Quality factor for band-pass filters designed with CPW resonators using J/K-inverter

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Abstract: - This paper describes the way of determining quality factor for the case of coplanar waveguide resonators using J/K-inverters. The model for single case resonator is simulated, realized and measured using numerical formulas, ADS Momentum software and vector network analyzer.

Key-Words: - coplanar waveguide, band-pass filter, resonators, J/K-inverter

1 Introduction

Quality factor for a resonant circuit can be computed using different methods. The majority of the methods are based on resonator input impedance because this is frequency variable and it can be represented almost by a perfect circle. Input impedance can be measured with vector network analyzer.



Fig. 1 Admittances definition used for Q factor

Generally, we can use the Kajfez and Hwan method but for high losses circuits it advisable to use Sun and Chao critical point method.

All these methods involve the use of Smith diagram. Taking into account the using of vector network analyzer for S parameters, we present an algorithm for determining Q factor, based on these parameters [1], [2].

In the figure no. 1 we have represented the resonant circuit as being determined by parallel admittance Y_s (as for the case of $\lambda/2$ resonators):

$$Y_s = G_s + jB_s$$

$$B_s = \frac{4(W_E - W_H)}{VV *}$$
(1)

where W_E and W_H are electric field and magnetic field energy and V is the voltage. It can be shown that:

$$\frac{\partial B_s}{\partial \omega} = 4 \frac{W_E + W_H}{VV *} \tag{2}$$

where $(W_E + W_H)$ is medium energy stored in resonator.

The medium power dissipated into resonant system is given by:

$$P_{medium} = \frac{1}{2}VV * (G_s + Y_A)$$
(3)

Using above equations we obtain loaded Q as:

$$Q_s = \left[\frac{\omega}{2(G_s + Y_A)} \frac{\partial B}{\partial \omega} \right]_{\omega = \omega_0}$$
(4)

By this, Q_S can be determined using measured or simulated/computed [S] or [Y] parameters. Because Y parameters cannot be determined directly by vector network analyzer, we can use $[S] \leftrightarrow [Y]$ transformation formulas.

2 Designing CPW resonators using J/K-inverter

The factor that affects the most filter performance is resonator coupling. Matthaei, Young and Jones had proposed a method for mutual coupling by J invertors (admittance) or K invertors (impedance) [3].

J and K invertors can be represented by element admittance, or impedance.

By this we can obtain an general equation for resonator coupling as:

$$k_{i,i+1} = \frac{J_{i,i+1}}{\beta} \quad \text{and} \quad k_{i,i+1} = \frac{K_{i,i+1}}{\alpha} \quad (5)$$

for parallel or series resonators, where the coupling between (i) and (i+1) resonators is represented by $k_{i,i+1}$.

In case of π type LC resonators, a capacitor is used in series between resonators to realize coupling and coupling coefficient is given by:

$$k_{\pi} = \frac{\omega_0 C_{\pi}}{\beta} = \frac{\omega_0 C_{\pi}}{2\omega_0 C} = \frac{C_{\pi}}{2C} \tag{6}$$

External quality factor is defined to make the difference between load and source and associated losses with every resonator.

In case of strong coupling:

$$k > \frac{1}{Q_{ext}} + \frac{1}{Q_u} \tag{7}$$

where Q_{ext} is external quality factor and Q_u is unloaded quality factor. It is possible to find resonators coupling coefficient:

$$k_{c} = \sqrt{\left(\frac{f_{\max 1} - f_{\max 2}}{f_{0}}\right)^{2} + \left(\frac{1}{Q_{ext}} + \frac{1}{Q_{u}}\right)^{2}}$$
(8)

where f_{max1} and f_{max2} are the frequencies relative to amplitude response poles of symmetrical FTB filter. To realize J invertor by using CPW, we have to cut a slot into central line of coplanar waveguide, this slot being characterized by:

$$\varphi = -\tan^{-1} \left(\frac{2B_b}{Y_0} + \frac{B_a}{Y_0} \right) - \tan^{-1} \left(\frac{B_a}{Y_0} \right)$$

$$\frac{J}{Y_0} = \left| \tan \left\{ \frac{\varphi}{2} + \tan^{-1} \left(\frac{B_a}{Y_0} \right) \right\} \right|$$
(11)

where $B_a = \omega C_a$ and $B_b = \omega C_b$, C_a and C_b are series and parallel capacitors associated with the slot. For i resonator the total electrical length is given by:

$$\theta_i = \pi + \left(\frac{\varphi_{i-1,i}}{2} + \frac{\varphi_{i,i+1}}{2}\right) \tag{12}$$

3 Practical results and simulations

The results for analysis and synthesis are for the case of CPW with substrate with $\varepsilon_r = 4.4$, height = 1.575 mm, thickness = 35 μ m, feed slot width = 100 μ m, tg δ = 0.005 and f_{design} = 7 GHz.

$$C_{1}(w) = C_{2}(w) = [-0.82 \cdot 10^{-2} \cdot w + 0.41] \cdot \\ \cdot \ln[s \cdot (2.23 \cdot 10^{-3} \cdot w + 2.68 \cdot 10^{2})] \quad [fF] \quad (13)$$
$$C_{3}(w) = \frac{0.605 \cdot w + 1}{s + 0.016 \cdot w + 6.9} \quad [fF]$$

In Fig. 2 we have represented the model for discontinuity in central coplanar line, simulations for

design of equivalent circuit using the numerical formulas form equations (13), [4]-[6].



Fig. 2 Discontinuity in central coplanar line

The design start from the case of an single resonator with length 11.35 mm realized with J invertor using CPW, shown in Fig. 3 along with the magnified photo of feed line slot.



Fig. 3 Single J-inverter CPW resonator and practical result for feed line slot

Along the influences due to practical manufacturing (as we can see there was a difference from design of extra 35 μ m in feed line slot width) there is a influence on quality factor due to the way of coupling with the feed line itself, in this case microstrip line.

We have choused via-hole type connection with microstrip line, supposed by cylindrical form with $r = 50 \mu m$ radius.



Fig. 4 Design and practical results for J-inverter type CPW single resonator



Fig. 5 External quality factor for coupling between CPW and microstrip line

4 Conclusion

Simulations for the case presented have been done with ADS Momentum software [7] and practical measurements of parameters have been done with HP 8714C vector network analyzer.

Because resonant frequency is known (as 7 GHz) we have realized different simulations for different distances between resonator and feed line. The optimum for this is given by a compromise between insertion loss and signal-noise ratio.

This has to taken into consideration because at lower distances signal-noise ratio will be insignificant and insertion loss will be smaller compared with the case for higher distances. From this result we can obtain external quality factor. Generally, loaded quality factor can be determined using the results for S_{21} parameter. For weak resonator coupling is more useful to measure S_{21} because it is not possible to for accurate measuring of S_{11} parameter. The loaded quality factor can be determined using 3 dB method with S_{21} parameter values.

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