertical tube with aiding and opposing flows for a wide range of Reynolds number (700-25000) at different constant Grashof numbers. He showed that the variation of the Nusselt number with increasing Reynolds number is quite different for flows with aiding and opposing buoyancy and established the asymptotic behaviour of Nu at low and high values of Re (Fig. 3). Parlatan et al (1996) studied the behaviour of the friction factor and the heat transfer coefficient for mixed convection with Reynolds numbers between 4000 and 9000. They showed that with increasing buoyancy the friction factor increases by as much as 25% for aiding flow while it decreases by 25% for opposing flow. They also showed that with increasing buoyancy the heat transfer coefficient increases monotonically by as much as 40% for opposing flow while for aiding flow it first decreases by 50% and then increases above the corresponding value for forced convection. This behaviour can be successfully described by expressing the Nusselt number as a function of the buoyancy parameter $Bo \sim Gr/(Re^{3.425}Pr^{0.8})$ as illustrated qualitatively in Figure 4. Aicher & Martin (1997) presented empirical correlations for the Nusselt number in turbulent mixed convection which take into account the laminarization (or relaminarization in the case of low Reynolds number for which the unheated flow is laminar) of turbulent flow that can occur during intense heating of an upward flow. Their results confirm the monotonic increase of the Nusselt number for opposing buoyancy and its initial decrease for aiding buoyancy. According to these authors, their correlations predict the experimental results from the literature and from their own work with an accuracy better than ±20%. Later, Celata et al (1998) investigated heat transfer in turbulent upward mixed

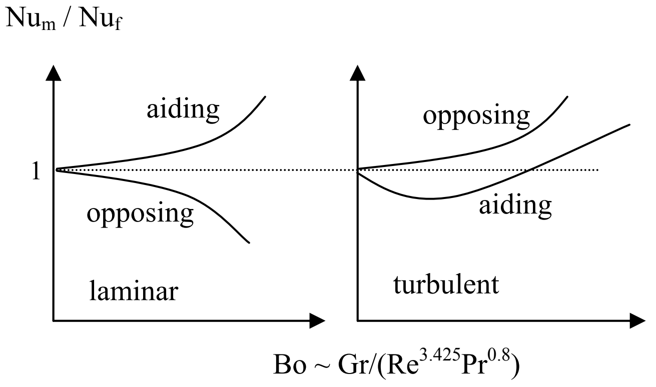

Figure 4. Qualitative variation of mixed convection Nusselt number

convection of water in a vertical channel for conditions ranging from forced to mixed convection with emphasis on the effects of the length to diameter ratio. Their experimental data confirms the reduction in the heat transfer rate for mixed convection in upward heated flow, which they attribute to the laminarization effect in the near-wall region. They also presented a correlation for the Nusselt number which provides a good prediction of their experimental results and those by Aicher & Martin (1997). Shehata & McEligot (1998) conducted experiments for air flowing upwards in vertical tubes with high heating rate. Different Re-Gr combinations were tested to obtain conditions considered to be turbulent, sub-turbulent and laminarizing. They presented the axial distribution of the wall temperature, the local bulk Reynolds number, the acceleration parameter, the buoyancy parameter, the local Nusselt number and the pressure defect as well as the mean velocity and temperature profiles. This data is extremely valuable for the development of turbulence models. Recently Behzadmehr et al (in press) reported unsteady phenomena in the case of airflows in a tube with UHF at Re= 1000, 1300 and 1600. They established that (i) this instability occurs when Gr/Re>1500 and is of thermal buoyant origin, (ii) the FFT of the temperature signal shows a peak at 0.45 Hz and (iii) that the velocity fluctuations follow this frequency and its multiples.

All of the previously discussed studies considered flow in a simple pipe of circular cross section. Analogous studies have also been conducted for flow in annular passages. Thus, Wu et al (2002) determined local heat transfer coefficients for ascending and descending turbulent mixed convection of water in a vertical passage of annular cross-section having a uniformly heated inner surface and an unheated outer surface. The results, for Re between 2000 and 12000, are qualitatively similar to those for uniformly heated tubes. With downward flow, heat transfer is systematically enhanced in comparison with that for forced convection and a fully developed thermal condition is achieved more and more readily. With upward flow, heat transfer is either impaired or enhanced depending on the strength of the buoyancy. In general, thermal development is then non-monotonic and a fully developed thermal condition is not readily achieved.

3 Modeling

The general three dimensional forms of the governing equations in cylindrical coordinates are:

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}\left(\rho r v_r\right)+\frac{1}{r}\frac{\partial}{\partial \theta}\left(\rho v_\theta\right)+\frac{\partial}{\partial z}(\rho w)=0 \qquad (2)$$

$$\begin{aligned}&\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}+w\frac{\partial v_r}{\partial z}\\&=f_r-\frac{1}{\rho}\frac{\partial P}{\partial r}+\nu\left\{\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r v_r\right)\right]+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial^2 v_r}{\partial z^2}\right\}\end{aligned} \qquad (3)$$

$$\begin{aligned}&\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}-\frac{v_r v_\theta}{r}+w\frac{\partial v_\theta}{\partial z}\\&=f_\theta-\frac{1}{\rho r}\frac{\partial P}{\partial \theta}+\nu\left\{\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r v_\theta\right)\right]+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right\}\end{aligned} \qquad (4)$$

$$\begin{aligned}&\frac{\partial w}{\partial t}+v_r\frac{\partial w}{\partial r}+\frac{v_\theta}{r}\frac{\partial w}{\partial \theta}+w\frac{\partial w}{\partial z}\\&=f_z-\frac{1}{\rho}\frac{\partial P}{\partial z}+\nu\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}+\frac{\partial^2 w}{\partial z^2}\right\}\end{aligned} \qquad (5)$$

$$\begin{aligned}&\rho c\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+w\frac{\partial T}{\partial z}\right)\\&=\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\lambda\frac{\partial T}{\partial \theta}\right)+\frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right)+\Phi\end{aligned} \qquad (6)$$

To close the system the equation of state and constitutive relations for the fluid as well as equations modeling turbulence and appropriate boundary conditions are necessary.

It is possible to simplify these equations by applying one or more of the following assumptions: steady state, laminar flow, negligible viscous dissipation, negligible axial diffusion (axially parabolic equations), Boussinesq hypothesis, fully developed flow.

The boundary condition for velocity at the solid-fluid interface is the usual no-slip condition while for the temperature a uniform wall temperature (UWT) or uniform heat flux (UHF) is applied at this interface or at the outer surface of the duct (in this last case heat conduction in the solid wall can play a significant role).

The system of coupled PDEs can be solved either analytically (with many simplifying assumptions) or numerically.

4 Analytical Solutions

Ostroumov (1950), Hallman (1956), Hanratty et al (1958) and Morton (1960) presented analytical solutions for steady state, laminar, fully developed, axial flow in a vertical tube ($v = w = 0$) with UHF.

Hallman (1956) neglected viscous dissipation and pressure work. He used the Boussinesq hypothesis for density and assumed constant viscosity, conductivity and specific heat. He applied the UHF condition at the solid-fluid interface and included a uniformly distributed heat source in the fluid. The non-dimensional expressions for the velocity and temperature were given in terms of first kind, zero order real and imaginary Bessel functions and depend on the Prandtl number, the ratio of the Grashof and Reynolds numbers as well as on the non-dimensional power density of the heat source. The solution includes analytical expressions for Re(-dp/dz) and Nu. It shows that the effect of free convection is to decrease radial temperature differences and to increase the Nusselt number with respect to those for pure forced convection. Free convection also changes the axial pressure gradient from that for pure forced convection but this change may be in either direction depending on the value of the heat source.

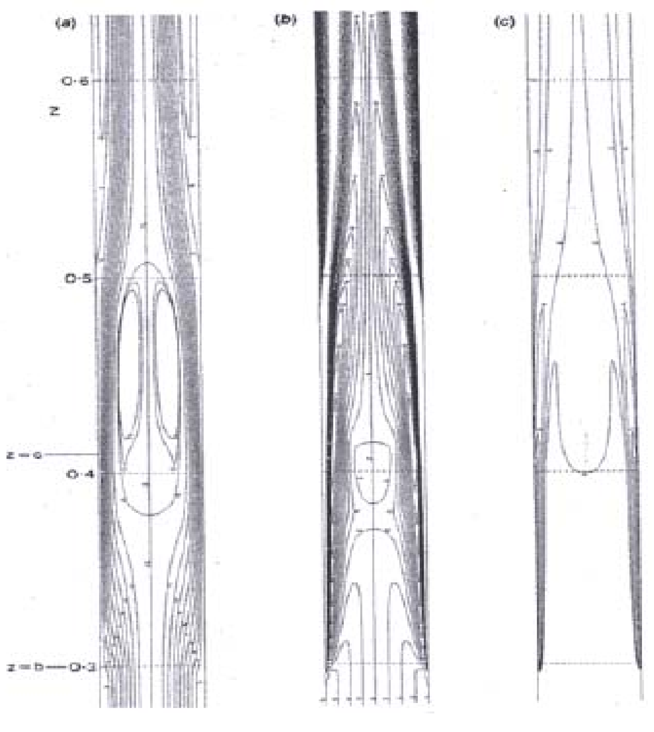

Figure 5. Contour plots of (a) streamlines, (b) vorticity (c) temperature (Morton et al 1989)

Later, Barletta et al (2001) included viscous dissipation and solved the momentum balance equation and the energy balance equation by means of a perturbation method. They obtained analytical expressions for the velocity and temperature profiles and established that, for aiding buoyancy with a constant value of Gr/Re, viscous dissipation reduces the value of the Nusselt number and increases the value of the Fanning friction factor.

Recently, Ben Mansour et al (2006) used a power series expansion of the axial velocity to solve the coupled PDEs for fully developed laminar mixed convection in a vertical tube with UHF, uniform internal volumetric heat source and viscous dissipation. Explicit analytical expressions were obtained for the temperature and velocity profiles as well as for the pressure gradient. The solution was then used to calculate the entropy generation rate. It was observed that the entropy generation rate due to friction is more important than that due to heat transfer by conduction.

5 Numerical Solutions

Some of the numerical studies of laminar and turbulent mixed convection published since the review by Jackson et al (1989) are summarized here.

5.1 Laminar flow

Morton et al (1989) used the elliptical governing equations to study mixed convection in vertical tubes with constant wall temperature. They used the finite difference method and found that flow reversal appears near the centre of the tube for heated upward flow and near the wall for cooled upward flows. Figure 5 shows the resulting streamlines, vorticity and temperature contours. Heggs et al (1990) studied the effects of conduction in the wall on the development of the recirculation zone. They thus showed that upstream axial conduction in the wall causes significant preheating of the fluid and that therefore the recirculation region at the centerline begins to form further upstream. Wang et al (1994) analysed mixed convection flow at low Péclet number in the thermal entrance region and presented the regime of flow reversal for both heating and cooling cases in the Pe-$|Gr/Re|$ plane. Lee and Yan (1996) studied transient mixed convection flow in a tube subject to external convection. They took into consideration wall conduction and wall heat capacity and found that neglecting these effects causes a significant error, particularly during the early transient period. They also showed that increasing the outside Nu causes a greater inside Nu and accelerates the attainment of the steady state condition. They also showed that the wall thickness has a small influence on the distribution of Nu at steady

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state conditions and that the duct material does not have a significant effect on the establishment of steady state condition. Nesreddine et al (1997) studied the effect of variable physical properties in laminar aiding and opposing mixed convection in vertical tubes. They considered that the fluid viscosity and thermal conductivity vary with absolute temperature according to simple power laws while the density varies linearly with the temperature and the heat capacity is constant. Their comparison between results calculated with variable properties and with the Boussinesq approximation showed substantial differences in the temperature and velocity fields when the Gr is high. Figure 6 shows that the friction factor is under-predicted by the Boussinesq approximation for aiding flow and it is over-predicted for opposing flow. However, the effects on the heat transfer performance remain negligible except for cases with flow reversal. Later, Nesreddine et al (1998) studied the effects of axial diffusion in the entrance region of thin vertical tubes with UHF. Their results reveal that axial diffusion plays a significant role in preheating the fluid upstream from the entrance of the heat transfer region for both aiding and opposing flow. They established criteria that determine (i) when the upstream boundary conditions can be applied at the entrance of the heated section and (ii) when the elliptical formulation is necessary to describe the flow field accurately.

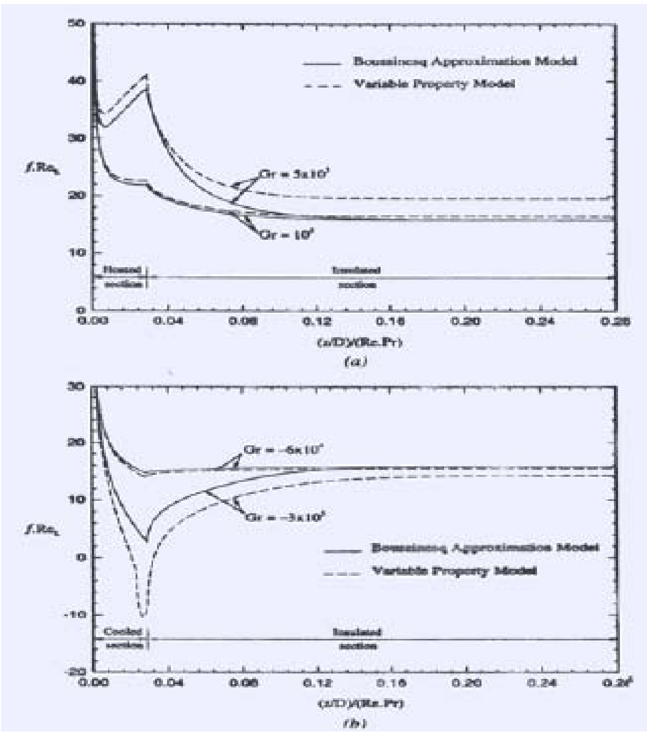

Figure 6. Effects of property variation on the friction factor for (a) heating and (b) cooling (Nesreddine et al 1997)

Su & Chung (2000) studied the stability of fully developed mixed convection in a vertical tube with UHF. Their results (Fig. 7) suggest that this flow can become unstable at low Reynolds and Rayleigh numbers irrespective of the Prandtl number. They described two instability mechanisms for aiding flow: thermal shear instability for flows with Prandtl numbers less than 0.3 and assisted buoyant instability for flows with Prandtl numbers greater than 0.3. For opposing flow the Pr effect is less significant and there are three instability mechanisms: the Rayleigh-Taylor instability for flows with extremely low Reynolds numbers (Re≤5), the opposed thermal buoyant instability at somewhat larger Reynolds numbers and the thermal-shear instability at even higher Reynolds numbers. They mentioned that the instability in aiding and opposing flows can be attributed to the appearance of an inflection point and flow separation for fluids with Pr of order one.

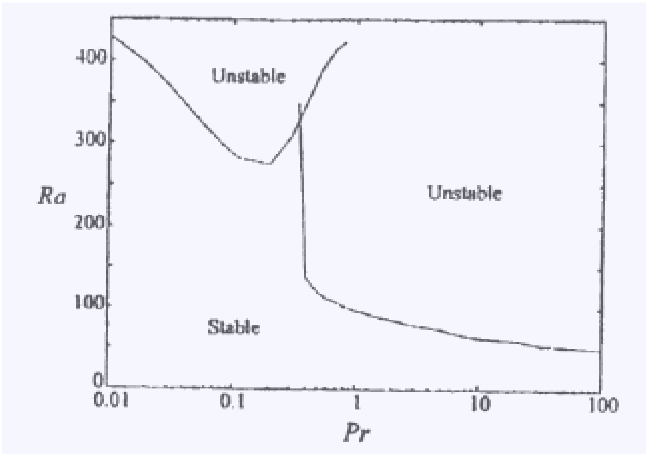

Figure 7. Stability map in Pr-Ra space for assisted flow, Re=1000 (Su & Chung 2000)

Behzadmehr et al (2001) solved the coupled, elliptic PDEs for developing upward heated flow and showed, for the first time numerically, the occurrence of flow reversal at the centerline and also at the region between the wall and the centerline (Fig. 8) as predicted analytically for fully developed conditions (see for instance Hallman, 1958).

Zghal et al (2001) solved the elliptic equations for upward flow of air in a tube with a uniformly heated zone preceded and followed by adiabatic zones. They mapped the conditions leading to flow reversal and to important upstream diffusion on a Péclet-Richardson chart (Fig. 9) and showed that for large values of Pe the critical value of Ri depends on the heating length L_2. However, for any given value of Pe there exists a characteristic length L_u such that for $L_2 > L_u$ the critical value of Ri depends only on Pe; as shown in Figure 9 heated gas flow. The results show that the low Reynolds number k-ε turbulence model by Launder & Sharma (1974) performed best in predicting axial wall temperature profiles. Overall agreement between the measured velocity and temperature distributions and those calculated using this turbulence model is good. Later Behzadmehr et al (2003) presented a numerical correlation for the Nusselt number at low Reynolds numbers. It is valid for both laminar and also turbulent flow regime in the range of $1000 \leq Re \leq 1500$ and $Gr \leq 5x10^7$.

$$Nu = 4.36\left(1 + \frac{Gr^{0.468}}{750 + 0.24\,Re}\right) \qquad (7)$$

They also presented a map, for Re=1000 and Re=1500, which indicates the conditions for laminar, turbulent or re-laminarized regimes as well as those for positive or negative pressure difference between the tube inlet and outlet (Figure 14).

You et al (2003) used direct numerical simulation to study fully developed turbulent mixed convection. They found that both the skin friction coefficient and the heat transfer coefficient first decreased and then increased with increasing heat flux in upward heated flow. In the case of downward heated flow, C_f was nearly unchanged but Nu_m increased with increasing heat flux. They thus confirmed the tendencies shown in Figures 3, 4 and 10.

6 Conclusion

The hydrodynamic and thermal fields for mixed convection in vertical ducts are very different from the corresponding ones for forced convection. The main conclusions of the present literature review are that:

- Fully developed mixed convection can be turbulent for Reynolds numbers as low as 1000.
- The phenomenon of relaminarization can occur when the Grashof number is high.
- The Nusselt number for fully developed mixed convection is significantly influenced by the relative direction between the buoyancy force and the imposed flow; this influence is qualitatively different for laminar and turbulent flow and depends on the Prandtl number.

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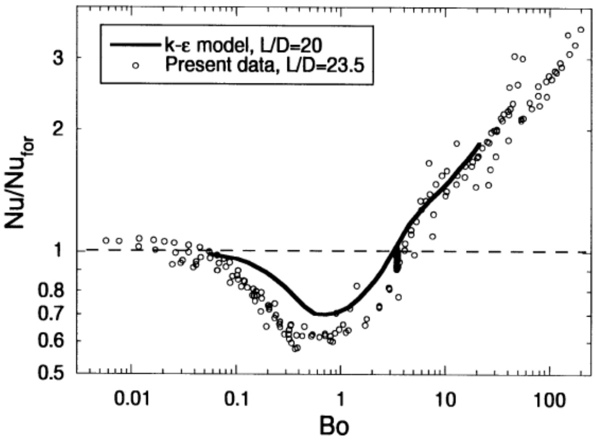


Figure 10. Experimental and numerical Nusselt number, turbulent fully developed aiding flow (Celata et al 1998)

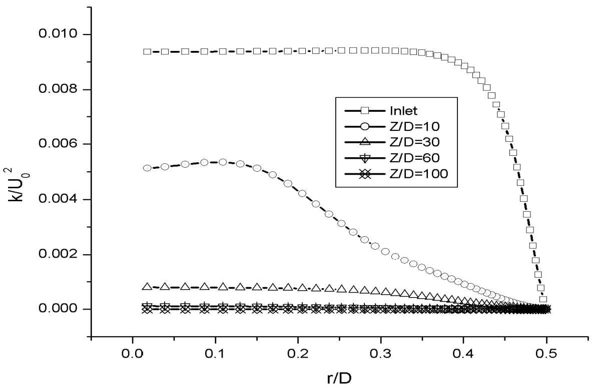


Figure 11. Axial evolution of turbulent kinetic energy for Pr=0.7, Re=380, Gr=12630 (Behzadmehr et al 2002)

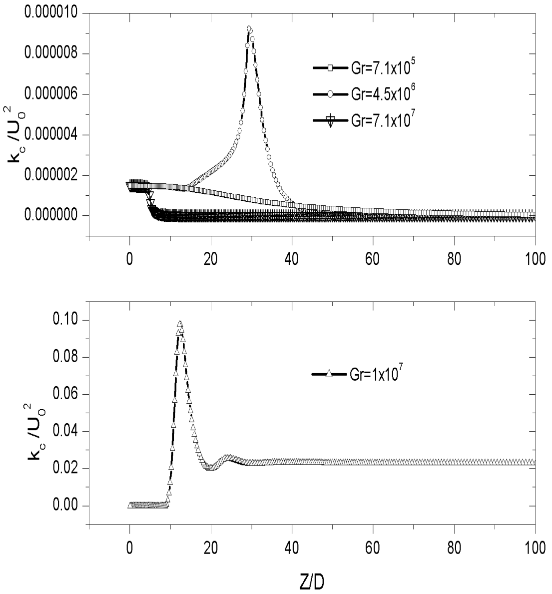


Figure 12. Axial evolution of centerline turbulent kinetic energy for Pr=0.7, Re=1000 (Behzadmehr et al 2002)

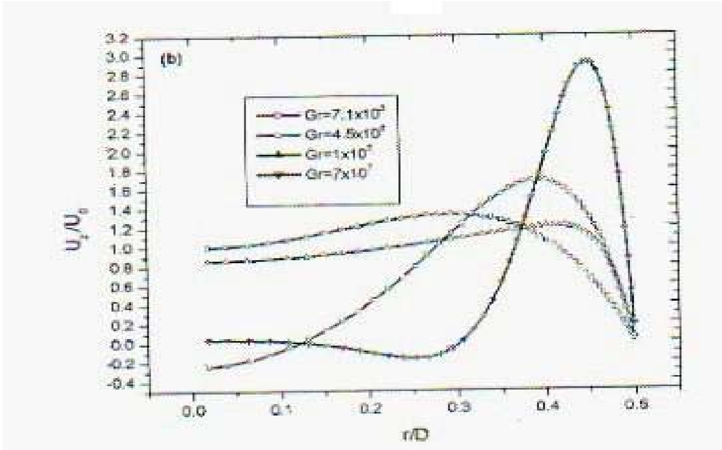
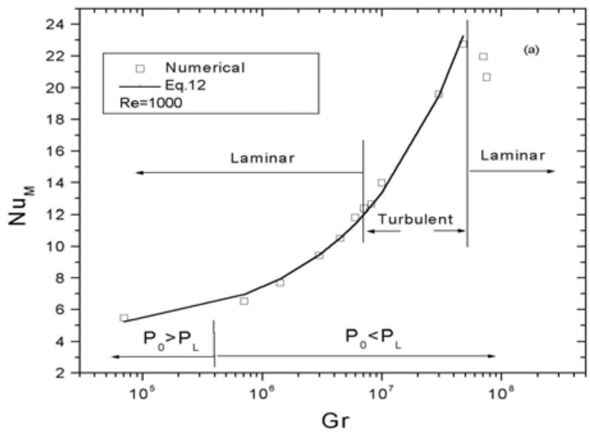


Figure 13. Fully developed velocity profiles for laminar and turbulent regimes (Behzadmehr et al 2002)



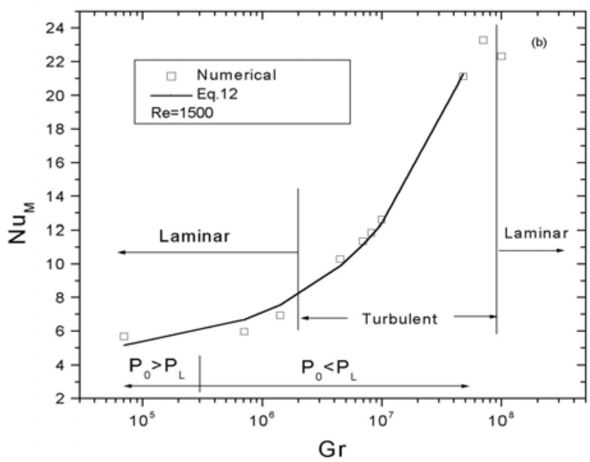


Figure 14. Nusselt number in the fully developed region for Pr=0.7 (Behzadmehr et al 2003)