Axial Slot Array Antenna Mounted on a Non-Confocal Dielectric-Coated Elliptic Cylinder

A. HELALY* and A. SEBAK
Electrical and Computer Engineering Dept
Concordia University
Montreal, QC, H3G 1M8, CANADA
* Engineering Mathematics and Physics Dept, Cairo University, Giza, EGYPT

Abstract- This paper presents the analytical solution for the radiation by axial slots array antenna on a non-confocal dielectric-coated elliptic cylinder. The analysis is based on the eigen function technique and the addition theorem of Mathieu functions. The excited apertures are assumed to generate a TM polarized wave. Calculated azimuthal radiation patterns are presented for different antenna and coating parameters.

Key-Words: Axial slot antennas, non-confocal coating, analytical methods.

1 Introduction
Slot antennas mounted on cylindrical structures are potentially useful antenna elements. The radiation properties of such antennas have been studied extensively [1]-[6]. However, we are not aware of any previous work on the radiation by multi-slotted antenna on a conducting elliptic cylinder coated with a non-confocal dielectric. Arrays of antennas mounted on vertical structures are useful for the broadcasting of waves and for base stations. The elliptic geometry is becoming more popular since it allows a better control of the polarization characteristics and facilitates the design by changing both eccentricity and focal length to tune the parameters of interest. In addition, the non-uniform coating offers another degree of design freedom. Also, elliptic cylindrical cavities excite more propagating modes compared to circular cavities [7]. For the analysis of such antennas, the homogeneous Helmholtz equation in the elliptic coordinates is employed. Solving field problems of structures with elliptical geometries requires the computation of Mathieu and modified Mathieu functions [8]. These are the eigen solutions of the wave equation in elliptical coordinates. The analytical development of the solution for far field patterns of the multi-slotted elliptic cylinder with concentric coating have been presented in [5].

The studies presented in this paper are focused on the analysis and design of a finite array of axial slots on a non-confocal dielectric-coated conducting elliptic cylinder. The work is an extension of the formulation for a single slot antenna on an elliptic cylinder [4]. One of the motivations for this work is to study the effect of a dielectric layer on the performance of antennas mounted on a space shuttle. We assumed that the aperture field i.e., the excitation voltages, is a specified function on each slot. The fields in the dielectric coating and exterior regions are expanded in terms of Mathieu and modified Mathieu functions with unknown expansion coefficients. These unknown expansion coefficients are determined by enforcing appropriate boundary conditions at the interface between the dielectric coating and free space and at the slotted cylinder. In order to generate numerical results and from a practical point of view, the series solution must be truncated in a suitable fashion to obtain a finite size matrix. The order of such a matrix depends on the electrical size and properties of the coating region.

2 Antenna Geometry
The configuration of the axial array antenna mounted on elliptic cylinder is shown in Fig. 1. The antenna radiates through a non-confocal dielectric coating (region II) to free space (region I). Two systems of coordinates are used. The global coordinates at the center of the outer surface of the dielectric coating are identified by (x, y). The local coordinates at the center of the slotted conducting cylinder are (ẋ, ẏ). The semi major and
semi minor axes for the conducting cylinder are denoted by \( a_c \) and \( b_c \). The corresponding parameters for the coating region are \( a \) and \( b \). The focal length of the dielectric outer surface is \( 2F \) and that of the conducting cylinder is denoted by \( 2F_c \). The major axis of the cross section of the conducting cylinder is inclined by an angle \( \beta \) with respect to the x axis and its center is located at \((d, \psi)\) with respect to the global coordinates. The elliptical coordinates \((u, \nu, z)\) are employed throughout with
\[
x = F \cosh(u) \cos(\nu)\]
\[
y = F \sinh(u) \sin(\nu)\]
where \( F \) is the semi focal length of the corresponding elliptical cross section.

![Configuration of axial slotted-elliptic cylinder array antenna coated with a non-confocal dielectric.](image)

3 Series Solution
This paper considers the TM polarization in which the non zero field components are \( E_z, H_\rho \) and \( H_\phi \). It is assumed that the filed distribution on the slots depends on the angular variable \( \nu \). In region I and using the global coordinates \((u, \nu)\), the electric field component is given by
\[
E^{(I)}_z = \sum_{m=0}^{\infty} B_m R^{(4)}(c_0, \xi) S_m(c_0, \eta) + \sum_{m=1}^{\infty} B_m R^{(4)}(c_0, \xi) S_m(c_0, \eta)
\]
where \( \xi = \cosh(u), \eta = \cos(\nu) \) and \( c_0 = k_0 F, k_0 = 2\pi / \lambda \) and \( \lambda \) is the free space wavelength. \( S_m \), \( S_m \) are, respectively, the even and odd angular Mathieu functions of order \( m \), \( R^{(4)}(c_0, \xi) \), \( R^{(4)}(c_0, \xi) \) are the even and odd modified Mathieu functions of fourth kind. The corresponding magnetic field components are given by
\[
H_\eta = \frac{-j}{2\omega \mu_0 h} \frac{\partial E_z}{\partial \xi}
\]
\[
H_\xi = \frac{-j}{2\omega \mu_0 h} \frac{\partial E_z}{\partial \xi}
\]
where \( h = F \sqrt{\frac{\xi^2 - 1}{\xi^2 - \eta^2}} \).

The electric field component in region II must vanish on the conducting part of the core cylinder except at the slots. In region II and using the local coordinates, we may express it in the form
\[
E^{(II)}_z = \sum_{m=0}^{\infty} \left[ C_m R^{(1)}(c_3, \xi_c) + D_m R^{(2)}(c_3, \xi_c) \right] S_m(c_3, \eta_c)
\]

\[
E^{(II)}_z = \sum_{m=0}^{\infty} \left[ C_m R^{(1)}(c_3, \xi_c) + D_m R^{(2)}(c_3, \xi_c) \right] S_m(c_3, \eta_c)
\]

where \( \xi_c = \cosh(u_c), \eta_c = \cos(\nu_c) \) and \( c_3 = kF_c \) with \( k = k_0 \sqrt{\varepsilon_r} \). \( R^{(4)}(c_0, \xi) \), \( R^{(4)}(c_0, \xi) \) are the even and odd modified Mathieu functions of the \( \ell \)th kind. The corresponding magnetic field components are given by (2) but using the local coordinates. In (1) and (3) \( B, C \) and \( D \) are unknown expansion coefficients to be determined by enforcing proper boundary conditions on the antenna and the interface between the dielectric and free space.

4 Boundary Conditions
The three sets of unknown expansion coefficients are determined by applying boundary conditions on the two surfaces: the core conducting cylinder and the interface between the coating material and free space.

4.1 Core (antenna) boundary
On the conducting surface \( \xi = \xi_2 \), the tangential electric field component must vanish except on the slots. Using the local coordinates, the boundary condition may be expressed as:
\[ E_z^{(II)} = \begin{cases} F(\eta_c) & \text{on the slots} \\ 0 & \text{otherwise} \end{cases} \quad (4) \]

Where

\[ F(\eta_c) = \sum_{i=1}^{N} F_i(\eta_c) \quad (5) \]

is the aperture field distribution on the slots.

On the \(i\)th slot, it is assumed that the excitation voltage is varying as

\[ V = V e^{j\delta} \cos(\pi(v - v_{oi})/2\Delta), \quad (6) \]

\( V e^{j\delta} \) is the applied voltage, \( v_{oi} \) is the angular coordinate of its center, \( \Delta_i \) is its width and \( \eta_c = \cos(\nu_c) \) in the \((x_c, y_c)\) local coordinates. Substituting (3) and (5) in (4), multiplying the results by \( S_{o_n}(c_3, \eta_c) \) and applying the orthogonality of Mathieu functions \([9]\), the terms involving even functions decouple completely from those of odd functions. Thus, one obtains two subsystems of linear equations for the even and odd unknown expansion coefficients. For the even functions the resulting subsystem of equation is given by, \( n = 0, 1, 2, \ldots, \)

\[ \sum_{m=0}^{\infty} M_{enm}(c_3, c_0)B_{enm}\Re_{enm}(c_0, \xi_1) = 0 \quad (7) \]

\[ Ne_n(c_3) \Re_n^{(1)}(c_3, \xi_1) \]

For the odd functions case and for \( n = 1, 2, 3, \ldots \)

\[ N_o_n(c_3)[C_{on}\Re_n^{(1)}(c_3, \xi_2) + D_{on}\Re_n^{(2)}(c_1, \xi_2)] = \int_{\text{slots}} F(\eta_c)S_{o_n}(c_3, \eta_c)d\eta_c = F_o(c_3) \quad (8) \]

\[ Ne_n(c_3)\Re_n^{(1)}(c_3, \xi_1) \]

And \( F_e, F_o \) are the slots excitation vector and given by:

\[ F_{en}^{(II)}(c_3) = \sum_{i=1}^{N} \int_{\text{slot}} F_i(\eta_c)S_{o_n}(c_3, \eta_c)d\eta \quad (9) \]

### 4.2 Coating – free space boundary

On the outer surface of the dielectric coating, i.e., \( \xi = \xi_1 \), the continuity of both \( E \) and \( H \) field tangential components must be enforced. The field components inside the coating region can be expressed in terms of the global coordinate system using the transition theorem of Mathieu functions \([10]\). Again using the orthogonal properties of Mathieu functions, one obtains from the tangential \( E_z \) filed component:

\[ \sum_{m=0}^{\infty} M_{enm}(c_3, c_0)B_{enm}\Re_{enm}(c_0, \xi_1) = 0 \quad (10) \]

\[ Ne_n(c_3) \Re_n^{(1)}(c_3, \xi_1) \]

\[ \sum_{m=0}^{\infty} W_{enm}C_{enm} + \sum_{m=1}^{\infty} W_{enm}Co_{nm} \]

\[ + Ne_n(c_3) \Re_n^{(2)}(c_3, \xi_1) \]

\[ \sum_{m=0}^{\infty} W_{enm}D_{enm} + \sum_{m=1}^{\infty} W_{enm}Do_{nm} \]

And from the tangential \( H_z \) filed component, we obtain:

\[ \frac{\mu_1}{\mu_0} \sum_{m=0}^{\infty} M_{enm}(c_3, c_0)B_{enm}\Re_{enm}(c_0, \xi_1) = 0 \quad (11) \]

\[ Ne_n(c_3) \Re_n^{(1)}(c_3, \xi_1) \]

\[ \sum_{m=0}^{\infty} W_{enm}C_{enm} + \sum_{m=1}^{\infty} W_{enm}Co_{nm} \]

\[ + Ne_n(c_3) \Re_n^{(2)}(c_3, \xi_1) \]

\[ \sum_{m=0}^{\infty} W_{enm}D_{enm} + \sum_{m=1}^{\infty} W_{enm}Do_{nm} \]

\[ \sum_{m=0}^{\infty} Mo_{nm}(c_3, c_0)[B_{m}\Re_{m}(c_0, \xi_1)] = 0 \quad (12) \]

\[ Ne_n(c_3)\Re_n^{(1)}(c_3, \xi_1) \]

\[ \sum_{m=0}^{\infty} W_{enm}C_{enm} + \sum_{m=1}^{\infty} W_{enm}Co_{nm} \]

\[ + Ne_n(c_3)\Re_n^{(2)}(c_3, \xi_1) \]

\[ \sum_{m=0}^{\infty} W_{enm}D_{enm} + \sum_{m=1}^{\infty} W_{enm}Do_{nm} \]

\[ \int_0^{2\pi} [S\Re_n^{(1)}(c_3, \eta)]^2 dv, \quad \eta = \cos \nu \quad (9) \]

And \( Fe, Fo \) are the slots excitation vector and given by:

\[ F_{en}^{(II)}(c_3) = \sum_{i=1}^{N} \int_{\text{slot}} F_i(\eta_c)S_{o_n}(c_3, \eta_c)d\eta \quad (10) \]
\[
\frac{\mu_1}{\mu_0} \sum_{m=0}^{\infty} M_{nm}(c_1, c_0) [B_0^m R_0^{(4)}(c_0, \xi_1)] \\
= No_n(c_1) R_{n}^{(1)}(c_1, \xi_1) \cdot \sum_{m=0}^{\infty} W O_{nm} C_{e_m} + \sum_{m=1}^{\infty} W O_{nm} C_{n_m} \\
+ No_n(c_1) R_{n}^{(2)}(c_1, \xi_1) \cdot \sum_{m=0}^{\infty} W O_{nm} C_{e_m} + \sum_{m=1}^{\infty} W O_{nm} C_{n_m} \\
\]  
(14)

Where \( c_1 = k_1 F \), the prime denotes derivative with respect to \( \xi \) and

\[
M_{mn}(c_i, c_j) = \int_{\sigma} S_{\sigma}^{(c_i, \eta)} S_{\sigma}^{(c_j, \eta)} dv, \quad (15)
\]

Equations (7)-(8) and (11–14) constitute a system of linear equations of unknown expansion coefficients of the fields: namely: \( B_e_m, B_o_m, C_e_m, C_o_m, D_e_m \) and \( D_o_m \). Expressions for the transformation coefficients \( W E_{nm} \) and \( W O_{nm} \) are given in [10].

### 4.3 Radiated Power Density

Employing the asymptotic expansion of \( R_{e_m}^{(4)} \) and \( R_{o_m}^{(4)} \) in equation (1), the far radiated field, which is of particular interest, is given by;

\[
E_z = \sqrt{j/(k\rho)} e^{-j\rho} \sum_{m} j^m [B_e_m S_e_m(c_0, \eta) \\
+ B_o_m S_o_m(c_0, \eta)] \\
(16)
\]

The time-average power density radiated by the antenna is then given by:

\[
P(\rho, \phi) = \frac{1}{120\pi k\rho} \left[ \sum_{m} j^m [B_e_m S_e_m(c_0, \cos \phi) \\
+ B_o_m S_o_m(c_0, \cos \phi)] \right]^2 \\
(17)
\]

### 5 Numerical Results

For illustration purposes, we consider the case of two slots each with width \( \Delta=2\xi \). For all considered cases, the electrical and geometrical parameters are given in each figure caption. Fig. 2 shows the radiation pattern when the two slots are on the minor axis of the conducting cylinder. The pattern is similar to that of a pair of dipoles. The effect of location of the slots pair on the pattern is shown in Fig. 3. For this case the outer boundary of the coating dielectric is of circular shape. Fig. 4 represents the effect of the excitation phase on the radiation pattern. It is seen that the main beam direction is steered 90° when the phase is switched from the in-phase status to out of phase condition.

### 6 Conclusion

An analytic solution is given for the radiation by an array of axial slots mounted on a conducting elliptic cylinder coated with a non-confocal dielectric. The computed results show the flexibility of the antenna to control the shape and direction of its radiation pattern.
Fig. 3: Radiation pattern for an axially slotted elliptic cylinder \((a_c=0.25\lambda, \ a_c/b_c=2)\) coated with a circular dielectric \((\varepsilon_r=4, \ a=b=0.3\lambda, \ d=0, \ \psi=0, \ \beta=0)\). The two slots are fed with out of phase equal amplitude voltages: effect of the slots locations.

Fig. 4: Radiation patterns for an axially slotted elliptic cylinder \((a_c=0.25\lambda, \ a_c/b_c=2)\) coated with a dielectric \((\varepsilon_r=4, \ a=b=0.3\lambda, \ d=0, \ \psi=0, \ \beta=0)\). The two slots are located at the ends of the major axis: in-phase and out of phase excitation comparison.

References:


