Abstract: The paper is focus on the performance of fast frequency hopped spread spectrum by using M-ary frequency shift keying (MFSK) modulations. In fast FH systems, the frequency-hop rate $R_h$ is some multiple of the symbol rate. Basically, each (M-ary) symbol interval is subdivided into $N$ subintervals, which are called chips and one of $M$ frequencies is transmitted in each subinterval. The probability of error for noncoherent detection of binary FSK for each hop with $p = 0.01$ is discussed in this paper.

Key words: Spread spectrum, frequency hopping, wireless, CDMA.

1 Introduction

Recently, a new technology has emerged in the commercial marketplace - spread spectrum technology - which provides secure digital communications for business transactions. Important conditions to consider are frequency range, bandwidth, number of users in the bandwidth, distance between users and the network access point, and outside sources of interference.

Spread spectrum (SS) techniques offer one way to accomplish this objective. Current applications of spread spectrum technology include wireless LANs (local area networks), bar code scanners, and microphones. This technology improves the efficiency and effectiveness of business processes, many of which are finding that wireless communications are requisite for success. In SS signal, the transmitted signal bandwidth is much greater than the information bandwidth [1-3]. And some function other than the information being transmitted is employed to determine the resultant transmitted bandwidth. Spread spectrum uses wide band, noise-like signals. Thus, as SS signals are noise-like, they are hard to detect.

Spread spectrum signals are also hard to intercept or demodulate. Further, SS signals are harder to jam (interfere with) than narrowband signals. The use of special pseudo noise codes in SS communications makes signals appear wide band and noise-like. It is this very characteristic that makes SS signals possess the quality of Low Probability of Intercept. SS signals are hard to detect on narrow band equipment because the signal's energy is spread over a bandwidth of maybe 100 times the information bandwidth.

2 Frequency hopping spread spectrum (FHSS)

Frequency hopping spread spectrum (FHSS) inherently makes a radio signal more secure. FHSS is preferable when there are many users in a bandwidth because there is very little noise generated. This low signal to noise ratio also causes less degradation of the signal over distance, causing FHSS to be the better method when wireless devices are farther apart. FHSS is the only technique that can effectively account for outside sources of interference. It is reported that a weather station continuously broadcasts at 50 MHz within the spectrum 30-88 MHz will be deleted from the hopping
pattern which further assuring the station not to interfere with the transmission [2–4]. It also makes efficient use of the radio spectrum by allowing many more users per bandwidth than if each user were individually assigned a frequency. The use of FHSS has been shown to increase data rates and have positive effects on the power required for signal broadcasting. When FHSS is not used, the security vulnerabilities to a wireless network are greatly increased. Some recent trends in the wireless LAN industry illustrate the need for frequency hopping. The model of FHSS is shown in Fig. 1.

![Model of FHSS](image)

**Fig.1 Model of FHSS**

### 2.2. Applications of FH Spread Spectrum

FH spread spectrum is a viable alternative to DS spread spectrum for protection against jamming and for CDMA. In CDMA systems based on frequency hopping, each transmitter/receiver pair is assigned its own pseudorandom FH pattern. Aside from this distinguishing feature, the transmitters and receivers of all users may be identical, i.e. they have identical encoders, decoders, modulators and demodulators. CDMA systems based on FH spread spectrum signals are particularly attractive for mobile (land, air, sea) users because timing (synchronization) requirements are not a stringent as in DS spread spectrum systems. In addition, frequency synthesis techniques and associated hardware have been developed that make it possible to frequency hop over bandwidths that are significantly larger, by one or more orders of magnitude, than those currently possible with DS spread spectrum signals. Consequently, larger processing gains are possible by FH, which more than offset the loss in performance inherent in noncoherent detection of the FSK-type signals.

FH is also effective against jamming signals. FSK system that employs coding, or simply repeats the information symbol on multiple hops (repetition coding) is very effective against a partial band jammer. As a consequence, the jammer’s threat is reduced to that of an equivalent broadband noise jammer whose transmitter power is spread across the channel bandwidth $W$.

### 2.3 Fast Frequency Hopping

In fast FH systems, the frequency-hop rate $R_h$ is some multiple of the symbol rate. Basically, each ($M$-ary) symbol interval is subdivided into $N$ subintervals, which are called chips and one of $M$ frequencies is transmitted in each subinterval. The hope rate $R_h$ is selected sufficiently high so that a potential jammer does not have sufficient time to detect the presence of the transmitted frequency and to synthesize a jamming signal that occupies the same bandwidth.

To recover the information at the receiver, the received signal is first dehopped by mixing it with the hopped carrier frequency. This operation removes the hopping pattern and brings the received signal in all subintervals (chips) to a common frequency band that encompasses the $M$ possible transmitted frequencies, which are sampled at the end of each subinterval and passed to the detector. The detection of the FSK signals is noncoherent. Hence, decisions are based on the magnitude of the matched filter (or correlator) outputs. Since each symbol is transmitted over $N$ chips, the decoding may be performed either on the basis of hard decisions or soft decisions [5].

### 3 Mathematical Equations

Suppose that binary FSK is used to transmit binary symbols, and each symbol is transmitted over $N$ frequency hops, where $N$ is odd. The probability of error for noncoherent detection of binary FSK for each hop is:

$$ p = \frac{1}{2} e^{-p_b/2N} \quad (1) $$

where

$$ p_b = \frac{E_b}{N} \quad (2) $$

is the SNR/chip and $E_b$ is the total bit energy. The decoder decides in favor of the transmitted frequency that is large in at least $(N+1)/2$ chips. Thus, the decision is made on the basis of a majority vote given decisions on the $N$ chips. Consequently, the probability of a bit error is

$$ P_2 = \sum_{m=(N+1)/2}^{N} \binom{N}{m} p^m (1-p)^{N-m} \quad (3) $$
where \( p \) is given by (1). It may be noted that the error probability \( P_2 \) for hard decisions decoding of \( N \) chips will be higher than the error probability for a single hop/bit FSK system, which is given by (4).

\[
P = \frac{1}{2} e^{-pb/2}
\]

(4)

where the SNR/bit \( pb \) is the same in the two systems. However the opposite is expected to be true in the presence of a jammer.

The alternative to hard decision decoding is soft decision decoding in which the magnitudes (or magnitudes squared) of the corresponding matched filter outputs are summed over the \( N \) chips and a single decision is made based on the frequency giving the largest output. For example, if binary orthogonal FSK is used to transmit the information, the two soft decision metrics for the \( N \) chips based on square-law combing is

\[
DM_1 = \sum_{k=1}^{N} \left| \frac{E_{b}}{N} + v_{1k} \right|^2
\]

\[
DM_2 = \sum_{k=1}^{N} \left| v_{2k} \right|^2
\]

(5)

where \( \{v_{1k}\} \) and \( \{v_{2k}\} \) are the noise components from the two matched filters for the \( N \) chips. Since frequency \( f_i \) is assumed to have transmitted, a decision error occurs when \( DM_1 > DM_2 \). The probability of this event error for additive Gaussian noise may be obtained in closed form, although its derivation is cumbersome.

The final results is

\[
P_2 = \frac{1}{2^{2N-1}} e^{-pb/2} \sum_{i=0}^{N-1} K_i \left( \frac{pb}{2} \right)
\]

(6)

where the set \( \{K_i\} \) are constants that may be expressed as:

\[
K_i = \frac{1}{i!} \sum_{r=0}^{N-1-i} \left( \begin{array}{c} 2N-1 \end{array} \right)
\]

(7)

The error probability for soft-decision decoding given by (6) is lower than that for hard-decision decoding given by (3) for the same \( E_b/N_o \). The difference in performance is the loss in hard-decision decoding. However (6) is higher than the error probability for single-hop FSK, which is given by (4) for the AWGN channel. The difference in performance between (4) and (6) for the same SNR is due to the noncoherent combining loss of the system.

If soft decision decoding is used in the presence of partial-band jamming or interference, it is important to scale (or normalize) the matched filter outputs in each hop, so that a strong jammer or interference that falls within the transmitted signal band in any hop does not dominate the output of the combiner. A good strategy in such a case is to normalize or clip the matched filter outputs from each hop if their values exceed some threshold that is set near (slightly above) the mean of the signal-plus-noise power level.

4 M-ary FSK

In an M-ary FSK scheme, the transmitted signals are defined by:

\[
s_i(t) = \sqrt{\frac{2E}{T}} \cos \left( \frac{\pi}{T} (nc+i)t \right); 0 \leq t \leq T
\]

(8)

where \( i = 1,2,3,...,M \), and the carrier frequency \( f_c = \frac{n_c}{2T} \) for some fixed integer \( n_c \). Transmitted signals are of equal duration \( T \) and have equal energy \( E \). Since the individual signal frequencies are separated by \( 1/2T \) Hertz, the signals in Eq. (8) are orthogonal; that is

\[
\int_0^T s_i(t)s_j(t)dt = 0, \quad i \neq j
\]

(9)

For coherent \( M \)-ary FSK, the optimum receiver consists of a bank of \( M \) correlators or matched filters, with the signals in Eq. (8) providing the pertinent references. At the sampling times \( t = kT \), the receiver makes decisions based on the largest matched filter output. An upper bound for the probability of symbol error maybe obtained by applying:

\[
P_e = P_e(m_i) \leq \frac{1}{2} \sum_{k=1}^{M} \text{erfc} \left( \frac{d_k}{2\sqrt{N_o}} \right)
\]

(10)

based on the union bound. The resulting bound is given by:

\[
P_e \leq \frac{1}{2}(M-1) \text{erfc} \left( \frac{E}{\sqrt{2N_o}} \right)
\]

(11)
For a fixed $M$, this bound becomes increasingly tight as $E/N_o$ is increased. Indeed, it becomes a good approximation to $P_e$ for values of $P_e \leq 10^{-3}$. Moreover, for $M=2$ (i.e., binary FSK), the bound of Eq. (11) becomes an equality.

Coherent detection of $M$-ary FSK requires the use of exact phase references, but providing these at the receiver can be costly and difficult to maintain. We may avoid the need for such a provision by using noncoherent detection, which results in a slightly inferior performance. In a noncoherent receiver, the individual matched filters are followed by envelope detectors that destroy the phase information. The probability of symbol error for the noncoherent detection of $M$-ary FSK is given by

$$P_e = \sum_{k=1}^{M-1} \frac{(-1)^{k+1}}{k+1} \binom{M-1}{k} \exp\left(-\frac{kE}{(k+1)N_o}\right)$$ (12)

where $\binom{M-1}{k}$ is a binomial coefficient, denoted by

$$\binom{M-1}{k} = \frac{(M-1)!}{(M-1-k)!k!}$$ (13)

The leading term of the series in Eq. (12) provides an upper bound on the probability of symbol error for the noncoherent detection of $M$-ary FSK.

$$P_e \leq \frac{1}{2}(M-1)\exp\left(\frac{E}{2N_o}\right)$$ (14)

For fixed $M$, this bound becomes increasingly close to the actual value of $P_e$, as $E/N_o$ is increased. Indeed, for $M=2$ (i.e., binary FSK), the bound of Eq. (14) becomes an equality.

5 Results and Discussion

The probability of error for noncoherent detection of binary FSK for each hop is shown in Fig. 2 and 3 respectively with $p = 0.01$. It is observed from the simulation results that the probability of error is greatly affected by increasing the number of chips keeping the value of $p = 0.01$ constant.

Alternatively, we may monitor the noise power level and scale the matched filter outputs for each hop by the reciprocal of the noise power level. Thus, the noise power levels from the matched filter outputs are normalized. Therefore, with proper scaling, a fast FH spread spectrum system will not be as vulnerable to partial-band jamming or interference because the transmitted information per bit is distributed (or spread) over $N$ frequency hops.

6 Conclusion

Coherent detection of $M$-ary FSK requires the use of exact phase references, but providing these at the receiver can be costly and difficult to maintain. We may avoid the need for such a provision by using noncoherent detection, which results in a slightly inferior performance. In a noncoherent receiver, the individual matched filters are followed by envelope detectors that destroy the phase information. FH is effective against jamming signals. FSK system that employs coding, or simply repeats the information symbol on multiple hops (repetition coding) is very effective against a partial band jammer. As a consequence, the jammer’s threat is reduced to that of an equivalent broadband noise jammer whose transmitter power is spread across the channel bandwidth $W$. 
References